Effective Field Theories for Hot and Dense Matter

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Abstract. The lecture is divided in two parts. The first one deals with an introduction to the physics of hot, dense many-particle systems in quantum field theory [1,2]. The basics of the path integral approach to the partition function are explained for the example of chiral quark models. The QCD phase diagram is discussed in the meanfield approximation while QCD bound states in the medium are treated in the rainbow-ladder approximation (Gaussian fluctuations). Special emphasis is devoted to the discussion of the Mott effect, i.e. the transition of bound states to unbound, but resonant scattering states in the continuum under the influence of compression and heating of the system. Three examples are given: (1) the QCD model phase diagram with chiral symmetry restoration and color superconductivity [3], (2) the Schrödinger equation for heavy-quarkonia [4], and (2) Pions [5] as well as Kaons and D-mesons in the finite-temperature Bethe-Salpeter equation [6]. We discuss recent applications of this quantum field theoretical approach to hot and dense quark matter for a description of anomalous J/ψ suppression in heavy-ion collisions [7] and for the structure and cooling of compact stars with quark matter interiors [8].


References

**EFFECTIVE FIELD THEORIES FOR HOT AND DENSE MATTER (I)**

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- Introduction: Many-particle Systems and Quantum Field Theory
- Partition function for QCD: Lattice Simulations vs. Resonance Gas
- Bound states and Mott effect, Color superconductivity
  - Heavy Quarkonia - Schrödinger Equation
  - Chiral quark model - Color superconductivity
  - Pions, Kaons, D-mesons - Chiral Quark Model
- Application 1: $J/\psi$ suppression in Heavy-Ion Collisions
- Application 2: Quark Matter in Compact Stars
- Summary / Outlook to Lecture II

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**MANY PARTICLE SYSTEMS & QUANTUM FIELD THEORY**

<table>
<thead>
<tr>
<th>Elements</th>
<th>Bound states</th>
<th>System</th>
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</thead>
<tbody>
<tr>
<td>humans, animals</td>
<td>couples, groups, parties</td>
<td>society</td>
</tr>
<tr>
<td>molecules, crystals</td>
<td>(bio)polymers</td>
<td>animals, plants</td>
</tr>
<tr>
<td>atoms</td>
<td>molecules, clusters, crystals</td>
<td>solids, liquids, ...</td>
</tr>
<tr>
<td>ions, electrons</td>
<td>atoms</td>
<td>plasmas</td>
</tr>
<tr>
<td>nucleons, mesons</td>
<td>nuclei</td>
<td>nuclear matter</td>
</tr>
<tr>
<td>quarks, anti-quarks</td>
<td>nucleons, mesons</td>
<td>quark matter</td>
</tr>
</tbody>
</table>

Highly Compressed Matter $\iff$ Pauli Principle

*Partition function:* 

$Z = \text{Tr} \left\{ e^{-\beta(H - \mu_i Q_i)} \right\}$
**Partition function for Quantum Chromodynamics (QCD)**

- Partition function as a Path Integral (imaginary time $\tau = it$, $0 \leq \tau \leq \beta = 1/T$) ⇒
  
  \[ Z[T,V,\mu] = \int D\bar{\psi}D\psi DA \exp \left\{ -\int_0^\beta d\tau \int_V d^3x \mathcal{L}_{QCD}(\psi,\bar{\psi},A) \right\} \]

- QCD Lagrangian, non-Abelian gluon field strength:
  
  \[ F_{\mu\nu}^a(A) = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc}A^b_\mu A^c_\nu \]

- L$_{\text{QCD}}(\psi,\bar{\psi},A) = \bar{\psi}(i\gamma^\mu(\partial_\mu - igA_\mu) - m - \gamma^0\mu)\psi - \frac{1}{4}F_{\mu\nu}^a(A)F^{a,\mu\nu}(A) \]

- Numerical evaluation: Lattice gauge theory simulations (Bielefeld group)

- Equation of state: $\varepsilon(T) = -\partial\ln Z[T,V,\mu] / \partial\beta$
- Phase transition at $T_c = 170$ MeV
- Problem: Interpretation?

\[ \varepsilon/T^4 = \frac{\pi^2}{30}N_c \sim 1 \text{ (ideal pion gas)} \]
\[ \varepsilon/T^4 = \frac{\pi^2}{30}(N_G + \frac{7}{8}N_Q) \sim 15.6 \text{ (quarks and gluons)} \]

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**Phasediagram of QCD: Lattice Simulations**

- **Early universe**
- **RHIC LHC**
- **Neutron stars**
- **Critical point?**
- **Deconfinement and chiral transition**
- **Baryon stars**
- **Color Superconductor?**
- **Quarks and Gluons**
- **Hadrons**
- **Net Baryon Density**

