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Effective Field Theories for Hot and Dense Matter

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Abstract. The lecture is divided in two parts. The first one deals with an introduction to the physics of hot, dense many-particle systems in quantum field theory [1,2]. The basics of the path integral approach to the partition function are explained for the example of chiral quark models. The QCD phase diagram is discussed in the meanfield approximation while QCD bound states in the medium are treated in the rainbow-ladder approximation (Gaussian fluctuations). Special emphasis is devoted to the discussion of the Mott effect, i.e. the transition of bound states to unbound, but resonant scattering states in the continnum under the influence of compression and heating of the system. Three examples are given: (1) the QCD model phase diagram with chiral symmetry restoration and color superconductivity [3], (2) the Schrödinger equation for heavy-quarkonia [4], and (2) Pions [5] as well as Kaons and D-mesons in the finite-temperature Bethe-Salpeter equation [6]. We discuss recent applications of this quantum field theoretical approach to hot and dense quark matter for a description of anomalous J/ψ supression in heavy-ion collisions [7] and for the structure and cooling of compact stars with quark matter interiors [8].

The second part provides a detailed introduction to the Polyakov-loop Nambu–Jona-Lasinio model [9] for thermodynamics and mesonic correlations [10] in the phase diagram of quark matter. Important relationships of low-energy QCD like the Gell-Mann–Oakes–Renner relation are generalized to finite temperatures. The effect of including the coupling to the Polyakov-loop potential on the phase diagram and mesonic correlations is discussed. An outlook is given to effects of nonlocality of the interactions [11] and of mesonic correlations in the medium [12] which go beyond the meanfield description.

References

1. J.I. Kapusta and C. Gale, *Finite Temperature Field Theory*, Cambridge University Press (2006).
2. K. Yagi, T. Hatsuda and Y. Miake, *Quark-Gluon Plasma*, Cambridge University Press (2005).
3. D. Blaschke, S. Fredriksson, H. Grigorian, A. M. Oztas and F. Sandin, Phys. Rev. D **72**, 065020 (2005).
4. D. Blaschke, O. Kaczmarek, E. Laermann and V. Yudichev, Eur. Phys. J. C **43**, 81 (2005).
5. D. Zablocki, D. Blaschke and R. Anglani, AIP Conf. Proc. **1038**, 159 (2008).
6. D. Blaschke, G. Burau, Yu. L. Kalinovsky and V. L. Yudichev, Prog. Theor. Phys. Suppl. **149**, 182 (2003).
7. R. Rapp, D. Blaschke and P. Crochet, "Charmonium and bottomonium production in heavy-ion collisions," arXiv:0807.2470.
8. D. Blaschke, T. Klaehn and F. Weber, in: *Astronomy and Relativistic Astrophysics*, World Scientific, Singaprore (2010), pp. 31-47; [arXiv:0808.1279 [astro-ph]].
9. C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D **73**, 014019 (2006).
10. H. Hansen, W. M. Alberico, A. Beraudo, A. Molinari, M. Nardi and C. Ratti, Phys. Rev. D **75**, 065004 (2007).
11. D. Gomez Dumm, D. B. Blaschke, A. G. Grunfeld and N. N. Scoccola, Phys. Rev. D **78**, 114021 (2008).
12. D. Blaschke, M. Buballa, A. E. Radzhabov and M. K. Volkov, Yad. Fiz. **71**, 2012 (2008).

EFFECTIVE FIELD THEORIES FOR HOT AND DENSE MATTER (I)

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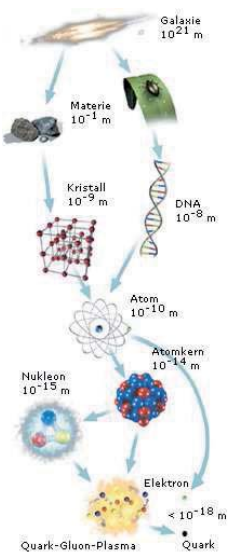
- Introduction: Many-particle Systems and Quantum Field Theory
- Partition function for QCD: Lattice Simulations vs. Resonance Gas
- Bound states and Mott effect, Color superconductivity
  - Heavy Quarkonia - Schrödinger Equation
  - Chiral quark model - Color superconductivity
  - Pions, Kaons, D-mesons - Chiral Quark Model
- Application 1: J/ψ suppression in Heavy-Ion Collisions
- Application 2: Quark Matter in Compact Stars
- Summary / Outlook to Lecture II

MANY PARTICLE SYSTEMS & QUANTUM FIELD THEORY



Elements	Bound states	System
humans, animals	couples, groups, parties	society
molecules, crystals	(bio)polymers	animals, plants
atoms	molecules, clusters, crystals	solids, liquids, ...
ions, electrons	atoms	plasmas
nucleons, mesons	nuclei	nuclear matter
quarks, anti-quarks	nucleons, mesons	quark matter

Highly Compressed Matter $\Leftrightarrow$ Pauli Principle

Partition function: $Z = \mathrm{Tr}\left\{\mathrm{e}^{-\beta(H-\mu_i Q_i)}\right\}$

PARTITION FUNCTION FOR QUANTUM CHROMODYNAMICS (QCD)

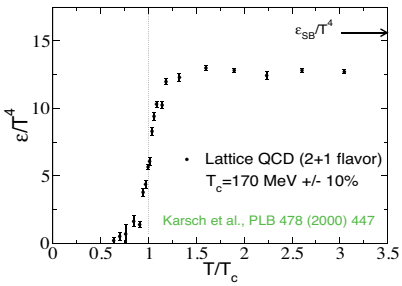
- Partition function as a Path Integral (imaginary time $\tau = i\,t$, $0 \leq \tau \leq \beta = 1/T$) $\Rightarrow$ PS I

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A \exp\left\{-\int_0^\beta d\tau \int_V d^3x\, \mathcal{L}_{QCD}(\psi,\bar{\psi},A)\right\}$$

- QCD Lagrangian, non-Abelian gluon field strength: $F^a_{\mu\nu}(A) = \partial_\mu A^a \nu - \partial_\nu A^a_\mu + g\, f^{abc}[A^b_\mu, A^c_\nu]$

$$\mathcal{L}_{QCD}(\psi,\bar{\psi},A) = \bar{\psi}[i\gamma^\mu(\partial_\mu - igA_\mu) - m - \gamma^0\mu]\psi - \frac{1}{4}F^a_{\mu\nu}(A)F^{a,\mu\nu}(A)$$

- Numerical evaluation: Lattice gauge theory simulations (Bielefeld group)

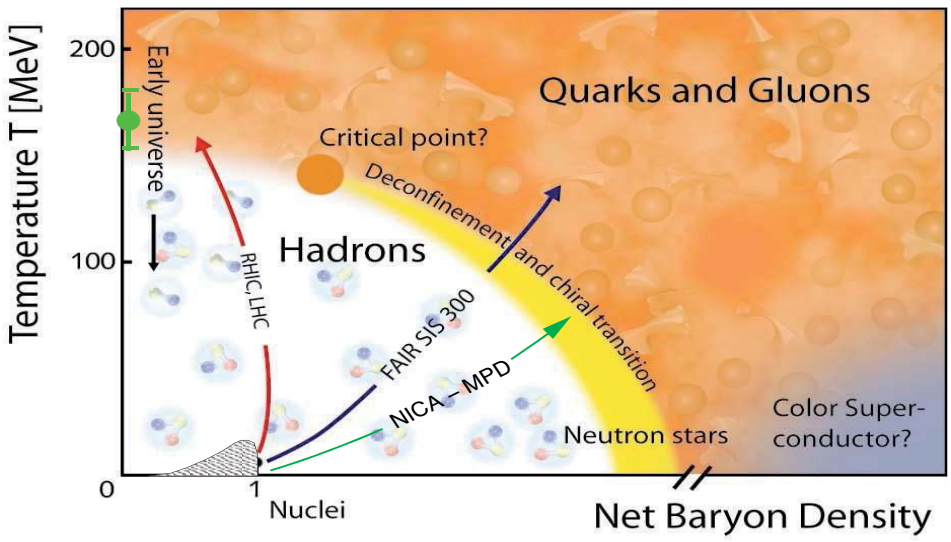


- Equation of state: $\varepsilon(T) = -\partial \ln Z[T,V,\mu]/\partial\beta$
- Phase transition at $T_c = 170$ MeV
- Problem: Interpretation ?

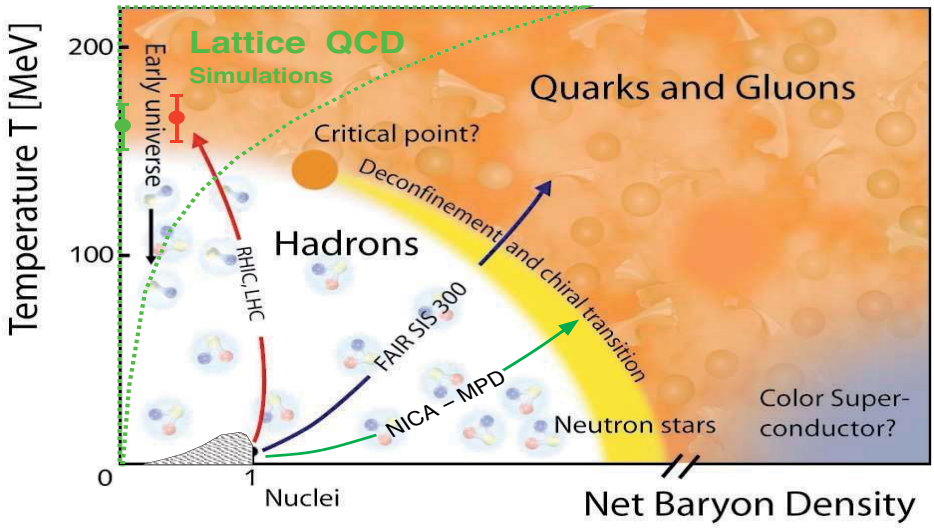
$\varepsilon/T^4 = \frac{\pi^2}{30}N_\pi \sim 1$ (ideal pion gas)

