Condensates and correlations in nuclear matter

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Abstract. Nuclei in dense matter are influenced by the medium. Solving an A-particle Schroedinger equation including the effects of self-energy and Pauli blocking, a quasiparticle description is introduced. Deriving thermodynamic properties, this approach contains the NSE at low densities as well as mean-field approaches at high densities. Consequences for the symmetry energy, the phase transition, the determination of thermodynamic parameters from cluster yields and astrophysical applications are discussed.

HISS: Dense Matter in HIC and Astrophysics, Dubna, 18./21. 7. 08

Condensates and Correlations in Nuclear Matter

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Universität Rostock

Problems:

- Warm Dilute Matter: Nuclear matter at subsaturation densities:

Temperature $T \leq 16 \text{ MeV} = E_s/A$, baryon density $n_B \leq 0.17 \text{ fm}^{-3} = n_s$.

- Formation of clusters (nuclei in matter):

$A = 1, 2, 3, 4$: deuterons (d), tritons (t), helions (h), alphas (α)

- Low-density, low-temperature limit:

Virial expansion, non-interacting nuclides, quantum condensates

- Transition to higher densities:

Medium effects, quasiparticles,

interpolation between Beth-Uhlenbeck and DBHF / RMF

Refs:

Particle clustering and Mott transition in nuclear matter at finite temperatures,

G. Röpke, M. Schmidt, L. Münchow, H. Schulz: NPA **399**, 587-602 (1983).

Generalized Beth-Uhlenbeck Approach for Hot Nuclear Matter,

M. Schmidt, G. Röpke, H. Schulz: Annals of Physics **202**, 57 - 99 (1990).

Outline

- Schrödinger equation with medium corrections:
Self-energy and Pauli blocking
- Composition of the nuclear gas:
Generalized Beth-Uhlenbeck equation
- Quantum condensates:
Pairing and quartetting
- Composition and the EoS of nuclear matter
(astrophysics: supernovae explosions)
- Cluster formation in dilute nuclei
(Hoyle state and THSR wave function)
- Symmetry energy in the low-density region
(heavy ion collisions: cluster abundances)

Low-density EoS and astrophysics

- H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi,
Progr. Theor. Phys. **100**, 1013 (1998); Nucl. Phys. **A637** 435 (1998).
- G. Röpke, A. Grigo, K. Sumiyoshi, and Hong Shen,
in: Superdense QCD Matter and Compact Stars, Ed. D. Blaschke and
A. Sedrakian, NATO Science Series, Springer, Dordrecht (2006),
pp. 75 - 91;
Physics of Particles and Nuclei Letters **2**, 275 (2005).
- J.M.Lattimer and F. D. Swesty,
Nucl. Phys. **A 535**, 331 (2001).
- C. J. Horowitz and A. Schwenk,
Nucl. Phys. **A 776**, 55 (2006).

Neutron stars

Neutron Star

interior topology

$M \sim 1.5 M_{\odot}$

Structure of a Neutron star

Neutron Star

density zones & shell thickness

Supernova

Crab nebula, 1054 China, PSR 0531+21

Supernova collapse: spherically symmetric simulations

A. Arcones
Neutrino driven winds
Talk 25. 2. 08 Ladek

Core-collapse supernovae

Density.

electron fraction, and

temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

K.Sumiyoshi et al.,
Astrophys.J. **629**, 922 (2005)

Parameter range: Explosion

40 m_solar

T. Fischer, On the possible fate of massive progenitor stars, Talk 25.2.08 Ladek

Phase diagram: nuclear matter

Symmetric nuclear matter: Phase diagram

Correlations in low-density matter

- Ideal fermion gas of protons and neutrons?
- Formation of bound states: nuclei
- Medium modifications: quasiparticle concept
- Relativistic mean field approach, nucleons
- Cluster - mean field approach