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01003-p.3
PHASE DIAGRAM OF QCD: LATTICE SIMULATIONS

LATTICE QCD EoS VS. RESONANCE GAS

Ideal hadron gas mixture ...

\[ \varepsilon(T) = \sum_{i=\pi,\rho} g_i \int \frac{d^3p}{(2\pi)^3} \exp\left(\sqrt{p^2 + m_i^2}/T\right) + \delta_i \]

missing degrees of freedom below and above \( T_c \)

Resonance gas ...


\[ \varepsilon(T) = \sum_{i=\pi,\rho} \varepsilon_i(T) + \sum_{r=M,B} \frac{g_r}{\tau} \int dm \rho(m) \int \frac{d^3p}{(2\pi)^3} \exp\left(\sqrt{p^2 + m^2}/T\right) + \delta_r \]

\[ \rho(m) \sim m^3 \exp(m/T_H) \] ... Hagedorn mass spectrum

too many degrees of freedom above \( T_c \)
**LATTICE QCD EOS AND MOTT-HAGEDORN GAS**

\[
\varepsilon_R(T, \{\mu_j\}) = \sum_{i=\pi,K,...} \varepsilon_i(T, \{\mu_i\}) + \sum_{r=M,B} g_r \int dm \int d^3p \left( \frac{2\pi^3}{(2\pi)^3} \frac{\sqrt{p^2 + s}}{\exp\left(\frac{\sqrt{p^2 + s} - \mu_r}{T}\right)} \right) + \delta_r
\]

Hagedorn mass spectrum: \(\rho(m)\)

**Spectral function** for heavy resonances:

\[
A(s, m; T) = N_s \frac{m \Gamma(T)}{(s - m^2)^2 + m^2 \Gamma^2(T)}
\]

Ansatz with Mott effect at \(T = T_H = 180\) MeV:

\[
\Gamma(T) = B \Theta(T - T_H) \left( \frac{m}{T_H} \right)^{2.5} \left( \frac{T}{T_H} \right)^6 \exp\left(\frac{m}{T_H}\right)
\]

No width below \(T_H\): Hagedorn resonance gas

Apparent phase transition at \(T_c \sim 150\) MeV

Blaschke & Bugaev, Fizika B13, 491 (2004)


Blaschke & Yudichev, in preparation

Bugaev, Petrov, Zinovjev, arXiv:0812.2189

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**HADRONIC CORRELATIONS ABOVE \(T_c\): LATTICE QCD**

Hadron correlators \(G_H \Rightarrow \) spectral densities \(\rho_H(\omega, T)\)

\[
G_H(\tau, T) = \int_0^\infty d\omega \rho_H(\omega, T) \frac{\cosh(\omega(\tau - T/2))}{\sinh(\omega/2T)}
\]

Maximum entropy method

Karsch et al. PLB 530 (2002) 147

Result:

**Correlations persist above \(T_c\)**!


\(J/\psi\) and \(\eta_c\) survive up to \(T \sim 1.6T_c\)

Asakawa, Hatsuda; PRL 92 (2004) 012001

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Hadron correlations in the phase diagram of QCD
HEAVY QUARK POTENTIAL FROM LATTICE QCD

Color-singlet free energy $F_1$ in quenched QCD

$$\langle \text{Tr}[L(0)L'(r)] \rangle = \exp[-F_1(r)/T]$$

Long- and short-range parts

$$F_1(r, T) = F_{1,\text{long}}(r, T) + V_{1,\text{short}}(r) e^{-\mu(T)r^2}$$

$$F_{1,\text{long}}(r, T) = \text{'screened' confinement pot.}$$

$$V_{1,\text{short}}(r) = \frac{4\alpha(r)}{3} r, \quad \alpha(r) = \text{running coupling. (1)}$$

<table>
<thead>
<tr>
<th>Quarkonium $(q \bar{q})$</th>
<th>1S</th>
<th>1P</th>
<th>2S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charmonium $(c \bar{c})$</td>
<td>$J/\psi (3097)$</td>
<td>$\chi_{c1} (3510)$</td>
<td>$\psi (3686)$</td>
</tr>
<tr>
<td>Bottomonium $(b \bar{b})$</td>
<td>$\Upsilon (9460)$</td>
<td>$\chi_{b1} (9892)$</td>
<td>$\Upsilon' (10023)$</td>
</tr>
</tbody>
</table>

$\Rightarrow$ Wong (DM 2, 5); Lombardo (DM 3, 7)