$\varepsilon/T^4 = \frac{\pi^2}{30}(N_G + \frac{7}{8}N_Q) \sim 15.6$ (quarks and gluons)

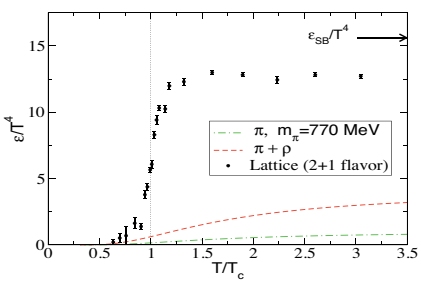
PHASEDIAGRAM OF QCD: LATTICE SIMULATIONS



PHASEDIAGRAM OF QCD: LATTICE SIMULATIONS



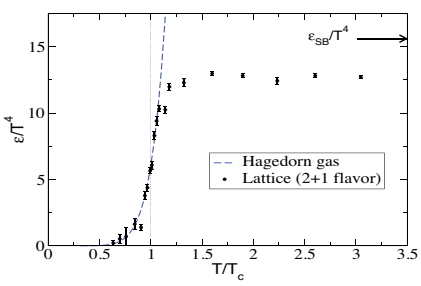
LATTICE QCD EOS VS. RESONANCE GAS



Ideal hadron gas mixture ...

$$\varepsilon(T) = \sum_{i=\pi,\rho,\ldots} g_i \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2+m_i^2}}{\exp(\sqrt{p^2+m_i^2}/T)+\delta_i}$$

missing degrees of freedom below and above T_c



Resonance gas ...
Karsch, Redlich, Tawfik, Eur.Phys.J. C29, 549 (2003)

$$\varepsilon(T) = \sum_{i=\pi,\rho,\ldots} \varepsilon_i(T) + \sum_{r=M,B} g_r \int dm\, \rho(m) \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2+m^2}}{\exp(\sqrt{p^2+m^2}/T)+\delta_r}$$

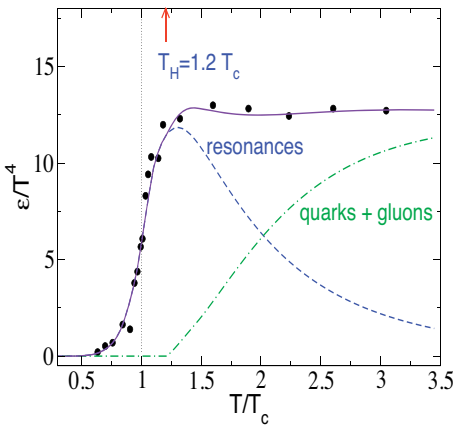
$\rho(m) \sim m^\beta \exp(m/T_H)$ ... Hagedorn mass spektrum

too many degrees of freedom above T_c

Lattice QCD EoS and Mott-Hagedorn Gas

$$\varepsilon_{\rm R}(T,\{\mu_j\}) = \sum_{i=\pi,K,\ldots} \varepsilon_i(T,\{\mu_i\}) + \sum_{r=M,B} g_r \int_{m_r} dm \int ds\, \rho(m) A(s,m;T) \int \frac{d^3p}{(2\pi)^3} \frac{\sqrt{p^2+s}}{\exp\left(\frac{\sqrt{p^2+s}-\mu_r}{T}\right)+\delta_r}$$

Hagedorn mass spectrum: $\rho(m)$



Spectral function for heavy resonances:

$$A(s,m;T) = N_s \frac{m\Gamma(T)}{(s-m^2)^2 + m^2\Gamma^2(T)}$$

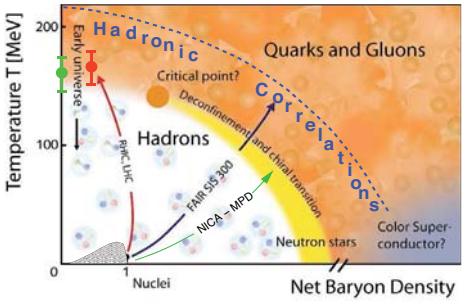
Ansatz with Mott effect at $T = T_H = 180$ MeV:

$$\Gamma(T) = B\Theta(T-T_H)\left(\frac{m}{T_H}\right)^{2.5}\left(\frac{T}{T_H}\right)^{6}\exp\left(\frac{m}{T_H}\right)$$

No width below T_H: Hagedorn resonance gas
Apparent phase transition at $T_c \sim 150$ MeV

Blaschke & Bugaev, Fizika B13, 491 (2004)
Prog. Part. Nucl. Phys. 53, 197 (2004)
Blaschke & Yudichev, in preparation
Bugaev, Petrov, Zinovjev, arXiv:0812.2189

Hadronic Correlations above T_c: Lattice QCD

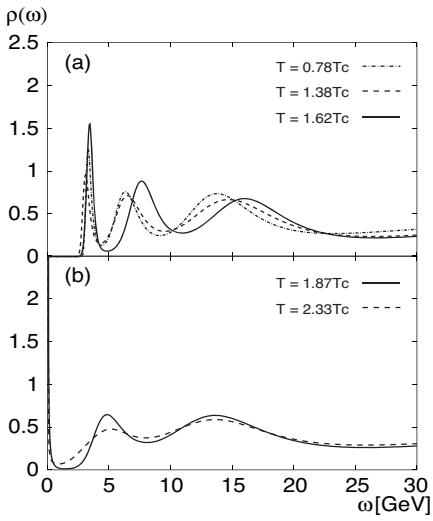


Hadron correlators $G_H \Longrightarrow$ spectral densities $\rho_H(\omega,T)$

$$G_H(\tau,T) = \int_0^\infty d\omega \rho_H(\omega,T) \frac{\cosh(\omega(\tau - T/2))}{\sinh(\omega/2T)}$$

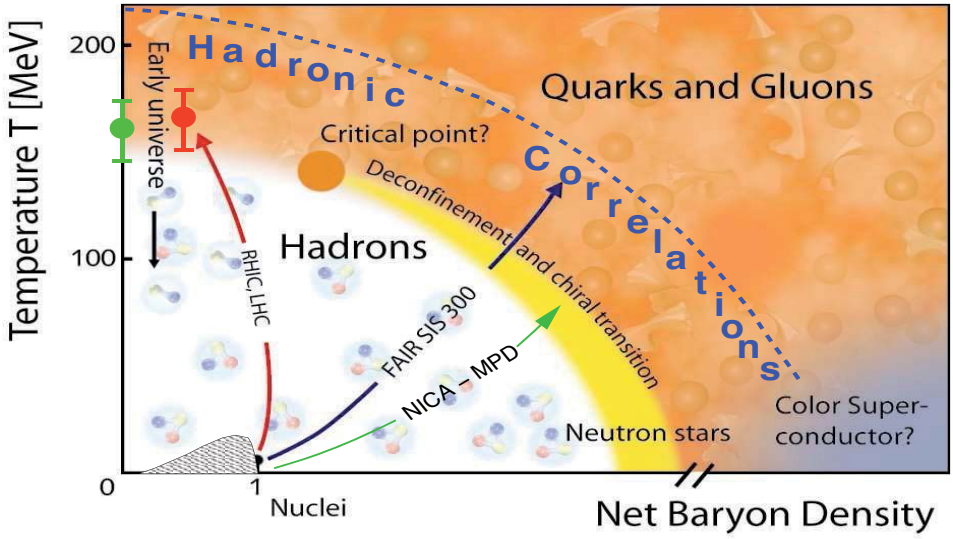
Maximum entropy method
Karsch et al. PLB 530 (2002) 147

Result:
Correlations persist above T_c !
Karsch et al. NPA 715 (2003)

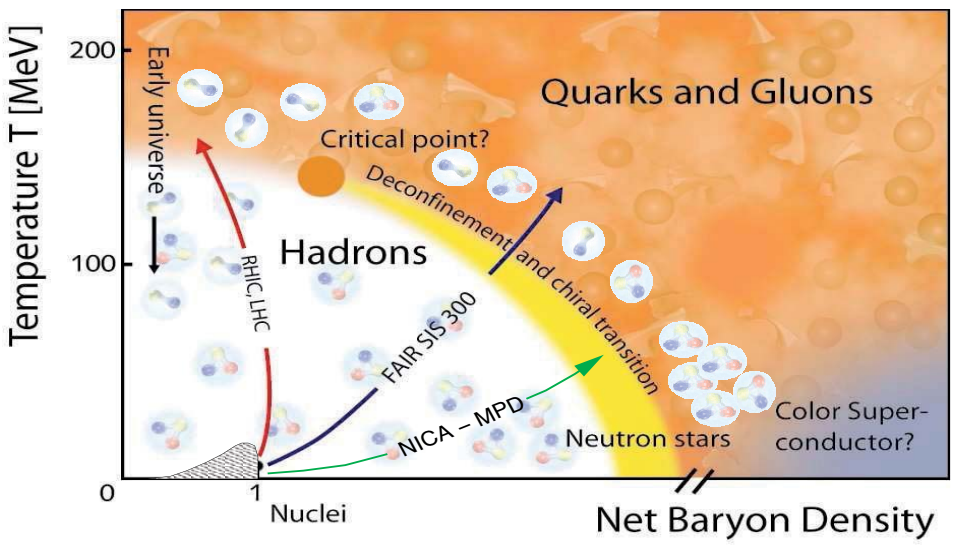


J/ψ and η_c survive up to $T \sim 1.6T_c$
Asakawa, Hatsuda; PRL 92 (2004) 012001

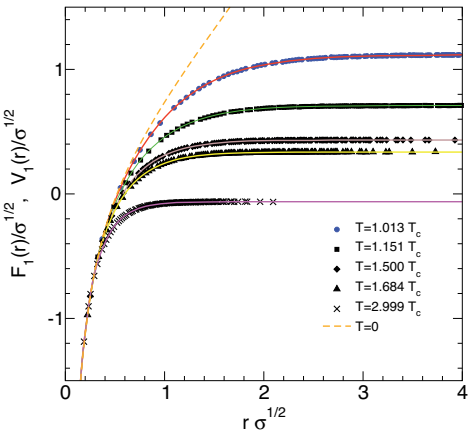
HADRONIC CORRELATIONS IN THE PHASEDIAGRAM OF QCD



HADRONIC CORRELATIONS IN THE PHASEDIAGRAM OF QCD



Heavy Quark Potential from Lattice QCD



Blaschke, Kaczmarek, Laermann, Yudichev, EPJC 43, 81 (2005); [hep-ph/0505053]