Properties of light clusters

	binding energy	mass	spin	rms-radius
n	0	939.565 MeV/c ²	1/2	0.34 fm
p	0	938.783 MeV/c ²	1/2	0.87 fm
d	-2.225 MeV	1876.12 MeV/c ²	1	2.17 fm
t	-8.482 MeV	2809.43 MeV/c ²	1/2	1.70 fm
h	-7.718 MeV	2809.41 MeV/c ²	1/2	1.87 fm
α	-28.3 MeV	3728.40 MeV/c ²	0	1.63 fm

Ideal mixture of reacting nuclides

$$n_p(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A, \nu, K} Z_A f_A \{ E_{A, \nu K} - Z_A \mu_p - (A - Z_A) \mu_n \}$$

$$n_n(T, \mu_p, \mu_n) = \frac{1}{V} \sum_{A, \nu, K} (A - Z_A) f_A \{ E_{A, \nu K} - Z_A \mu_p - (A - Z_A) \mu_n \}$$

mass number A ,
 charge Z_A ,
 energy $E_{A, \nu, K}$,
 ν internal quantum number,
 $\sim K$ center of mass momentum

$$f_A(z) = \frac{1}{\exp(z/T) - (-1)^A}$$

Composition of nuclear matter

Virial expansion

- excited nuclei
- resonances
- scattering phase shifts (no double counting)
- virial expansions
- quantum statistical approach

Beth-Uhlenbeck formula

rigorous results at low density: virial expansion

Beth-Uhlenbeck formula

$$\begin{aligned}
 n(T, \mu) = & \frac{1}{V} \sum_p e^{-(E(p)-\mu)/k_B T} \\
 & + \frac{1}{V} \sum_{nP} e^{-(E_{nP}-2\mu)/k_B T} \\
 & + \frac{1}{V} \sum_{\alpha P} \int_0^\infty \frac{dE}{2\pi} e^{-(E+P^2/4m-2\mu)/k_B T} \frac{d}{dE} \delta_\alpha(E) \\
 & + \dots
 \end{aligned}$$

$\delta_\alpha(E)$: scattering phase shifts, channel α

Alpha-particle fraction in the low-density limit

symmetric matter, $T=4$ MeV

Horowitz & Schwenk (2006), Lattimer & Swesty, (2001), Shen et al. (1998))

alpha - fraction at $T = 4$ MeV

C.J.Horowitz, A.Schwenk, Nucl. Phys. A **776**, 55 (2006)

Nucleon-nucleon interaction

- general form:

$$V_{\alpha}(p, p') = \sum_{i,j=1}^N w_{\alpha i}(p) \lambda_{\alpha ij} w_{\alpha j}(p') \quad \text{uncoupled}$$

and

$$V_{\alpha}^{LL'}(p, p') = \sum_{i,j=1}^N w_{\alpha i}^L(p) \lambda_{\alpha ij} w_{\alpha j}^{L'}(p') \quad \text{coupled}$$

p, p' in- and outgoing relative momentum
 $\alpha \dots$ channel
 $N \dots$ rank
 $\lambda_{\alpha ij}$ coupling parameter
 L, L' orbital angular momentum

Separable nucleon-nucleon interaction

- examples:

- Yamaguchi potential

Y. Yamaguchi, Phys. Rev. 95, 1628 (1954)

rank = 1, uncoupled, only S -waves

- PARIS-potential, PESTN

J. Haidenbauer and W. Plessas, Phys. Rev. C 30, 1822 (1984)

consideration of partial waves up to $L = 2$

- BONN-potential, BESTN

Plessas et al., Few-Body Syst. Suppl. 7, 251 (1994)

consideration of partial waves up to $L = 3$

Many-particle theory

- equilibrium correlation functions

e.g. equation of state $n(\beta, \mu) = \frac{1}{\Omega_0} \sum_1 \langle a_1^\dagger a_1 \rangle$

density matrix $\langle a_1^\dagger a_1^\dagger \rangle = \int \frac{d\omega}{2\pi} e^{-i\omega t} f_1(\omega) A(1, 1', \omega)$