SCHROEDINGER EQN: BOUND & SCATTERING STATES

Quarkonia bound states at finite $T$:

$$[-\nabla^2/m_Q + V_{\text{eff}}(r, T)]\psi(r, T) = E_B(T)\psi(r, T)$$

Binding energy vanishes $E_B(T_{\text{Mott}}) = 0$: Mott effect

Scattering states:

$$\frac{d\delta_S(k, r, T)}{dr} = -\frac{m_QV_g}{k} \sin(kr + \delta_S(k, r, T))$$

Levinson theorem:

Phase shift at threshold jumps by $\pi$ when bound state $\rightarrow$ resonance at $T = T_{\text{Mott}}$

Blaschke, Kaczmarek, Laermann, Yudichev
EPJC 43, 81 (2005); [hep-ph/0505053]
**Chiral Model Field Theory for Quark Matter**

- Partition function as a Path Integral (imaginary time $\tau = i t$)

$$Z[T, V, \mu] = \int D\bar{\psi} D\psi \exp\left\{ -\int d^3 x [\bar{\psi}(i\gamma^\mu \partial_\mu - m - \gamma^0 \mu)\psi - L_{\text{int}}]\right\}$$

- Current-current interaction (4-Fermion coupling)

$$L_{\text{int}} = \sum_{M=\pi, \sigma, \ldots} G_M (\psi \Gamma_M \psi)^2 + \sum_D G_D (\bar{\psi} C \Gamma_D \psi)^2$$

- Bosonization (Hubbard-Stratonovich Transformation)

$$Z[T, V, \mu] = \int D M_M D \Delta_D^\dagger D \Delta_D \exp\left\{ -\sum_M \frac{M_M^2}{4G_M} - \sum_D \frac{\Delta^2_D}{4G_D} + \frac{1}{2} \text{Tr ln} S^{-1}[\{M_M\}, \{\Delta_D\}]\right\}$$

- Collective (stochastic) fields: Mesons ($M_M$) and Diquarks ($\Delta_D$)

- Systematic evaluation: **Mean fields + Fluctuations**
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)
  - Higher order fluctuations: hadron-hadron interactions
**NJL MODEL FOR NEUTRAL 3-FLAVOR QUARK MATTER**

Thermodynamic Potential $\Omega(T, \mu) = -T \ln Z[T, \mu]$

$$\Omega(T, \mu) = \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2 + |\Delta_{us}|^2 + |\Delta_{ds}|^2}{4G_D} - T \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \text{Tr} \ln \left( \frac{1}{T} S^{-1}(i\omega_n, \vec{p}) \right) + \Omega_e - \Omega_0.$$ 

Inverse Nambu–Gorkov Propagator

$$S^{-1}(i\omega_n, \vec{p}) = \begin{bmatrix} 
\gamma_\mu p^\mu - M(\vec{p}) + \mu \gamma^0 & \widehat{\Delta}(\vec{p}) \\
\widehat{\Delta}(\vec{p}) & \gamma_\mu p^\mu - M(\vec{p}) - \mu \gamma^0
\end{bmatrix},$$

Fermion Determinant (Tr ln D = ln det D): $\ln \text{det}[\beta S^{-1}(i\omega_n, \vec{p})] = 2 \sum_{a=1}^{18} \ln \{\beta^2 [\omega_n^2 + \lambda_a(\vec{p})^2]\}$

Result for the thermodynamic Potential (Mean-field approximation)

$$\Omega(T, \mu) = \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2 + |\Delta_{us}|^2 + |\Delta_{ds}|^2}{4G_D} - \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \sum_{a=1}^{18} \left[ \lambda_a + 2T \ln \left( 1 + e^{-\lambda_a/T} \right) \right] + \Omega_e - \Omega_0.$$ 

Color and electric charge neutrality constraints: $n_Q = n_s = n_3 = 0$, $n_i = -\partial \Omega / \partial \mu_i = 0$.

Equations of state: $P = -\Omega$, etc.

**ORDER PARAMETERS: Masses and Diquark Gaps**

Masses ($M$) and Diquark gaps ($\Delta$) as a function of the chemical potential at $T = 0$

Left: Gap in excitation spectrum ($T = 0$)  
Right: 'Gapless' excitations ($T = 60$ MeV)
**MOTT EFFECT: NJL MODEL PRIMER**

RPA-type resummation of quark-antiquark scattering in the mesonic channel $M$,

\[
D_M(p_0, P; T) \sim |1 - J_M(p_0, P; T)|^{-1},
\]

defines Meson propagator

by the complex polarization function $J_M$

→ Breit-Wigner type spectral function

\[
A_M(p_0, P; T) = \frac{1}{\pi} \text{Im} \ D_M(p_0, P; T)
\]

\[
\sim \frac{1}{\pi} \frac{\Gamma_M(T)M_M(T)}{(s - M_M^2(T))^2 + \Gamma_M^2(T)}
\]

For $T < T_{\text{Mott}}$, $\Gamma \to 0$, i.e. bound state

\[
A_M(p_0, P; T) = \delta(s - M_M^2(T))
\]

Light meson sector:

Charm meson sector:
Blaschke, Burau, Kalinovsky, Yudichev,

**PHASEDIAGRAM OF QCD: HEAVY-ION COLLISIONS**

Lattice QCD Simulations

Early universe

PHC LHC

Critical point?