Color-singlet free energy F_1 in quenched QCD

$$\langle \mathrm{Tr}[L(0)L^\dagger(r)]\rangle = \exp[-F_1(r)/T]$$

Long- and short- range parts

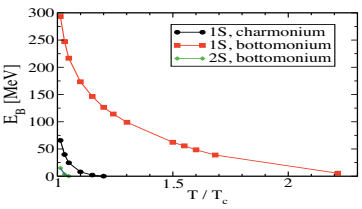
$$F_1(r,T) = F_{1,\mathrm{long}}(r,T) + V_{1,\mathrm{short}}(r)e^{-(\mu(T)r)^2}$$

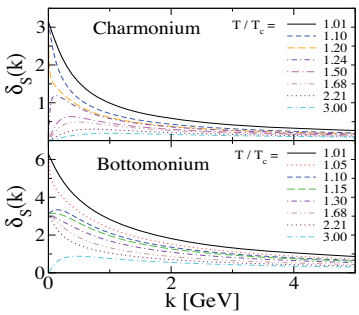
$$\begin{aligned} F_{1,\mathrm{long}}(r,T) &= \text{'screened'confinementpot.} \\ V_{1,\mathrm{short}}(r) &= -\frac{4}{3}\frac{\alpha(r)}{r},\ \alpha(r) = \mathrm{runningcoupl.} \quad (1) \end{aligned}$$

Quarkonium ($Q\bar{Q}$)	1S	$1P_1$	2S
Charmonium ($c\bar{c}$)	J/ψ(3097)	χ_{c1}(3510)	ψ' (3686)
Bottomonium ($b\bar{b}$)	Υ (9460)	χ_{b1} (9892)	Υ' (10023)

$\Longrightarrow$ Wong (DM 2, 5); Lombardo (DM 3, 7)

Schroedinger Eqn: Bound & Scattering States





Quarkonia bound states at finite T:

$$[-\nabla^2/m_Q + V_{\mathrm{eff}}(r,T)]\psi(r,T) = E_B(T)\psi(r,T)$$

Binding energy vanishes $E_B(T_{\mathrm{Mott}}) = 0$: Mott effect

Scattering states:

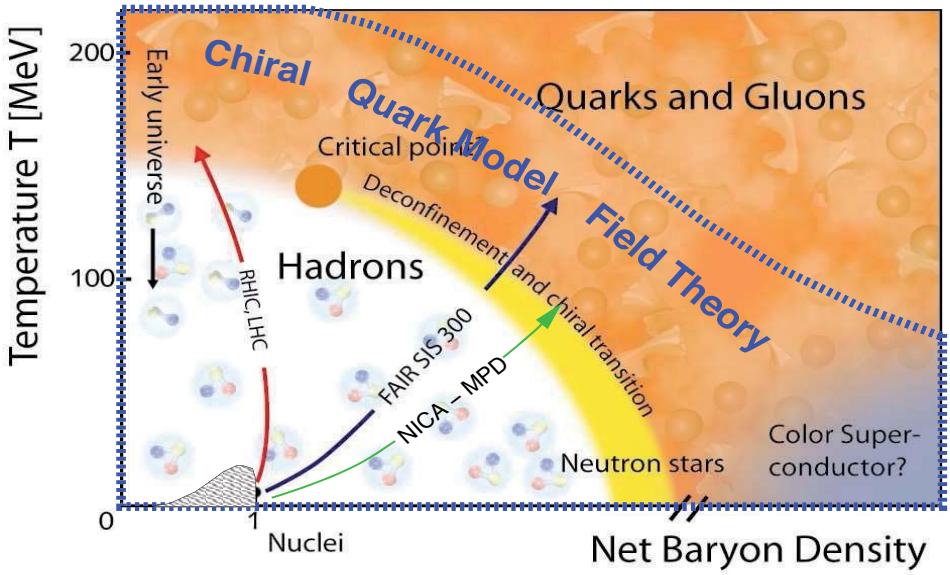
$$\frac{d\delta_S(k,r,T)}{dr} = -\frac{m_Q V_{\mathrm{eff}}}{k}\sin(kr + \delta_S(k,r,T))$$

Levinson theorem:

Phase shift at threshold jumps by π when bound state $\rightarrow$ resonance at $T = T_{\mathrm{Mott}}$

Blaschke, Kaczmarek, Laermann, Yudichev EPJC 43, 81 (2005); [hep-ph/0505053]

PHASEDIAGRAM OF QCD: CHIRAL MODEL FIELD THEORIES



CHIRAL MODEL FIELD THEORY FOR QUARK MATTER

- Partition function as a Path Integral (imaginary time $\tau = i\,t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{-\int^{\beta} d\tau \int_V d^3x[\bar{\psi}(i\gamma^\mu\partial_\mu - m - \gamma^0\mu)\psi - \mathcal{L}_{\rm int}]\right\}$$

- Current-current interaction (4-Fermion coupling)

$$\mathcal{L}_{\rm int} = \sum_{M=\pi,\sigma,\ldots} G_M(\bar{\psi}\Gamma_M\psi)^2 + \sum_D G_D(\bar{\psi}^C\Gamma_D\psi)^2$$

- Bosonization (Hubbard-Stratonovich Transformation)

$$Z[T,V,\mu] = \int \mathcal{D}M_M\mathcal{D}\Delta_D^\dagger\mathcal{D}\Delta_D \exp\left\{-\sum_M \frac{M_M^2}{4G_M} - \sum_D \frac{|\Delta_D|^2}{4G_D} + \frac{1}{2}{\rm Tr}\,\ln S^{-1}[\{M_M\},\{\Delta_D\}]\right\}$$

- Collective (stochastic) fields: Mesons (M_M) and Diquarks (Δ_D)
- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)
  - Higher order fluctuations: hadron-hadron interactions

NJL MODEL FOR NEUTRAL 3-FLAVOR QUARK MATTER

Thermodynamic Potential $\Omega(T,\mu) = -T \ln Z[T,\mu]$

$$\Omega(T,\mu) \;=\; \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D} - T\sum_n \int \frac{d^3p}{(2\pi)^3}\frac{1}{2}\mathrm{Tr}\ln\left(\frac{1}{T}S^{-1}(i\omega_n,\vec{p})\right) + \Omega_e - \Omega_0.$$

$$\mathrm{InverseNambu-GorkovPropagator} \quad S^{-1}(i\omega_n,\vec{p}) = \begin{bmatrix} \gamma_\mu p^\mu - M(\vec{p}) + \mu\gamma^0 & \widehat{\Delta}(\vec{p}) \\ \widehat{\Delta}^\dagger(\vec{p}) & \gamma_\mu p^\mu - M(\vec{p}) - \mu\gamma^0 \end{bmatrix},$$

$$\widehat{\Delta}(\vec{p}) = i\gamma_5\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\Delta_{k\gamma}g(\vec{p}) \;\;;\;\; \Delta_{k\gamma} = 2G_D\langle\bar{q}_{i\alpha}i\gamma_5\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}g(\vec{q})q^C_{j\beta}\rangle.$$

Fermion Determinant (Tr ln D = ln det D): $\mathrm{ln det}[\beta\, S^{-1}(i\omega_n,\vec{p})] = 2\sum_{a=1}^{18}\ln\{\beta^2[\omega_n^2+\lambda_a(\vec{p})^2]\}$.

Result for the thermodynamic Potential (Meanfield approximation)

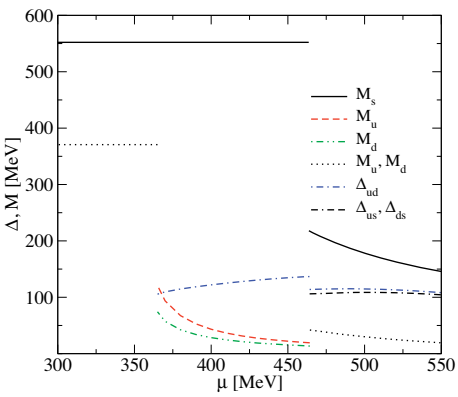
$$\Omega(T,\mu) \;=\; \frac{\phi_u^2+\phi_d^2+\phi_s^2}{8G_S} + \frac{|\Delta_{ud}|^2+|\Delta_{us}|^2+|\Delta_{ds}|^2}{4G_D} - \int \frac{d^3p}{(2\pi)^3}\sum_{a=1}^{18}\left[\lambda_a + 2T\ln\left(1+e^{-\lambda_a/T}\right)\right] + \Omega_e - \Omega_0.$$

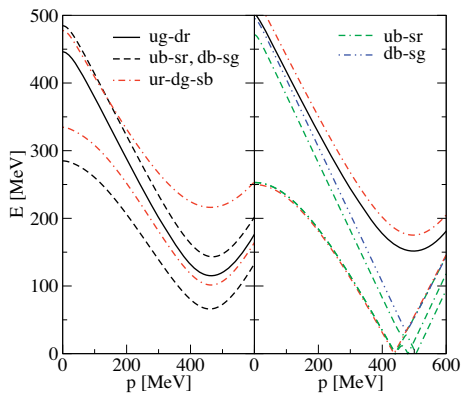
Color and electric charge neutrality constraints: $n_Q = n_8 = n_3 = 0$, $n_i = -\partial\Omega/\partial\mu_i = 0$,
Equations of state: $P = -\Omega$, etc.