- Spectral function

$$A(1, 1', \omega) = \text{Im} [G(1, 1', \omega + i\eta) - G(1, 1', \omega - i\eta)]$$

- Matsubara Green function

$$G(1, 1', iz_\nu), \quad z_\nu = \frac{\pi\nu}{\beta} + \mu, \quad \nu = \pm 1, \pm 3, \dots$$

$$1 \equiv \{\mathbf{p}_1, \sigma_1, c_1\}, \quad f_1(\omega) = \frac{1}{e^{\beta(\omega - \mu)} + 1}, \quad \Omega_0 - \text{volume}$$

Many-particle theory

- Dyson equation and self energy (homogeneous system)

$$G(1, iz_\nu) = \frac{1}{iz_\nu - E(1) - \Sigma(1, iz_\nu)}$$

- Evaluation of $\Sigma(1, iz_\nu)$:

perturbation expansion, diagram representation

$$A(1, \omega) = \frac{2\text{Im} \Sigma(1, \omega + i0)}{[\omega - E(1) - \text{Re} \Sigma(1, \omega)]^2 + [\text{Im} \Sigma(1, \omega + i0)]^2}$$

approximation for
self energy

→

approximation for
equilibrium correlation functions

alternatively: simulations, path integral methods

Different approximations

- Expansion for small $\text{Im } \Sigma(1, \omega + i\eta)$

$$A(1, \omega) \approx \frac{2\pi\delta(\omega - E^{\text{quasi}}(1))}{1 - \frac{d}{dz} \text{Re } \Sigma(1, z)|_{z=E^{\text{quasi}}-\mu_1}} \frac{P}{-2\text{Im } \Sigma(1, \omega + i\eta) \frac{d}{d\omega} \omega + \mu_1 - E^{\text{quasi}}(1)}$$

quasiparticle energy $E^{\text{quasi}}(1) = E(1) + \text{Re } \Sigma(1, \omega)|_{\omega=E^{\text{quasi}}}$

- chemical picture: bound states $\hat{=}$ new species

summation of ladder diagrams, Bethe-Salpeter equation

Medium effects

Quasiparticle approximation

Lagrangian: non-linear sigma

$$\begin{aligned}
 \mathcal{L} = & \bar{\psi}_i [i\gamma_\mu \partial^\mu - m_i - g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \tau_a \rho_a^\mu] \psi_i \\
 & + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
 & - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2 \\
 & - \frac{1}{4} R_{\mu\nu}^a R^{a\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu^a \rho_a^\mu .
 \end{aligned}$$

TM1 parameters

- Shen et al. (1998)

Parameter	Numerical	value
m_p	938.783	MeV
m_n	939.565	MeV
m_σ	511.19777	MeV
m_ω	783.0	MeV
m_ρ	770.0	MeV
g_σ	10.02892	
g_ω	12.61394	
g_ρ	4.63219	
g_2	-7.23247	fm ⁻¹
g_3	0.61833	
c_3	71.30747	

Quasiparticle energies

- Skyrme
- relativistic mean field (RMF)

$$e_i(k) = \sqrt{(m_i c^2 - S(n_B, \delta, T))^2 + \hbar^2 c^2 k^2} + V_i(n_B, \delta, T)$$

$$n_i = \frac{1}{\pi^2} \int_0^\infty dk \frac{k^2}{\exp[e_i(k)/k_B T - \mu_i/k_B T] + 1}$$

Single particle modifications

- effective mass

$$m_i^* = m_i + g_\sigma \sigma_0,$$

- energy shift

$$E_p(k; T, \mu_p, \mu_n) = \sqrt{k^2 + m_p^{*2}} + g_\omega \omega_0 + g_\rho \rho_0,$$

$$E_n(k; T, \mu_p, \mu_n) = \sqrt{k^2 + m_n^{*2}} + g_\omega \omega_0 - g_\rho \rho_0$$

DD-RMF (Tipel, 2007)