Deconfinement and chiral transition

Neutron stars

Color Superconductor?

Net Baryon Density
The Picture: String-flip (Rearrangement) ↔ Pair correlation

- Strong correlations present: hadronic spectral functions above $T_c$ (lattice QCD)
- Finite width due to rearrangement collisions (higher order correlations)
- Liquid-like pair correlation function (nearest neighbor peak)

Statistical model describes composition of hadron yields in Heavy-Ion Collisions with few freeze-out parameters.

$$\ln Z(T, V; \{\mu\}) = \pm V \sum_i \frac{g_i}{2\pi} \int_0^{\infty} dp \frac{p^2}{2\pi} \ln[1 \pm \lambda_i \exp(-\beta \varepsilon_i(p))]$$

$$\lambda_i(T, \{\mu\}) = \exp[\beta (\mu_B B_i + \mu_S S_i + \mu_Q Q_i)]$$

QUANTUM KINETIC APPROACH TO J/ψ BREAKUP

Inverse lifetime for Charmonium states

$$ \tau^{-1}(p) = \Gamma(p) = \Sigma^<(p) + \Sigma^>(p) $$

$$ \Sigma^>(p, \omega) = \int_{p_1} d p_2 (2\pi)^4 \delta(p-p_1) |M|^2 G^>(p') G^<(p_1) G^<(p_2) $$

$$ G^>(p) = [1 \pm f_+(p)] \Lambda_+(p) \text{ and } G^<(p) = f_+(p) \Lambda_+(p) $$

$$ \tau^{-1}(p) = \int \frac{d^3 p'}{(2\pi)^3} \int ds' f_{A}(s', s') A_{u}(s') \nu a s \sigma(s') $$

In-medium breakup cross section

$$ \sigma(s) = \int ds_1 ds_2 A_{D_1}(s_1) A_{D_2}(s_2) \sigma(s; s_1, s_2) $$

Medium effects in spectral functions $$ A_0 $$

$$ A_0(s) = \frac{1}{\pi} \frac{\Gamma_0(T) M_0(T)}{\left(s - M_0^2(T)\right)^2 + \frac{1}{2} \left(M_0^2(T)\right)^2} \delta\left(s - M_0^2\right) $$

Blaschke et al., Heavy Ion Phys. 18 (2003) 49

“ANOMALOUS” J/ψ SUPPRESSION IN MOTT-HAGEDORN GAS

Survival probability for J/ψ

$$ S(E_T)/S_N(E_T) = \exp\left[-\int_0^T dt \, \tau^{-1}(n(t))\right] $$

Threshold: Mott effect for hadrons

Blaschke and Bugaev, Prog. Part. Nucl. Phys. 53 (2004) 197

In progress: full kinetics with gain processes (D-fusion), HIC simulation

In progress: full kinetics with gain processes (D-fusion), HIC simulation
PHASE DIAGRAM OF DEGENERATE QUARK MATTER

Temperature T [MeV]

Early universe
RHIC/LHC

Critical point
Deconfinement and chiral transition

Chiral Quark Model
Field Theory

Quarks and Gluons

Hadrons

Neutron stars
Color Superconductor?

Net Baryon Density
The phases are characterized by 3 gaps:

- **NQ**: $\Delta_{ud} = \Delta_{us} = \Delta_{ds} = 0$;
- **NQ-2SC**: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$, $0 \leq \chi_{2SC} \leq 1$;
- **2SC**: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$;
- **uSC**: $\Delta_{ud} \neq 0$, $\Delta_{us} \neq 0$, $\Delta_{ds} = 0$;
- **CFL**: $\Delta_{ud} \neq 0$, $\Delta_{us} \neq 0$, $\Delta_{ds} \neq 0$;

Result:

- Gapless phases only at high $T$,
- CFL only at high chemical potential,
- At $T \leq 25$-$30$ MeV: mixed NQ-2SC phase,
- Critical point $(T_c, \mu_c) = (48$ MeV, $353$ MeV),
- Strong coupling, $\eta = 1$, changes?.