ORDER PARAMETERS: MASSES AND DIQUARK GAPS

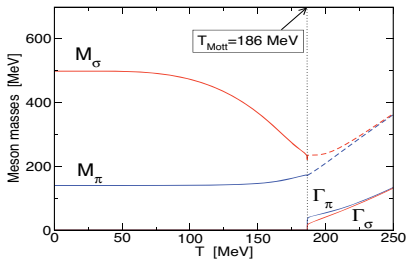
Masses (M) and Diquark gaps (Δ) as a function of the chemical potential at $T = 0$

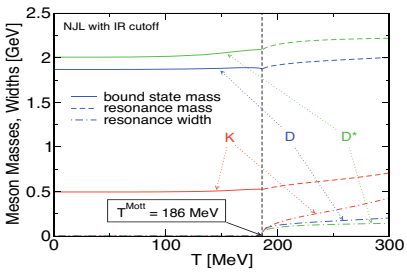
Left: Gap in excitation spectrum ($T = 0$)
Right: 'Gapless' excitations ($T = 60$ MeV)



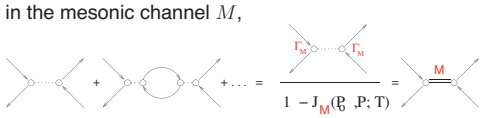


MOTT EFFECT: NJL MODEL PRIMER





RPA-type resummation of quark-antiquark scattering in the mesonic channel M,



defines Meson propagator

$$D_M(P_0, P; T) \sim [1 - J_M(P_0, P; T)]^{-1},$$

by the complex polarization function J_M
→ Breit-Wigner type spectral function

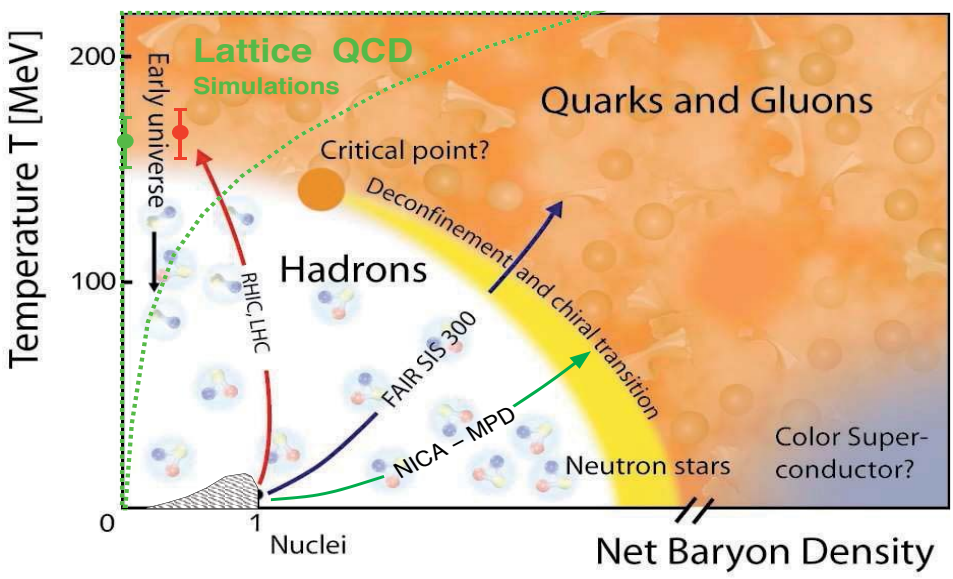
$$\begin{aligned} \mathcal{A}_M(P_0, P; T) &= \frac{1}{\pi}\mathrm{Im}\, D_M(P_0, P; T) \\ &\sim \frac{1}{\pi}\frac{\Gamma_M(T) M_M(T)}{(s - M_M^2(T))^2 + \Gamma_M^2(T) M_M^2(T)} \end{aligned}$$

For $T < T_{\mathrm{Mott}}$: $\Gamma \to 0$, i.e. bound state
$\mathcal{A}_M(P_0, P; T) = \delta(s - M_M^2(T))$

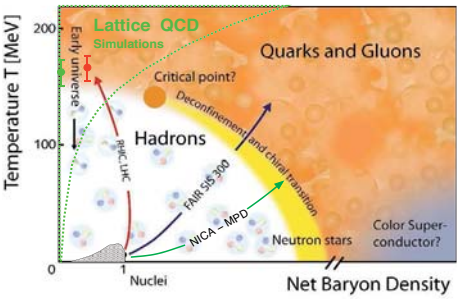
Light meson sector:
Blaschke, Burau, Volkov, Yudichev: EPJA **11** (2001) 319

Charm meson sector:
Blaschke, Burau, Kalinovsky, Yudichev,
Prog. Theor. Phys. Suppl. **149** (2003) 182

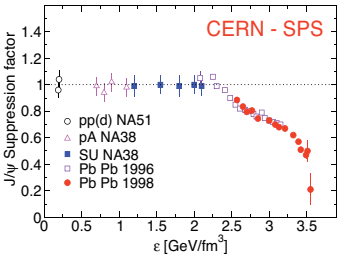
PHASEDIAGRAM OF QCD: HEAVY-ION COLLISIONS

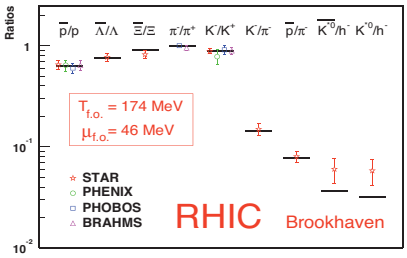


PHASEDIAGRAM OF QCD: LATTICE VS. HEAVY-ION COLLISIONS



QGP Signal: Anomalous J/ψ suppression





Statistical model describes composition of hadron yields in Heavy-Ion Collisions with few freeze-out parameters.

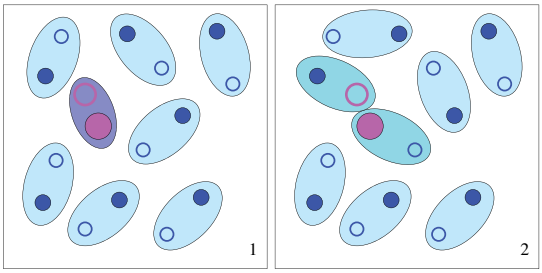
$$\ln Z[T,V,\{\mu\}] = \pm V \sum_i \frac{g_i}{2\pi^2} \int_0^\infty dp\, p^2 \ln[1 \pm \lambda_i \exp(-\beta\varepsilon_i(p))]$$

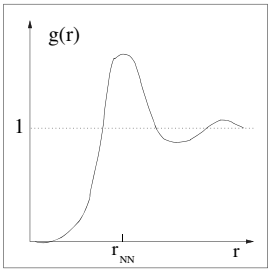
$$\lambda_i(T,\{\mu\}) = \exp[\beta(\mu_B B_i + \mu_S S_i + \mu_Q Q_i)]$$

Braun-Munzinger, Redlich, Stachel, in *QGP III* (2003)

A SNAPSHOP OF THE sQGP

The Picture: String-flip (Rearrangement) $\Longleftrightarrow$ Pair correlation



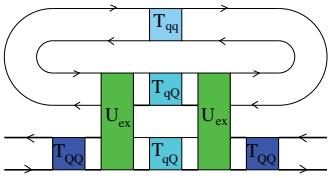


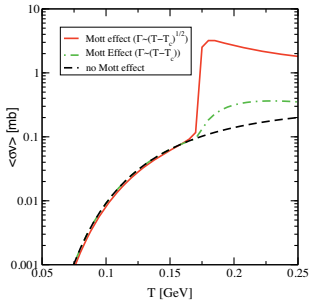
Horowitz et al. PRD (1985), D.B. et al. PLB (1985), Röpke, Blaschke, Schulz, PRD (1986)

Thoma,[hep-ph/0509154] Gelman et al., PRC 74 (2006)

- Strong correlations present: hadronic spectral functions above T_c (lattice QCD)
- Finite width due to rearrangement collisions (higher order correlations)
- Liquid-like pair correlation function (nearest neighbor peak)

QUANTUM KINETIC APPROACH TO J/ψ BREAKUP





Inverse lifetime for Charmonium states

$$\tau^{-1}(p) = \Gamma(p) = \Sigma^{>}(p) \mp \Sigma^{<}(p)$$

$$\Sigma^{\gtrless}(p,\omega) \;=\; \int_{p'}\int_{p_1}\int_{p_2} (2\pi)^4 \delta_{p,p';p_1,p_2}\, |\mathcal{M}|^2\, G^{\lessgtr}_{\pi}(p')\, G^{\gtrless}_{D_1}(p_1)\, G^{\gtrless}_{D_2}(p_2)$$

$G^{>}_h(p) = [1 \pm f_h(p)] A_h(p)$ and $G^{<}_h(p) = f_h(p) A_h(p)$

$$\tau^{-1}(p) = \int \frac{d^3\mathbf{p}'}{(2\pi)^3} \int ds' \quad f_\pi(\mathbf{p}', s')\, A_\pi(s') v_{\rm rel}\, \sigma^*(s)$$

In-medium breakup cross section

$$\sigma^*(s) = \int ds_1\, ds_2\, A_{D_1}(s_1)\, A_{D_2}(s_2)\, \sigma(s; s_1, s_2)$$