- scalar field

$$\begin{aligned}
 S(n_B, \delta, T) = & n_B[(4524.13 - 6.926T) - 14.5157/4\delta^2 + 0.833943/16\delta^4 - 9.00693/64\delta^6] \\
 & + n_B^2[-19190.7 - 2426.57/4\delta^2 - 317.732/16\delta^4 - 1547.38/64\delta^6] \\
 & + n_B^3[62169.5 + 2521.29/4\delta^2 + 3470.28/16\delta^4] \\
 & + n_B^4[-91005.1 + 3984.82/4\delta^2 - 9148.6/16\delta^4];
 \end{aligned}$$

- vector fields

$$\begin{aligned}
 V_p(n_B, \delta) = & V_n(n_B, -\delta) = n_B[3462.24 + 946.705/2\delta - 0.334508/4\delta^2] \\
 & + n_B^2[-11312.4 - 6246.21/2\delta - 6353.53/4\delta^2 - 0.099478/8\delta^3] \\
 & + n_B^3[20806.1 + 18717.6/2\delta + 29298./4\delta^2 - 0.490543/8\delta^3] \\
 & + n_B^4[352.371 - 24887.2/2\delta - 39807.4/4\delta^2 - 0.346218/8\delta^3].
 \end{aligned}$$

Quasiparticle approximation for nuclear matter

Equation of state for symmetric matter

RMF and DBHF

C. Fuchs et al. (2007)

Different approximations

- Expansion for small $\text{Im } \Sigma(1, \omega + i\eta)$

$$A(1, \omega) \approx \frac{2\pi\delta(\omega - E^{\text{quasi}}(1))}{1 - \frac{d}{dz} \text{Re } \Sigma(1, z)|_{z=E^{\text{quasi}} - \mu_1} - 2\text{Im } \Sigma(1, \omega + i\eta) \frac{d}{d\omega} \frac{P}{\omega + \mu_1 - E^{\text{quasi}}(1)}}$$

quasiparticle energy $E^{\text{quasi}}(1) = E(1) + \text{Re } \Sigma(1, \omega)|_{\omega=E^{\text{quasi}}}$

- chemical picture: bound states $\hat{=}$ new species

summation of ladder diagrams, Bethe-Salpeter equation

$$\begin{array}{c} \text{---} \boxed{G_L} \text{---} \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \end{array} + \dots$$

Different approximations

low density limit:

$$G_2^L(12, 1'2', i\lambda) = \sum_{n\mathbf{P}} \Psi_{n\mathbf{P}}(12) \frac{1}{i\omega_\lambda - E_{n\mathbf{P}}} \Psi_{n\mathbf{P}}^*(12)$$

$$\Sigma = \begin{array}{|c|} \hline T_2^L \\ \hline \end{array}$$

$$n(\beta, \mu) = \sum_1 f_1(E^{\text{quasi}}(1)) + \sum_{2, n\mathbf{P}}^{\text{bound}} g_{12}(E_{n\mathbf{P}}) \\ + \sum_{2, n\mathbf{P}} \int_0^\infty dk \delta_{k, \mathbf{p}_1 - \mathbf{p}_2} g_{12}(E^{\text{quasi}}(1) + E^{\text{quasi}}(2)) 2 \sin^2 \delta_n(k) \frac{1}{\pi} \frac{d}{dk} \delta_n(k)$$

- generalized Beth-Uhlenbeck formula
correct low density/low temperature limit:
mixture of free particles and bound clusters

Effective wave equation for the deuteron in matter

$$\left(\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}\right) \Psi_{n,P}(p_1, p_2) + \sum_{p_1', p_2'} (1 - f_{p_1} - f_{p_2}) V(p_1, p_2; p_1', p_2') \Psi_{n,P}(p_1', p_2') = E_{n,P} \Psi_{n,P}(p_1, p_2)$$