$\Rightarrow$ Zhuang (DM 12, 17)
Quark matter in compact stars: color superconducting

- Neutrinos carry energy off the star, Cooling evolution (schematic) by

\[
\frac{dT(t)}{dt} = -\epsilon_\gamma + \sum_{j=U\rightarrow e,\gamma} \epsilon_j^V
\]

- Most efficient process: Urca

\[
\bar{\nu} \rightarrow e^- \\
\nu \rightarrow \bar{e}
\]

- Exponential suppression by pairing gaps! \(\Delta \sim 10\ldots 100\text{ keV}\)

**Summary**

- Mott-Hagedorn model as alternative interpretation of Lattice data
- Microscopic formulation of the hadronic Mott effect within a chiral quark model
- Mesonic (hadronic) correlations important for \(T > T_c\)
- Step-like enhancement of threshold processes due to Mott effect
- Reaction kinetics for strong correlations in plasmas applicable @ SPS and RHIC
- Prospects for LHC: Plasma diagnostics with bottomonium

**Lecture II: NJL Model and Its Relatives**

- Polyakov-loop Nambu–Jona-Lasinio (NJL) model
- Nonlocal NJL models
- Schwinger-Dyson Equation approach at finite \(T, \mu\)
- Walecka model - towards a unified model of quark-hadron matter
EFFECTIVE FIELD THEORIES FOR HOT AND DENSE MATTER (II)
NJL MODEL AND ITS RELATIVES

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- NJL Model and Its Polyakov-Loop Extension:
  - Mesonic correlations - Mott Effect
  - Polyakov-Loop NJL Model

- Nonlocal, separable NJL Model
  - 3D Formfactors, 4D Formfactors and Phase Diagram
  - Rank-2 Extension - Schwinger-Dyson type Approach

- Summary / Outlook to a Unified Quark-Hadron Approach


HADRONIC CORRELATIONS IN THE PHASEDIAGRAM OF QCD
**CHIRAL MODEL FIELD THEORY FOR QUARK MATTER**

- Partition function as a Path Integral (imaginary time $\tau = i t$)
  \[
  Z[T, V, \mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp \left\{ - \int_{0}^{T} \int d^{3}x \left[ i\gamma^{\mu} \partial_{\mu} \psi - m - \gamma^{0}(\mu + \lambda_{8}\mu_{8} + i\lambda_{3}\phi_{3})\psi - L_{\text{int}} + U(\Phi) \right] \right\}
  \]
  Polyakov loop: $\Phi = N_{c}^{-1} \text{Tr} [\exp (i\beta\lambda_{3}\phi_{3})]$  \quad Order parameter for deconfinement

- Current-current interaction (4-Fermion coupling)
  \[ L_{\text{int}} = \sum_{M=\pi,\sigma,...} GM(\bar{\psi}\Gamma_{M}\psi)^{2} + \sum_{D} GD(\bar{\psi}C\Gamma_{D}\psi)^{2} \]

- Bosonization (Hubbard-Stratonovich Transformation)
  \[
  Z[T, V, \mu] = \int \mathcal{D}M_{M} \mathcal{D}\Delta_{D} \mathcal{D}\Delta_{D} \exp \left\{ - \sum_{M,D} \frac{M_{M}^{2}}{4M_{M}} - \frac{\left| \Delta_{D} \right|^{2}}{4G_{D}} + \frac{1}{2} \text{Tr} \ln S^{-1}\{ M_{M}, \{ \Delta_{D} \}, \Phi \} + U(\Phi) \right\}
  \]

- Collective quark fields: Mesons ($M_{M}$) and Diquarks ($\Delta_{D}$); Gluon mean field: $\Phi$

- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)
  - Higher order fluctuations: hadron-hadron interactions
$SU(N_c)$ pure gauge sector: Polyakov line

\[ L(\vec{x}) \equiv \mathcal{P} \exp \left[ i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right] \quad ; \quad A_4 = iA^0 = \lambda_3 \phi_3 + \lambda_8 \phi_8 \]

Polyakov loop

\[ l(\vec{x}) = \frac{1}{N_c} \text{Tr} L(\vec{x}) \quad \langle l(\vec{x}) \rangle = e^{-\beta \Delta F_Q(\vec{x})}. \]

$Z_{N_c}$ symmetric phase: $\langle l(\vec{x}) \rangle = 0 \quad \Rightarrow \quad \Delta F_Q \rightarrow \infty$ : Confinement!