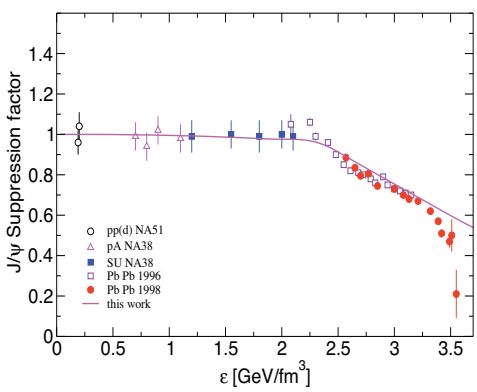
Medium effects in spectral functions A_h and $\sigma(s; s_1, s_2)$

$$A_h(s) \;=\; \frac{1}{\pi} \frac{\Gamma_h(T)\, M_h(T)}{(s - M_h^2(T))^2 + \Gamma_h^2(T) M_h^2(T)} \longrightarrow \delta(s - M_h^2)$$

resonance $\Leftarrow$ Mott-effect $\Leftarrow$ bound state

Blaschke et al., Heavy Ion Phys. 18 (2003) 49

"ANOMALOUS" J/ψ SUPPRESSION IN MOTT-HAGEDORN GAS



Survival probability for J/ψ

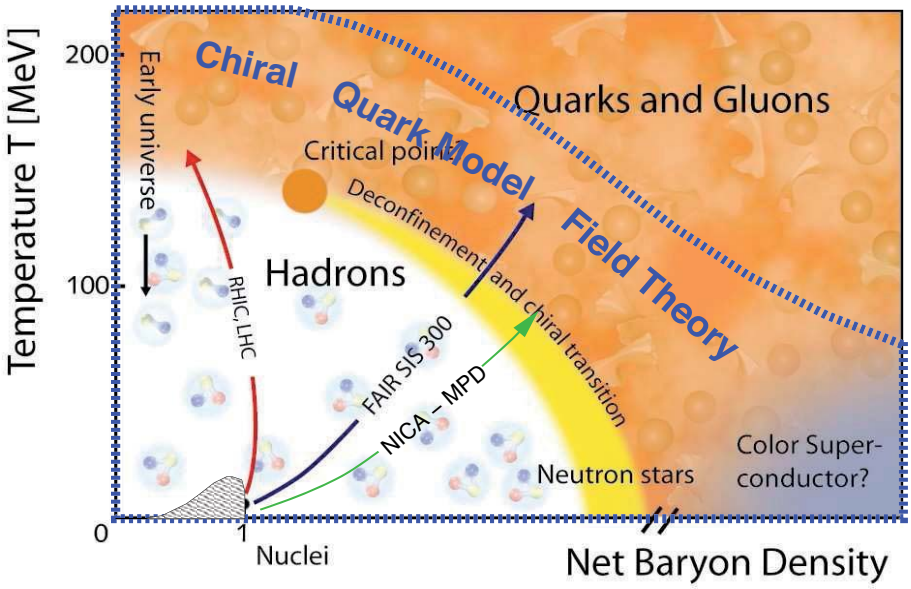
$$S(E_T)/S_N(E_T) \;=\; \exp\left[-\int_{t_0}^{t_f} dt\; \tau^{-1}(n(t))\right]$$

Threshold: Mott effect for hadrons

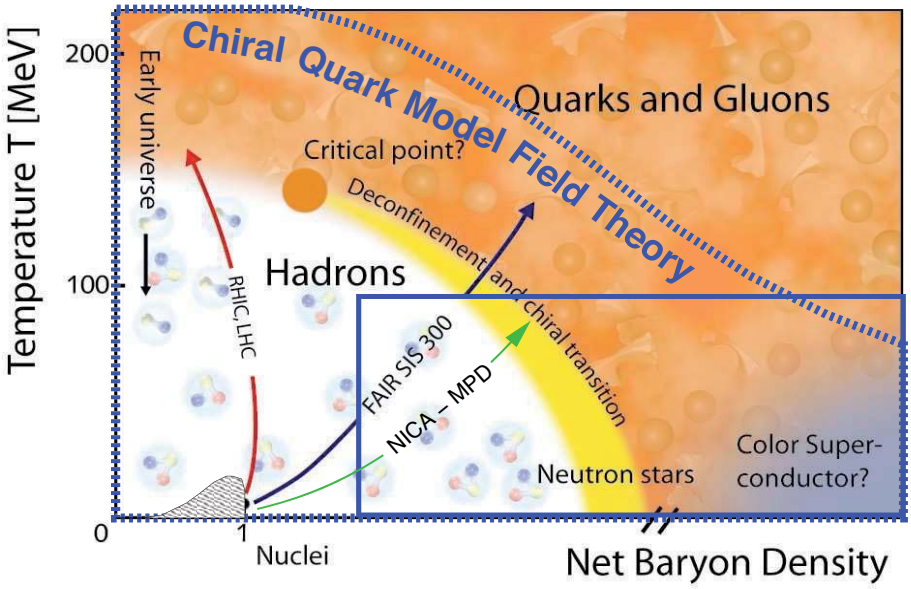
Blaschke and Bugaev, Prog. Part. Nucl. Phys. 53 (2004) 197

In progress: full kinetics with gain processes (D-fusion), HIC simulation

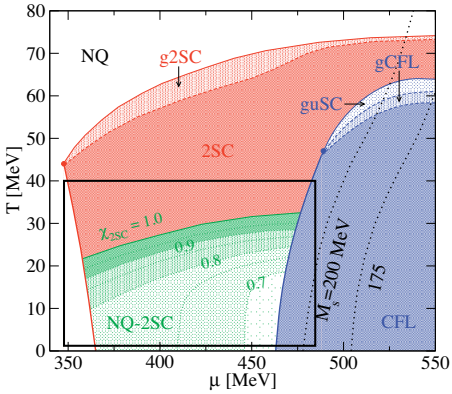
PHASEDIAGRAM OF DEGENERATE QUARK MATTER



PHASEDIAGRAM OF DEGENERATE QUARK MATTER



Quark matter in compact stars



Rüster et al: PRD 72 (2005) 034004
Blaschke et al: PRD 72 (2005) 065020
Abuki, Kunihiro: NPA 768 (2006) 118

The phases are characterized by 3 gaps:

- NQ: $\Delta_{ud} = \Delta_{us} = \Delta_{ds} = 0$;
- NQ-2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$, $0 \leq \chi_{2SC} \leq 1$;
- 2SC: $\Delta_{ud} \neq 0$, $\Delta_{us} = \Delta_{ds} = 0$;
- uSC: $\Delta_{ud} \neq 0$, $\Delta_{us} \neq 0$, $\Delta_{ds} = 0$;
- CFL: $\Delta_{ud} \neq 0$, $\Delta_{ds} \neq 0$, $\Delta_{us} \neq 0$;

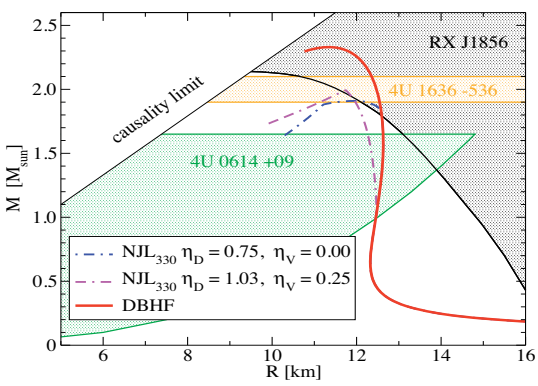
Result:

- Gapless phases only at high T,
- CFL only at high chemical potential,
- At T $\leq$25-30 MeV: mixed NQ-2SC phase,
- Critical point (T_c, μ_c)=(48 MeV, 353 MeV),
- Strong coupling, $\eta = 1$, changes?.

$\Longrightarrow$ Zhuang (DM 12, 17)

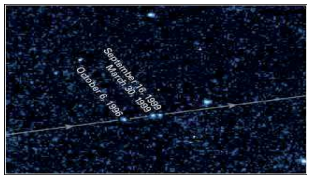
Quark matter in compact stars: Mass-Radius constraint

Solve TOV Eqn. $\rightarrow$ Hybrid stars fulfill constraint!

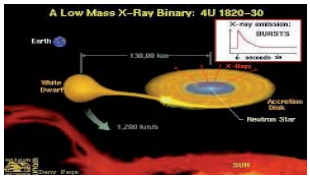


Klähn et al: Constraints on the high-density EoS ...
PRC 74 (2006); [nucl-th/0602038], [astro-ph/0606524]

- Isolated Neutron star RX J1856: M-R constraint from thermal emission

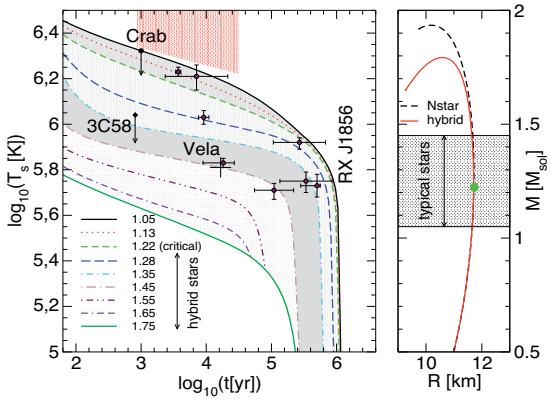


- Low-mass X-ray binary 4U 1636: Mass constraint from ISCO obs.