Fermi distribution function

$$f_p = \left[e^{(p^2/2m - \mu)/k_B T} + 1 \right]^{-1}$$

Deuterons in nuclear matter

$T=10 \text{ MeV}$, P : center of mass momentum

Scattering phase shifts in matter

Generalized Beth-Uhlenbeck formula

- quantum gas
- medium modifications

$$n(T, \mu) = n_1(T, \mu) + n_2(T, \mu)$$

contribution of free quasiparticles

$$n_1(T, \mu) = \frac{1}{V} \sum_p \frac{1}{e^{(E^{\text{HF}}(p) - \mu)/k_B T} + 1}$$

$(E^{\text{HF}}(p))$: quasiparticle energy)

contribution of two-particle correlations

$$n_2(T, \mu) = n_2^{\text{bound}}(T, \mu) + n_2^{\text{scatt}}(T, \mu)$$

Generalized Beth-Uhlenbeck formula

$$n_2^{\text{bound}}(T, \mu) = \frac{1}{V} \sum_{nP} \frac{1}{e^{(E_{nP}^{\text{mean field}} - 2\mu)/k_B T} - 1},$$

$$n_2^{\text{scatt}}(T, \mu) = \frac{1}{V} \sum_{\alpha P} \int_0^\infty \frac{dE}{2\pi} \frac{1}{e^{(E_{\alpha P}^{\text{mean field}} + E - 2\mu)/k_B T} - 1} \sin^2(\delta_\alpha(E)) \frac{d}{dE} \delta_\alpha(E)$$

Composition: (ionization degree)

correlated part $\frac{n_2(T, \mu)}{n(T, \mu)}$

Bose distribution function: Quantum condensates

Composition of symmetric nuclear matter

Fraction of correlated matter (virial expansion, Generalized Beth-Uhlenbeck approach, contribution of bound states, of scattering states, phase shifts)

H. Stein et al.,
Z. Phys. **A351**, 259 (1995)

Quantum condensate

Bose-Einstein-Condensation of deuterons (BEC)

Bardeen-Cooper Schrieffer pairing (BCS)

Correlations in the medium

$$\Sigma_2 = \begin{array}{c} \begin{array}{ccc} \text{(2x)} & \text{(2x)} & \text{(2x)} \\ \text{Diagram 1} & \text{Diagram 2} & \text{Diagram 3} \end{array} \\ + \\ \begin{array}{ccc} \text{(2x)} & \text{(2x)} & \text{(2x)} \\ \text{Diagram 4} & \text{Diagram 5} & \text{Diagram 6} \end{array} \end{array}$$

The diagrams represent various two-particle correlation functions in a medium, showing different topologies of particle interactions and loops. Each diagram is labeled with a factor of (2x), indicating a specific contribution to the sum Σ_2 .

Account of two-particle correlations in the medium

Quasiparticle energy shift

- Brueckner-Bethe-Goldstone (dashed)
- generalized Beth-Uhlenbeck (solid)
- incomplete Pauli-blocking (dotted)

Cluster decomposition of the self-energy

Few-particle Schoedinger equation in a dense medium

Four-particle Schrödinger equation with medium effects

$$\begin{aligned}
 & [E^{\text{HF}}(p_1) + E^{\text{HF}}(p_2) + E^{\text{HF}}(p_3) + E^{\text{HF}}(p_4)] \psi_{nP}(p_1, p_2, p_3, p_4) \\
 & + \sum_{p'_1 p'_2 p'_3 p'_4} \left\{ [1 - \underline{f(p_1)} - \underline{f(p_2)}] V(p_1 p_2, p'_1 p'_2) \delta_{p_3 p'_3} \delta_{p_4 p'_4} \right. \\
 & \quad + [1 - \underline{f(p_1)} - \underline{f(p_3)}] V(p_1 p_3, p'_1 p'_3) \delta_{p_2 p'_2} \delta_{p_4 p'_4} \\
 & \quad \left. + \text{permutations} \right\} \psi_{nP}(p'_1, p'_2, p'_3, p'_4) \\
 & = E_{nP} \psi_{nP}(p_1, p_2, p_3, p_4)
 \end{aligned}$$