Polyakov loop field:

\[ \Phi(\vec{x}) \equiv \langle l(\vec{x}) \rangle = \frac{1}{N_c} \text{Tr} \langle L(\vec{x}) \rangle \]

Potential for the PL-meanfield $\Phi(\vec{x}) = \text{const.}$, which fits quenched QCD lattice thermodynamics

\[ \frac{U(\Phi, \bar{\Phi}; T)}{T^4} = -\frac{b_2(T)}{2} \bar{\Phi} \Phi - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\bar{\Phi} \Phi)^2, \]

\[ b_2(T) = a_0 + a_1 \left( \frac{T_0}{T} \right) + a_2 \left( \frac{T_0}{T} \right)^2 + a_3 \left( \frac{T_0}{T} \right)^3. \]

\[
\begin{array}{ccccccc}
  a_0 & a_1 & a_2 & a_3 & b_2 & b_3 & b_4 \\
  6.75 & -1.95 & 2.625 & -7.44 & 0.75 & 7.5 & \\
\end{array}
\]

$T = 0.26 \text{ GeV} < T_0$  
“Color confinement”  

$T = 1.0 \text{ GeV} > T_0$  
“Color deconfinement”  

Critical temperature for pure gauge $SU_c(3)$ lattice simulations: $T_0 = 270 \text{ MeV}$.

Lagrangian for \( N_f = 2, N_c = 3 \) quark matter, coupled to the gauge sector

\[
\mathcal{L}_{PQCD} = \bar{q}(i\gamma^\mu D_\mu - m + i\gamma_5 q) + G_1 \left[ (\bar{q}q)^2 + (q^2)^2 \right] - U (\bar{\Phi}|A|, \Phi|A|, T),
\]

\( D^\mu = \partial^\mu - iA^\mu \); \( A^\mu = \delta^\mu_0 A_0 \) (Polyakov gauge), with \( A_0 = -iA_1 \)

Diagrammatic Hartree equation:

\[
S_0(p) = \frac{1}{\Lambda^2} = -(\hat{p} - m_0 + \gamma^0(\mu - iA_1))^{-1}; \quad S(p) = \frac{1}{\Lambda^2} = -(\hat{p} - m + \gamma^0(\mu - iA_1))^{-1}
\]

Dynamical chiral symmetry breaking \( \sigma = m - m_0 \neq 0 \)? Solve Gap Equation! \((E = \sqrt{p^2 + m^2})\)

\[
m - m_0 = 2G_1 T \sum_{n=-\infty}^{+\infty} \int \frac{d^3 p}{(2\pi)^3} \frac{-1}{\hat{p} - m + \gamma^0(\mu - iA_1)}
\]

\[
= 2G_1 N_f N_c \int \frac{d^3 p}{(2\pi)^3} \frac{2m}{E} \left[ 1 - f^+_{\Phi}(E) - f^-_{\bar{\Phi}}(E) \right]
\]

Modified quark distribution functions \((\Phi = \bar{\Phi} = 0; "\text{poor man's nucleon}"; E_N = 3E, \mu_N = 3\mu)\)

\[
f^+_{\Phi}(E) = \frac{\left( \Phi + 2\Phi e^{-\beta(E_\Phi - p)} \right) e^{-\beta(E_\Phi - p)} + e^{-3\beta(E_\Phi - p)}}{1 + 3 \left( \Phi + \Phi e^{-\beta(E_\Phi - p)} \right) e^{-\beta(E_\Phi - p)} + e^{-3\beta(E_\Phi - p)}} \rightarrow f^+_{\bar{\Phi}}(E) = \frac{1}{1 + e^{\beta(E_\bar{\Phi} + \mu_N)}/T}
\]
**POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (V)**

Mesonic currents

\[ J^P_p(x) = \bar{q}(x)i\gamma^5\tau^a q(x) \] (pion)

\[ J^S_p(x) = \bar{q}(x)q(x) - \langle \bar{q}(x)q(x) \rangle \] (sigma)

... and correlation functions

\[ C^{PP}_{ab}(q^2) \equiv \int d^4xe^{iqx} \langle 0 | T \left( J^P_p(x)J^P_P(0) \right) | 0 \rangle = C^{PP}(q^2)\delta_{ab} \]

\[ C^{SS}_{ab}(q^2) \equiv \int d^4xe^{iqx} \langle 0 | T \left( J^S_p(x)J^S_S(0) \right) | 0 \rangle \]

Schwinger-Dyson Equations, \( T = \mu = 0 \)

\[ C^{MM}(q^2) = \Pi^{MM}(q^2) + \sum_{M'} \Pi^{MM'}(q^2)(2G_1)C^{M'M}(q^2) \]

One-loop polarization functions

\[ \Pi^{MM'}(q^2) \equiv \int_{\Lambda} \frac{d^4p}{(2\pi)^4} \text{Tr} (\Gamma_{M'}S(p+q)\Gamma_M S(q)) \]