Quark matter in compact stars: Cooling constraint

Quark matter in compact stars: color superconducting

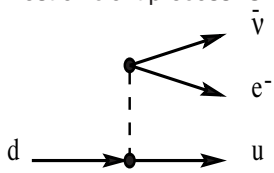


Popov et al: Neutron star cooling constraints ...
PRC 74, 025803 (2006); [nucl-th/0512098]

- Neutrinos carry energy off the star, Cooling evolution (schematic) by

$$\frac{dT(t)}{dt} = -\frac{\epsilon_\gamma + \sum_{j=Urca,\ldots} \epsilon_\nu^j}{\sum_{i=q,e,\gamma,\ldots} c_V^i}$$

- Most efficient process: Urca



- Exponential suppression by pairing gaps! $\Delta \sim 10...100$ keV

Summary

- Mott-Hagedorn model as alternative interpretation of Lattice data
- Microscopic formulation of the hadronic Mott effect within a chiral quark model
- Mesonic (hadronic) correlations important for $T > T_c$
- Step-like enhancement of threshold processes due to Mott effect
- Reaction kinetics for strong correlations in plasmas applicable @ SPS and RHIC
- Prospects for LHC: Plasma diagnostics with bottomonium

Lecture II: NJL model and its relatives

- Polyakov-loop Nambu–Jona-Lasinio (NJL) model
- Nonlocal NJL models
- Schwinger-Dyson Equation approach at finite T, μ
- Walecka model - towards a unified model of quark-hadron matter

EFFECTIVE FIELD THEORIES FOR HOT AND DENSE MATTER (II) NJL MODEL AND ITS RELATIVES

David Blaschke

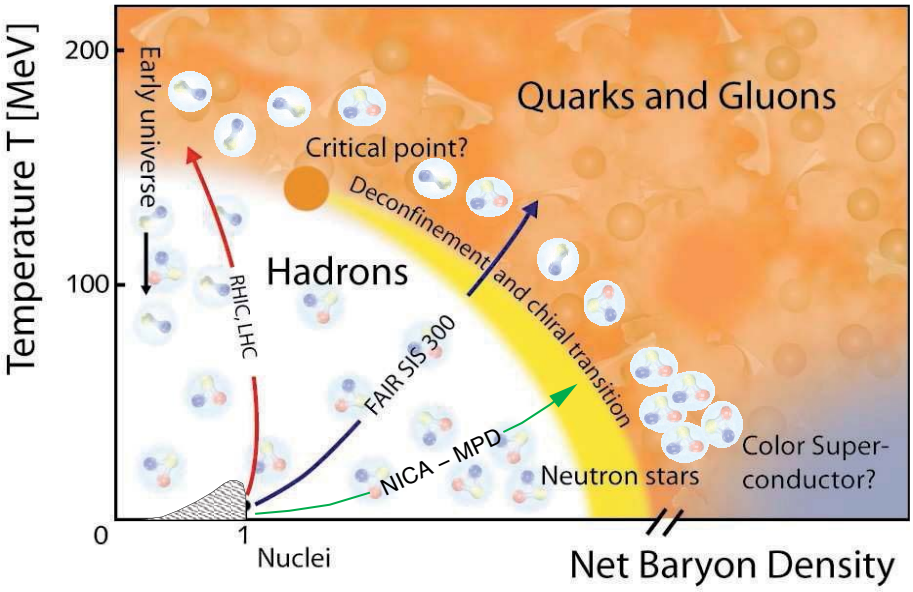
Institute for Theoretical Physics, University of Wroclaw, Poland

Bogoliubov Laboratory for Theoretical Physics, JINR Dubna, Russia

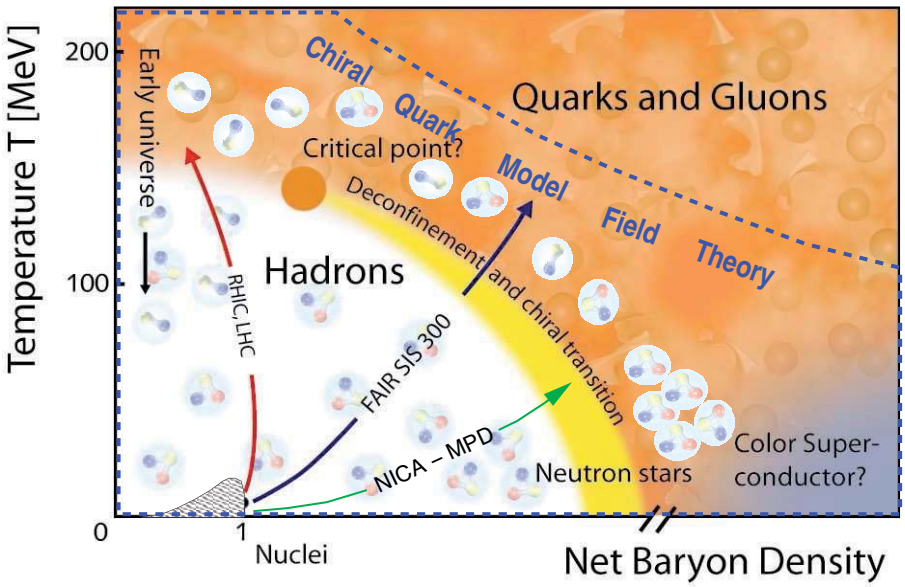
- NJL Model and its Polyakov-Loop Extension:
  - Mesonic correlations - Mott Effect
  - Polyakov-Loop NJL Model
- Nonlocal, separable NJL Model
  - 3D Formfactors, 4D Formfactors and Phase Diagram
  - Rank-2 Extension - Schwinger-Dyson type Approach
- Summary / Outlook to a Unified Quark-Hadron Approach

Literature: Hansen et al., Phys. Rev. D75, 065004 (2007); Gomez Dumm et al., Phys. Rev. D73, 114019 (2006); arXiv:0807.1660; Blaschke et al., arXiv:0705.0384; Schmidt et al., Phys. Rev. C50, 435 (1994); Zablocki at al., arXiv:0805.2687

HADRONIC CORRELATIONS IN THE PHASEDIAGRAM OF QCD



Hadronic Correlations in the Phasediagram of QCD



Chiral Model Field Theory for Quark Matter

- Partition function as a Path Integral (imaginary time $\tau = i\,t$)

$$Z[T,V,\mu] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left\{-\int^{\beta} d\tau \int_V d^3x[\bar{\psi}[i\gamma^\mu\partial_\mu - m - \gamma^0(\mu + \lambda_8\mu_8 + i\lambda_3\phi_3]\psi - \mathcal{L}_{\rm int} + U(\Phi)]\right\}$$

Polyakov loop: $\Phi = N_c^{-1}{\rm Tr}_c[\exp(i\beta\lambda_3\phi_3)]$ Order parameter for deconfinement

- Current-current interaction (4-Fermion coupling)

$$\mathcal{L}_{\rm int} = \sum_{M=\pi,\sigma,\ldots} G_M(\bar{\psi}\Gamma_M\psi)^2 + \sum_D G_D(\bar{\psi}^C\Gamma_D\psi)^2$$

- Bosonization (Hubbard-Stratonovich Transformation)

$$Z[T,V,\mu] = \int \mathcal{D}M_M\mathcal{D}\Delta_D^\dagger\mathcal{D}\Delta_D \exp\left\{-\sum_{M,D}\frac{M_M^2}{4G_M} - \frac{|\Delta_D|^2}{4G_D} + \frac{1}{2}{\rm Tr}\,\ln S^{-1}[\{M_M\},\{\Delta_D\},\Phi] + U(\Phi)\right\}$$

- Collective quark fields: Mesons (M_M) and Diquarks (Δ_D); Gluon mean field: Φ
- Systematic evaluation: Mean fields + Fluctuations
  - Mean-field approximation: order parameters for phase transitions (gap equations)
  - Lowest order fluctuations: hadronic correlations (bound & scattering states)
  - Higher order fluctuations: hadron-hadron interactions

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (I)

$SU(N_c)$ pure gauge sector: Polyakov line

$$L(\vec{x}) \equiv \mathcal{P} \exp\left[i \int_0^\beta d\tau \, A_4(\vec{x},\tau) \right] \; ; \quad A_4 = iA^0 = \lambda_3\phi_3 + \lambda_8\phi_8$$

Polyakov loop

$$l(\vec{x}) = \frac{1}{N_c}\mathrm{Tr}L(\vec{x}) \; , \quad \langle l(\vec{x})\rangle = e^{-\beta\Delta F_Q(\vec{x})}.$$

$\mathbf{Z}_{N_c}$ symmetric phase: $\langle l(\vec{x})\rangle = 0 \quad \Longrightarrow \Delta F_Q \to \infty$: Confinement !
Polyakov loop field:

$$\Phi(\vec{x}) \equiv \langle\langle l(\vec{x})\rangle\rangle = \frac{1}{N_c}\,\mathrm{Tr}_c\,\langle\langle L(\vec{x})\rangle\rangle$$

Potential for the PL-meanfield $\Phi(\vec{x})$ =const., which fits quenched QCD lattice thermodynamics

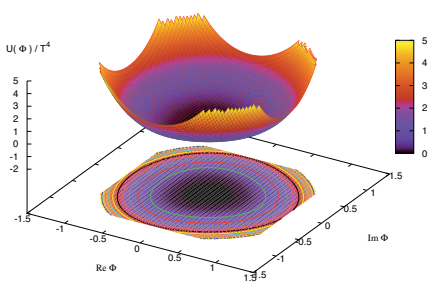
$$\frac{\mathcal{U}(\Phi,\bar{\Phi};T)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2 \; ,$$

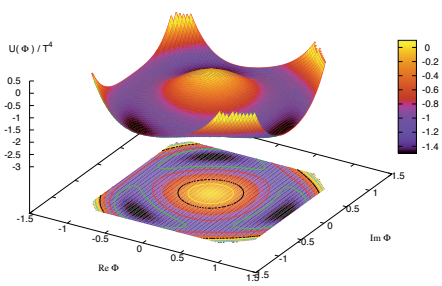
$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 .$$

a_0	a_1	a_2	a_3	b_3	b_4
6.75	-1.95	2.625	-7.44	0.75	7.5

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (II)

Temperature dependence of the Polyakov-loop potential $U(\Phi,\bar{\Phi};T)$





$T = 0.26$ GeV$< T_0$
"Color confinement"

$T = 1.0$ GeV$> T_0$
"Color deconfinement"

Critical temperature for pure gauge $SU_c(3)$ lattice simulations: $T_0 = 270$ MeV.