In-medium shift of binding energies of clusters

A. Sedrakian et al., PRC (2006), M. Beyer et al., Phys.Lett.B **488**, 247 (2000)

Composition of symmetric nuclear matter

T=10 MeV

G.Ropke, A.Grigo, K. Sumiyoshi, Hong Shen,
Phys.Part.Nucl.Lett. **2**, 275 (2005)

Light Cluster Abundances

S. Typel, 2007

Core-collapse supernovae

Density.

electron fraction, and

temperature profile

of a 15 solar mass supernova
at 150 ms after core bounce
as function of the radius.

Influence of cluster formation
on neutrino emission
in the cooling region and
on neutrino absorption
in the heating region ?

K. Sumiyoshi et al.,
Astrophys.J. **629**, 922 (2005)

Composition of supernova core

Mass fraction X of light clusters for a post-bounce supernova core

K. Sumiyoshi, G. Roepke
PRC, 2008

Quasiparticle approximation for nuclear matter

Low-density limit: alpha matter?

Break

- Shown today:
 - correlations
 - composition
 - medium effects
 - astrophysical relevance
- Monday's lecture:
 - quantum condensates
 - pairing and quartetting
 - Hoyle state and low-density isomers
 - low density limit of symmetry energy

HISS: Dense Matter in HIC and Astrophysics, Dubna, 18./21. 7. 08

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RMF and DBHF

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 n(T, \mu) = & \frac{1}{V} \sum_p e^{-(E(p)-\mu)/k_B T} \\
 & + \frac{1}{V} \sum_{n,p} e^{-(E_{np}-2\mu)/k_B T} \\
 & + \frac{1}{V} \sum_{\alpha,p} \int_0^\infty \frac{dE}{2\pi} e^{-(E+P^2/4m-2\mu)/k_B T} \frac{d}{dE} \delta_\alpha(E) \\
 & + \dots
 \end{aligned}$$

$\delta_\alpha(E)$: scattering phase shifts, channel α

Alpha-particle fraction in the low-density limit

symmetric matter, $T=4$ MeV

Horowitz & Schwenk (2006), Lattimer & Swesty, (2001), Shen et al. (1998))

Alpha-particle fraction in the low-density limit

symmetric matter, $T=20$ MeV

Alpha-particle fraction in the low-density limit

symmetric matter, $T=2, 4, 8$ MeV

Composition of symmetric nuclear matter

α -cluster-condensation (quartetting)

α -cluster-condensation (quartetting)

Alpha-condensate (quartetting) in 4n symmetric nuclei

- A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke, Phys. Rev. Lett. **87**, 192501 (2001).
- G. Röpke, A. Schnell, P. Schuck, and P. Nozieres, Phys. Rev. Lett. **80**, 3177 (1998).
- Y. Funaki, A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke, Phys. Rev. C **67**, 051306(R) (2003); Eur. Phys. J. **A 28**, 259 (2006).
- T. Yamada, P. Schuck, Phys. Rev. C **69**, 024309 (2004).

Self-conjugate 4n nuclei

^{12}C :

0^+ state at 0.39 MeV above the 3α threshold energy:

α cluster interact predominantly in relative S waves,
gaslike structure

α -particle condensation in low-density nuclear matter
($\rho \leq \rho_0/5$)

$n\alpha$ cluster condensed states

--- a general feature in $N = Z$ nuclei?

Self-conjugate 4n nuclei

$n\alpha$ nuclei: ^8Be , ^{12}C , ^{16}O , ^{20}Ne , ^{24}Mg , ...