Hartree quark propagator \( S(p) \)

**POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (VI)**

Example of the pion channel:

\[ \Pi^{PP}(q^2) = -4iN_fN_f \int_{\Lambda} \frac{d^4p}{(2\pi)^4} \frac{m^2-p^2+q^2/4}{[(p+q/2)^2-m^2][(p-q/2)^2-m^2]} = 4iN_fN_f I_1 - 2iN_fN_f q^2 I_2(q^2) \]

Loop Integrals:

\[ I_1 = \int_{\Lambda} \frac{d^4p}{(2\pi)^4} \frac{1}{p^2-m^2} \]

\[ I_2(q^2) = \int_{\Lambda} \frac{d^4p}{(2\pi)^4} \frac{1}{[(p+q)^2-m^2][(p-q)^2-m^2]} \]

With pseudoscalar decay constant \( f_p \) and gap equation for \( I_1 \)

\[ f_p^2(q^2) = -4iN_f m^2 I_2(q^2) \]

\[ I_1 = \frac{m - m_0}{8G_1 m N_f} \]

One obtains \( \Pi^{PP}(q^2) = \frac{m - m_0}{2G_1 m^2} + f_p^2(q^2) \frac{q^2}{m^2} \) ; \( \Pi^{SS}(q^2) = \frac{m - m_0}{2G_1 m^2} + f_p^2(q^2) \frac{q^2 - 4m^2}{m^2} \). In the chiral limit \( m_0 = 0 \), the correlation functions

\[ C^{MM}(q^2) = \Pi^{MM}(q^2) + \Pi^{MM}(q^2)(2G_1)C^{MM}(q^2) = \frac{\Pi^{MM}(q^2)}{1 - 2G_1\Pi^{MM}(q^2)} \]

have poles at \( q^2 = M_P^2 = 0 \) (Pion) and \( q^2 = M_S^2 = (2m)^2 \) (Sigma meson) \( \Rightarrow \) Check!
POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (VII)

Finite $T, \mu$: $p = (p_0, \vec{p}) \rightarrow (i\omega_n + \mu - iA_4, \vec{p})$; $i \int_{\Lambda} \frac{d^4p}{(2\pi)^4} \rightarrow -T \frac{1}{N_c} \text{Tr}_c \sum_n \int_{\Lambda} \frac{d^4p}{(2\pi)^4}$

$$I_1 = -i \int \frac{d^3p}{(2\pi)^3} \frac{1 - f(E_p - \mu) - f(E_p + \mu)}{2E_p}$$

$$I_2(\omega, \vec{q}) = i \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p 2E_{p+q}} \left[ f(E_p + \mu) + f(E_p - \mu) - f(E_{p+q} + \mu) - f(E_{p+q} - \mu) \right] \frac{\omega - E_{p+q} + E_p}{\omega - E_{p+q} - E_p}$$

For a meson at rest in the medium ($\vec{q} = 0$)

$$I_2(\omega, 0) = -i \int \frac{d^3p}{(2\pi)^3} \frac{1 - f(E_p + \mu) - f(E_p - \mu)}{E_p (\omega^2 - 4E_p^2)}$$

which develops an imaginary part

$$\Im(-iI_2(\omega, 0)) = \frac{1}{16\pi} \left( 1 - f(\frac{\omega}{2} - \mu) - f(\frac{\omega}{2} + \mu) \right) \sqrt{\frac{\omega^2 - 4m^2}{\omega^2}} \times \Theta(\omega^2 - 4m^2) \Theta(4\Lambda^2 + m^2 - \omega^2)$$

with the Pauli-blocking factor: $N(\omega, \mu) = (1 - f(\frac{\omega}{2} - \mu) - f(\frac{\omega}{2} + \mu))$

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (VIII)

Spectral function

$$F^{MM}(\omega, \vec{q}) \equiv \Im C^{MM}(\omega + i\eta, \vec{q}) = \Im \frac{\Pi^{MM}(\omega + i\eta, \vec{q})}{1 - 2G_1 \Pi^{MM}(\omega + i\eta, \vec{q})}.$$ 

$$F^{MM}(\omega) = \frac{\pi}{2G_1} \frac{1}{\pi} \frac{2G_1 \Im \Pi^{MM}(\omega + i\eta)}{(1 - 2G_1 \Re \Pi^{MM}(\omega))^2 + (2G_1 \Im \Pi^{MM}(\omega + i\eta))^2}.$$ 

(2)

For $\omega < 2m(T, \mu)$, $\Im \Pi = 0$: decay channel closed $\rightarrow$ bound state!