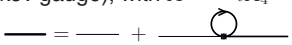
Hansen et al., Phys.Rev. D75, 065004 (2007)

Polyakov-loop Nambu–Jona-Lasinio Model (III)

Lagrangian for $N_f = 2$, $N_c = 3$ quark matter, coupled to the gauge sector

$$\mathcal{L}_{PNJL} = \bar{q}(i\gamma^\mu D_\mu - \hat{m} + \gamma_0\mu)q + G_1\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] - \mathcal{U}\left(\Phi[A], \bar{\Phi}[A]; T\right),$$

$D^\mu = \partial^\mu - iA^\mu$; $A^\mu = \delta^\mu_0 A^0$ (Polyakov gauge), with $A^0 = -iA_4$

Diagrammatic Hartree equation: 

$$S_0(p) = \text{——} = -(\not{p} - m_0 + \gamma^0(\mu - iA_4))^{-1}\ ;\ \ S(p) = \text{——} = -(\not{p} - m + \gamma^0(\mu - iA_4))^{-1}$$

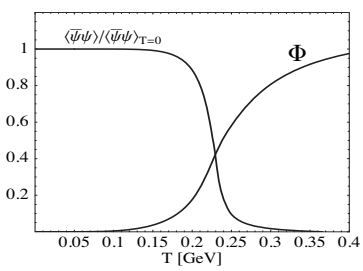
Dynamical chiral symmetry breaking $\sigma = m - m_0 \neq 0$? Solve Gap Equation! ($E = \sqrt{p^2 + m^2}$)

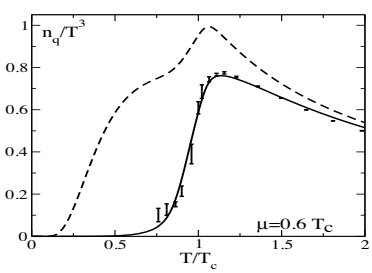
$$\begin{aligned} m - m_0 &= 2G_1 T\ \mathrm{Tr} \sum_{n=-\infty}^{+\infty} \int_\Lambda \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{-1}{\not{p} - m + \gamma^0(\mu - iA_4)} \\ &= 2G_1 N_f N_c \int_\Lambda \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{2m}{E}[1 - f^+_\Phi(E) - f^-_\Phi(E)] \end{aligned}$$

Modified quark distribution functions ($\Phi = \bar{\Phi} = 0$: "poor man's nucleon": $E_N = 3E$, $\mu_N = 3\mu$)

$$f^\pm_\Phi(E) = \frac{\left(\Phi + 2\bar{\Phi}e^{-\beta\left(E_p \mp \mu\right)}\right) e^{-\beta\left(E_p \mp \mu\right)} + e^{-3\beta\left(E_p \mp \mu\right)}}{1 + 3\left(\Phi + \bar{\Phi}e^{-\beta\left(E_p \mp \mu\right)}\right) e^{-\beta\left(E_p \mp \mu\right)} + e^{-3\beta\left(E_p \mp \mu\right)}} \longrightarrow f^\pm_0(E) = \frac{1}{1 + e^{\beta(E_N \mp \mu_N)}}$$

Polyakov-loop Nambu–Jona-Lasinio Model (IV)





Grand canonical thermodynamical potential

$$\begin{aligned} \Omega(T, \mu; \Phi, m) &= \frac{\sigma^2}{2G} - 6N_f \int \frac{\mathrm{d}^3p}{(2\pi)^3} E\,\theta\left(\Lambda^2 - \vec{p}^{\,2}\right) \\ &- 2N_f T \int \frac{\mathrm{d}^3p}{(2\pi)^3} \Big\{ \mathrm{Tr}_c \ln\left[1 + L\,\mathrm{e}^{-(E-\mu)/T}\right] \\ &+ \mathrm{Tr}_c \ln\left[1 + L^\dagger\,\mathrm{e}^{-(E+\mu)/T}\right] \Big\} + \mathcal{U}\left(\Phi, \bar{\Phi}, T\right) \end{aligned}$$

Appearance of quarks below T_c largely suppressed:

$$\begin{aligned} &\ln\det\left[1 + L\,\mathrm{e}^{-(E-\mu)/T}\right] + \ln\det\left[1 + L^\dagger\,\mathrm{e}^{-(E+\mu)/T}\right] \\ &= \ln\left[1 + 3\left(\Phi + \bar{\Phi}\mathrm{e}^{-(E-\mu)/T}\right)\mathrm{e}^{-(E-\mu)/T} + \mathrm{e}^{-3(E-\mu)/T}\right] \\ &+ \ln\left[1 + 3\left(\bar{\Phi} + \Phi\mathrm{e}^{-(E+\mu)/T}\right)\mathrm{e}^{-(E+\mu)/T} + \mathrm{e}^{-3(E+\mu)/T}\right]. \end{aligned}$$

Accordance with QCD lattice susceptibilities! Example:

$$\frac{n_q(T, \mu)}{T^3} = -\frac{1}{T^3}\frac{\partial\Omega(T, \mu)}{\partial\mu},$$

Ratti, Thaler, Weise, PRD 73 (2006) 014019.

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (V)

Mesonic currents

$$J_P^a(x) = \bar{q}(x) i\gamma_5 \tau^a q(x) \quad \text{(pion)} \; ; \quad J_S(x) = \bar{q}(x)q(x) - \langle \bar{q}(x)q(x) \rangle \quad \text{(sigma)}$$

... and correlation functions

$$C_{ab}^{PP}(q^2) \equiv i \int d^4x e^{iq.x} \langle 0 | T \left(J_P^a(x) J_P^{b\dagger}(0) \right) | 0 \rangle = C^{PP}(q^2)\delta_{ab}$$

$$C_{ab}^{SS}(q^2) \equiv i \int d^4x e^{iq.x} \langle 0 | T \left(J_S(x) J_S^{\dagger}(0) \right) | 0 \rangle$$

Schwinger-Dyson Equations, $T = \mu = 0$

$$C^{MM}(q^2) = \Pi^{MM}(q^2) + \sum_{M'} \Pi^{MM'}(q^2)(2G_1)C^{M'M}(q^2)$$

One-loop polarization functions

$$\Pi^{MM'}(q^2) \equiv \int_\Lambda \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left(\Gamma_M S(p+q) \Gamma_{M'} S(q)\right)$$

Hartree quark propagator $S(p)$

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (VI)

Example of the pion channel:

$$\Pi^{PP}(q^2) = -4iN_cN_f \int_\Lambda \frac{d^4p}{(2\pi)^4} \frac{m^2 - p^2 + q^2/4}{[(p+q/2)^2 - m^2][(p-q/2)^2 - m^2]} = 4iN_cN_f I_1 - 2iN_cN_f q^2 I_2(q^2)$$

Loop Integrals:

$$I_1 = \int_\Lambda \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m^2} \quad ; \quad I_2(q^2) = \int_\Lambda \frac{d^4p}{(2\pi)^4} \frac{1}{[(p+q)^2 - m^2]\,[p^2 - m^2]}$$

With pseudoscalar decay constant (f_P) and gap equation for I_1

$$f_P^2(q^2) = -4iN_c m^2 I_2(q^2) \quad ; \quad I_1 = \frac{m - m_0}{8iG_1 m N_c N_f},$$

One obtains $\Pi^{PP}(q^2) = \frac{m-m_0}{2G_1 m} + f_P^2(q^2)\frac{q^2}{m^2}$; $\Pi^{SS}(q^2) = \frac{m-m_0}{2G_1 m} + f_P^2(q^2)\frac{q^2-4m^2}{m^2}$. In the chiral limit ($m_0 = 0$), the correlation functions

$$C^{MM}(q^2) = \Pi^{MM}(q^2) + \Pi^{MM}(q^2)(2G_1)C^{MM}(q^2) = \frac{\Pi^{MM}(q^2)}{1 - 2G_1\Pi^{MM}(q^2)} \quad , \quad M = P, S \; ,$$

have poles at $q^2 = M_P^2 = 0$ (Pion) and $q^2 = M_S^2 = (2m)^2$ (Sigma meson) $\Longrightarrow$ Check !

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (VII)

Finite T, μ: $p = (p_0, \vec{p}) \to (i\omega_n + \mu - iA_4, \vec{p})$; $i\int_\Lambda \frac{d^4p}{(2\pi)^4} \to -T\frac{1}{N_c}\mathrm{Tr}_c \sum_n \int_\Lambda \frac{d^3p}{(2\pi)^3}$

$$\begin{aligned} I_1 &= -i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1-f(E_p-\mu)-f(E_p+\mu)}{2E_p} \\ I_2(\omega,\vec{q}) &= i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1}{2E_p 2E_{p+q}}\frac{f(E_p+\mu)+f(E_p-\mu)-f(E_{p+q}+\mu)-f(E_{p+q}-\mu)}{\omega-E_{p+q}+E_p} \\ &+i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1-f(E_p-\mu)-f(E_{p+q}+\mu)}{2E_p 2E_{p+q}}\left(\frac{1}{\omega+E_{p+q}+E_p}-\frac{1}{\omega-E_{p+q}-E_p}\right) \end{aligned} \quad (1)$$

For a meson at rest in the medium ($\vec{q}=0$)

$$I_2\left(\omega,\vec{0}\right) = -i\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{1-f(E_p+\mu)-f(E_p-\mu)}{E_p\left(\omega^2-4E_p^2\right)}$$

which develops an imaginary part

$$\Im m\,(-iI_2(\omega,0)) = \frac{1}{16\pi}\left(1-f\left(\frac{\omega}{2}-\mu\right)-f\left(\frac{\omega}{2}+\mu\right)\right)\sqrt{\frac{\omega^2-4m^2}{\omega^2}}\times\Theta(\omega^2-4m^2)\Theta(4(\Lambda^2+m^2)-\omega^2)$$

with the Pauli-blocking factor: $N(\omega,\mu) = \left(1-f\left(\frac{\omega}{2}-\mu\right)-f\left(\frac{\omega}{2}+\mu\right)\right)$

POLYAKOV-LOOP NAMBU–JONA-LASINIO MODEL (VIII)

Spectral function

$$\begin{aligned} F^{MM}(\omega,\vec{q}) &\equiv \Im m\, C^{MM}(\omega+i\eta,\vec{q}) = \Im m\,\frac{\Pi^{MM}(\omega+i\eta,\vec{q})}{1-2G_1\Pi^{MM}(\omega+i\eta,\vec{q})}. \\ F^{MM}(\omega) &= \frac{\pi}{2G_1}\frac{1}{\pi}\frac{2G_1\Im m\,\Pi^{MM}(\omega+i\eta)}{\left(1-2G_1\Re e\,\Pi^{MM}(\omega)\right)^2+\left(2G_1\Im m\,\Pi^{MM}(\omega+i\eta)\right)^2}. \end{aligned} \quad (2)$$