Single-particle shell model, or

Cluster type structures

ground state, excited states

$n\alpha$ break up at the threshold energy $E_{n\alpha}^{\text{thr}} = nE_\alpha$

Variational ansatz

$$|\Phi_{n\alpha}\rangle = (C_{\alpha}^{\dagger})^n |\text{vac}\rangle$$

α - particle creation operator

$$C_{\alpha}^{\dagger} = \int d^3R e^{-\vec{R}^2/R_0^2} \\ \times \int d^3r_1 \dots d^3r_4 \phi_{0s}(\vec{r}_1 - \vec{R}) a_{\sigma_1 \tau_1}^{\dagger}(\vec{r}_1) \dots \phi_{0s}(\vec{r}_4 - \vec{R}) a_{\sigma_4 \tau_4}^{\dagger}(\vec{r}_4)$$

with

$$\phi_{0s}(\vec{r}) = \frac{1}{(\pi b^2)^{3/4}} e^{-\vec{r}^2/(2b^2)}$$

Variational ansatz

total $n\alpha$ wave function

$$\langle \vec{r}_1 \sigma_1 \tau_1 \dots \vec{r}_{4n} \sigma_{4n} \tau_{4n} | \Phi_{n\alpha} \rangle \\ \propto \mathcal{A} \left\{ e^{-\frac{2}{B^2} (\vec{X}_1^2 + \dots + \vec{X}_n^2)} \phi(\alpha_1) \dots \phi(\alpha_n) \right\}$$

where $B^2 = (b^2 + 2R_0^2)$, $\vec{X}_i = \frac{1}{4} \Sigma_n \vec{r}_{in}$,
 $\phi(\alpha_i) = e^{-\frac{1}{8b^2} \Sigma_{m>n} (\vec{r}_{im} - \vec{r}_{in})^2}$ - internal α wave function

A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke, PRL **87**,
 192501 (2001)

3 alpha variational energy

4 alpha variational energy

Results

	E_k (MeV)	E_{exp} (MeV)	$E_k - E_{n\alpha}^{\text{thr}}$ (MeV)	$(E - E_{n\alpha}^{\text{thr}})_{\text{exp}}$ (MeV)	$\sqrt{\langle r^2 \rangle}$ (fm)	$\sqrt{\langle r^2 \rangle}_{\text{exp}}$ (fm)
^{12}C $k=1$	-85.9	-92.16 (0_1^+)	-3.4	-7.27	2.97	2.65
$k=2$	-82.0	-84.51 (0_2^+)	+0.5	0.38	4.29	
$E_{3\alpha}^{\text{thr}}$	-82.5	-84.89				
^{16}O $k=1$	-124.8	-127.62 (0_1^+)	-14.8	-14.44	2.59	2.73
	(-128.0)*		(-18.0)*			
$k=2$	-116.0	-116.36 (0_3^+)	-6.0	-3.18	3.16	
$k=3$	-110.7	-113.62 (0_5^+)	-0.7	-0.44	3.97	
$E_{4\alpha}^{\text{thr}}$	-110.0	-113.18				

Tabelle 1: Comparison of the generator coordinate method calculations with experimental values. $E_{n\alpha}^{\text{thr}} = nE_\alpha$ denotes the threshold energy for the decay into α -clusters, the values marked by * correspond to a refined mesh.

Decay of α -condensate at high densities

- i) The bound state dissolves into free nucleon states

Occupied phase space cannot be used to form bound states

Pauli blocking $V(12, 1'2')[1 - f(1) - f(2)]$

α -like bound state only below $\rho_0/10$

α -condensate survives to $\approx 0.3 \text{ fm}^{-3}$

Decay of α -condensate at high densities

ii) Reduction of the condensate due to repulsive interaction

Penrose, Onsager 1956:

Off-diagonal long range order (ODLRO)

density matrix $\rho(\vec{r}, \vec{r}') at $|\vec{r} - \vec{r}'| \rightarrow \infty$:$

$$\rho(\vec{r}, \vec{r}') = \Psi(\vec{r})\Psi(\vec{r}') + \gamma(|\vec{r} - \vec{r}'|)$$

hard core: known solution (Rosenbluth)