$$F^{MM}(\omega) = \frac{\pi}{2G_1} \delta \left( 1 - 2G_1 \Re \Pi^{MM}(\omega) \right) = \frac{\pi}{4G_1^2} \left| \frac{2G_1 \Pi^{MM}}{\partial \omega} \right|_{\omega = m_M} \delta(\omega - m_M).$$

The meson mass $m_M$ is the solution of

$$1 - 2G_1 \Re \Pi^{MM}(m_M) = 0$$

The decay width (inverse lifetime) is

$$\Gamma_M = 2G_1 \Im \Pi^{MM}(m_M)$$

01003-p.21
**Pion Correlations in the Phase Diagram**

![Graph showing phase diagram with various lines and shading]


**Color Neutrality in the PNJL Phase Diagram**

Color neutrality constraint: \( \tilde{\mu} = \mu_1 + \mu_8 \lambda_8 + i \phi_3 \lambda_3 \); \( \partial \Omega_{MF} / \partial \mu_8 = n_8 = n_r + n_g - 2n_b = 0 \)

Gap equations: \( \partial \Omega_{MF} / (\partial \sigma, \partial \Delta, \partial \phi_3) = 0 \)

![Graph showing phase diagram with various lines and shading]


01003-p.22
NONLOCAL POLYAKOV LOOP CHIRAL QUARK MODEL

2-flavor, rank-1, 4D separable order parameters:


3-flavor, rank-2, 4D separable susceptibilities:

\[
\begin{array}{c|c|c|c}
T\,[\text{GeV}] & 0.18 & 0.19 & 0.20 \\
\hline
\frac{d}{T} & 0.19 & 0.20 & \\
\end{array}
\]

D.B., Horvatic, Klabucar, in prep.

COMPLEX MASS POLE FIT TO LATTICE PROPAGATOR

S(p) sum of N pairs of complex conj. mass poles

\[
S(p) = \sum_{i=1}^{N} \frac{1}{Z_2} \left\{ \frac{z_i}{i \hat{p} + m_i} + \frac{z_i^*}{i \hat{p} + m_i^*} \right\} = -i \hat{p} \sigma_V(p^2) + \sigma_s(p^2)
\]

"Derivation" of the equivalent separable model (in Feynman-like gauge) \( D_{\mu\nu}(p-q) = \delta_{\mu\nu} D(p,q) \) and

\[
\begin{align*}
D(p,q) &= f_0(p^2) f_0(q^2) + f_1(p^2) p \cdot q f_1(q^2) \\
f_1(p^2) &= \frac{A(p^2) - 1}{a} ; \quad f_0(p^2) = \frac{B(p^2) - m_e}{b} \\
\end{align*}
\]

\[
\begin{align*}
b^2 &= \frac{16}{3} \int_{q}^{\Lambda} [B(q^2) - m_e] \sigma_s(q^2) \\
a^2 &= \frac{8}{3} \int_{q}^{\Lambda} [A(q^2) - 1] \frac{q^2}{4} \sigma_s(q^2)
\end{align*}
\]


\[
\begin{align*}
S(p)^{-1} &= i \hat{p} A(p^2) + B(p^2) \\
M(p^2) &= B(p^2)/A(p^2) \\
Z(p^2) &= 1/A(p^2)
\end{align*}
\]
Nucleons in the Nonlocal Chiral Quark Model

\[ Z_{\text{fluct}} = \int D\Delta^\dagger D\Delta \exp \left\{ -\frac{|\Delta|^2}{4G_D} - Tr \ln S - \frac{1}{2} [\Delta, \Delta^\dagger] \right\} \]

Cahill, ibid, 171; Reinhardt: PLB 244 (1990) 316; Buck, Alkofer, Reinhardt: PLB 286 (1992) 29

Summary

- Compressed nuclear matter: quarkyonic phase (QP)! Coexisting chiral symm. + conf.
- Similarities: Mott-Hagedorn picture, string-flip model, confining DSE
- Here: PNJL model as microscopic formulation of the QP
- Color singlet quark triplets in chiral phase for \( \mu > \mu_c \) (approx. massless baryons)
- Color neutrality by singlet projection = sum over color hexagon
- Prospects for CBM & NICA: dilepton enhancement (peak?) from diquark-antidiquark annih.
- Preparatory step to compact stars: single flavor CSL phase - OK with structure & cooling

Outlook: Next Steps ...

- Walecka model as limit of PNJL model: chiral transition effects in nuclear EoS
- Beyond meanfield: mesons and baryons in the PNJL, higher clusters: sextetting
- Astrophysics: Maximum mass & cooling of quarkyonic stars; quarkyonic supernovae