For $\omega < 2m(T,\mu)$, $\Im m\,\Pi = 0$: decay channel closed → bound state!

$$F^{MM}(\omega) = \frac{\pi}{2G_1}\delta\left(1-2G_1\Re e\,\Pi^{MM}(\omega)\right) = \frac{\pi}{4G_1^2\left|\frac{\partial\Re e\,\Pi^{MM}}{\partial\omega}\right|_{\omega=m_M}}\delta(\omega-m_M)\ .$$

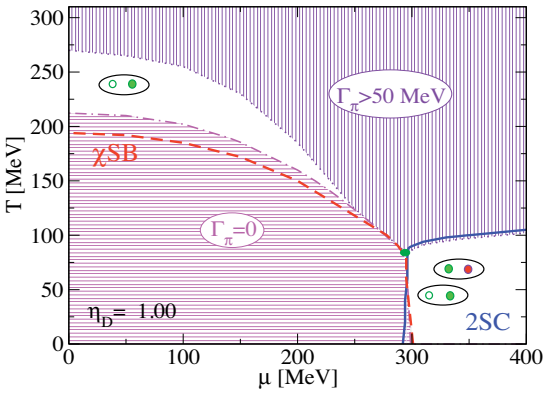
The meson mass m_M is the solution of

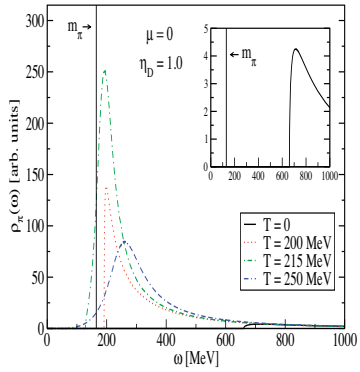
$$1-2G_1\Re e\,\Pi^{MM}(m_M) = 0$$

The decay width (inverse lifetime) is

$$\Gamma_M = 2G_1\Im m\,\Pi^{MM}(m_M)$$

Pion Correlations in the Phase Diagram



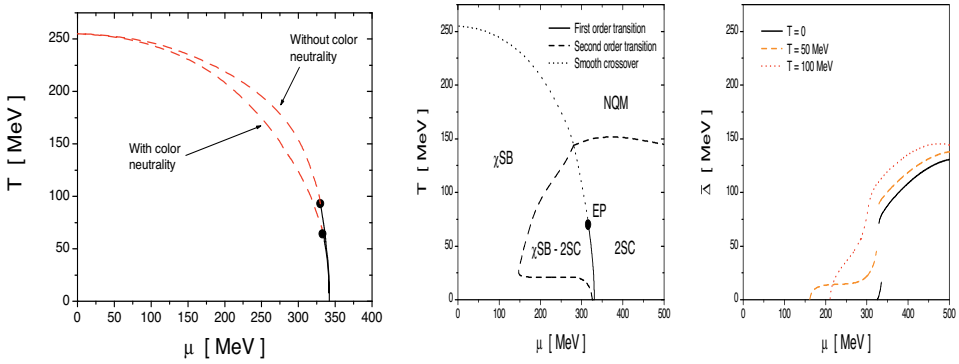


Zablocki, D.B., Anglani, arXiv:0805.2687 [hep-ph]

Color Neutrality in the PNJL Phase Diagram

Color neutrality constraint: $\tilde{\mu} = \mu \mathbf{1} + \mu_8 \lambda_8 + i\phi_3 \lambda_3$; $\partial\Omega_{MF}/\partial\mu_8 = n_8 = n_r + n_g - 2n_b = 0$

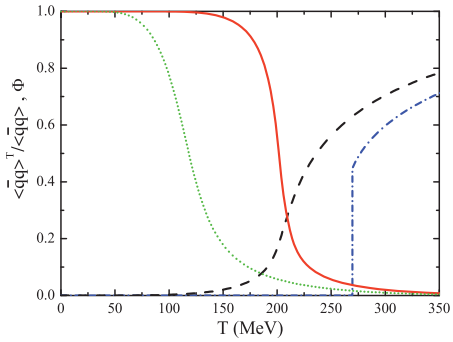
Gap equations: $\partial\Omega_{MF}/(\partial\sigma, \partial\Delta, \partial\phi_3) = 0$



Gomez-Dumm, D.B., Grunfeld, Scoccola, PRD 78, 114021 (2008) [arXiv:0807.1660]

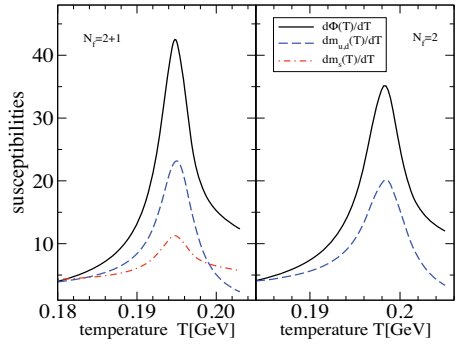
Nonlocal Polyakov Loop Chiral Quark Model

2-flavor, rank-1, 4D separable
order parameters:



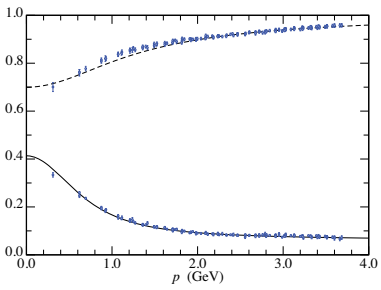
D.B., Buballa, Radzhabov, Volkov,
Yad. Fiz. 71 (2008); arXiv:0705.0384

3-flavor, rank-2, 4D separable
susceptibilities:



D.B., Horvatic, Klabucar, in prep.

Complex mass pole fit to Lattice propagator



Bhagwat, Pichowsky, Roberts, Tandy, Phys. Rev. **C68** (2003) 015203

$S(p)^{-1} = i\not{p}A(p^2) + B(p^2)$,

$M(p^2) = B(p^2)/A(p^2)$

$Z(p^2) = 1/A(p^2)$

$S(p)$ sum of N pairs of complex conj. mass poles

$$S(p) = \sum_{i=1}^{N} \frac{1}{Z_2}\left\{\frac{z_i}{i\not{p}+m_i} + \frac{z_i^*}{i\not{p}+m_i^*}\right\} = -i\not{p}\sigma_V(p^2) + \sigma_S(p^2)$$

Representation of the scalar amplitude

$$\sigma_S(p^2) = \sum_{i=1}^{N} Z_2^{-1}\left\{\frac{z_i\, m_i}{p^2+m_i^2} + \frac{z_i^*\, m_i^*}{p^2+m_i^{*2}}\right\}$$

"Derivation" of the equivalent separable model (in Feynman-like gauge) $D_{\mu\nu}(p-q) = \delta_{\mu\nu}\, D(p,q)$ and

$$\begin{aligned} D(p,q) &= f_0(p^2)\, f_0(q^2) + f_1(p^2)\, p\cdot q\, f_1(q^2) \\ f_1(p^2) &= \frac{A(p^2)-1}{a} \quad ; \quad f_0(p^2) = \frac{B(p^2)-m_c}{b} \end{aligned}$$

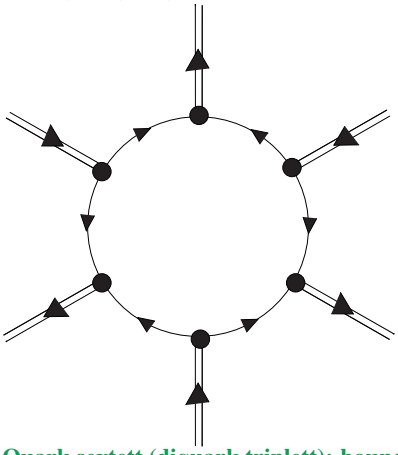
$$b^2 = \frac{16}{3}\int_q^{\Lambda} [B(q^2)-m_c]\sigma_s(q^2)$$

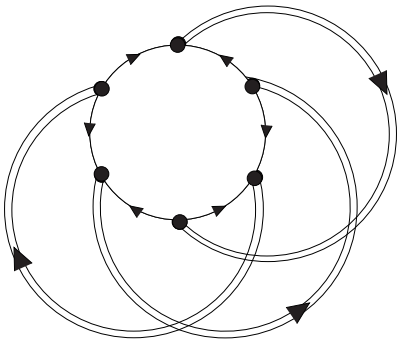
$$a^2 = \frac{8}{3}\int_q^{\Lambda} [A(q^2)-1]\frac{q^2}{4}\sigma_v(q^2)$$

Nucleons in the Nonlocal Chiral Quark Model

$$Z_{\text{fluct}} = \int D\Delta^\dagger D\Delta \exp\{-\frac{|\Delta|^2}{4G_D} - Tr\ lnS^{-1}[\Delta, \Delta^\dagger]\}$$

Cahill, Roberts, Prashifka: Aust. J. Phys. 42 (1989) 129, 161
Cahill, ibid, 171; Reinhardt: PLB 244 (1990) 316; Buck, Alkofer, Reinhardt: PLB 286 (1992) 29





Quark sextett (diquark triplett): bound by exchange forces? sextett condensate?

Summary

- Compressed nuclear matter: **quarkyonic phase** (QP)! Coexisting chiral symm. + conf.
- Similarities: Mott-Hagedorn picture, string-flip model, confining DSE
- Here: PNJL model as microscopic formulation of the QP
- Color singlet quark triplets in chiral phase for $\mu > \mu_c$ (approx. massless baryons)
- Color neutrality by singlet projection = sum over color hexagon
- Prospects for CBM & NICA: dilepton enhancement (peak?) from diquark-antidiquark annih.
- Preparatory step to **compact stars**: single flavor CSL phase - OK with structure & cooling

Outlook: next steps ...

- Walecka model as limit of PNJL model: chiral transition effects in nuclear EoS
- Beyond meanfield: mesons and baryons in the PNJL, higher clusters: sextetting
- Astrophysics: Maximum mass & cooling of quarkyonic stars; quarkyonic supernovae