Application to liquid ^4He :

filling factor $\approx 28\%$, condensate $\approx 8\%$

“excluded” volume for α particles $\approx 20 \text{ fm}^3$

Estimation of condensate fraction in zero temperature α -matter

$$n_0 = \frac{\langle \Psi | a_0^\dagger a_0 | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

destruction of the BEC of the ideal Bose gas:

thermal excitation, but also correlations

“excluded” volume for α -particles $\approx 20 \text{ fm}^3$
at nucleon density $\rho = 0.048 \text{ fm}^{-3}$ filling factor $\approx 28\%$
(liquid ^4He : 8% condensate),

destruction of the condensate at $\approx \rho_0/3$

Estimation of condensate fraction in zero temperature α -matter

α -cluster condensate in ^{12}C , ^{16}O :
 resonating group method
 occupation numbers of α -orbits in ^{12}C

	RMS radii	S-orbit	D-orbit	G-orbit
O_1^+ (g.s.)	2.44 fm	1.07	1.07	0.82
O_2^+	4.31 fm	2.38	0.29	0.16

80 % condensate at $1/8$ nuclear matter density

T. Yamada et al., P. Schuck: $(2.16 - \text{normal})/3 \approx 60\%$

Suppression of condensate fraction

Quasiparticle approximation for nuclear matter

Low-density limit: alpha matter?

Influence of cluster formation on the equation of state

Symmetry energy of a low density nuclear gas

- L. W. Chen, C. M. Ko, and B. A. Li,
Phys. Rev. Lett. **94**, 032701 (2005).
- T. Klähn *et al.*,
Phys. Rev. C **74**, 035802 (2006).
- C. J. Horowitz and A. Schwenk,
Nucl. Phys. **A 776**, 55 (2006).
- S. Kowalski *et al.*,
Phys. Rev. C **75**, 014601 (2007).

Symmetry energy and single nucleon potential used in the IBUU04 transport model

The x parameter is introduced to mimic various predictions by different microscopic Nuclear many-body theories using different Effective interactions

Single nucleon potential within the HF approach using a modified Gogny force:

$$U(\rho, \delta, \bar{p}, \tau, x) = A_u(x) \frac{\rho_\tau}{\rho_0} + A_l(x) \frac{\rho_\tau}{\rho_0} + B \left(\frac{\rho}{\rho_0} \right)^\sigma (1 - x \delta^2) - 8\tau x \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0^\sigma} \delta \rho_\tau \\ + \frac{2C_{\tau\tau}}{\rho_0} \int d^3 p' \frac{f_\tau(r, p')}{1 + (p - p')^2 / \Lambda^2} + \frac{2C_{\tau\tau'}}{\rho_0} \int d^3 p' \frac{f_{\tau'}(r, p')}{1 + (p - p')^2 / \Lambda^2}$$

$$\tau, \tau' = \pm \frac{1}{2}, A_l(x) = -121 + \frac{2Bx}{\sigma + 1}, A_u(x) = -96 - \frac{2Bx}{\sigma + 1}, K_0 = 211 \text{ MeV}$$

The momentum dependence of the nucleon potential is a result of the non-locality of nuclear effective interactions and the Pauli exclusion principle

C.B. Das, S. Das Gupta, C. Gale and B.A. Li, PRC 67, 034611 (2003).

B.A. Li, C.B. Das, S. Das Gupta and C. Gale, PRC 69, 034614; NPA 735, 563 (2004).

 $E_{\text{sym}}(\rho)$ predicted by microscopic many-body theories

EOS: $E(\rho, \delta) = E_0(\rho, 0) + E_{\text{sym}}(\rho) \delta^2 + o(\delta^4)$, where $\delta \equiv (\rho_n - \rho_p) / \rho$

$$E_{\text{sym}}(\rho) \approx E(\rho)_{\text{pure neutron matter}} - E(\rho)_{\text{symmetric nuclear matter}}$$

A.E. L. Dieperink, Y. Dewulf, D. Van Neck, M. Waroquier and V. Rodin, Phys. Rev. C68 (2003) 064307

Symmetry energy and symmetry free energy

Symmetry energy for $T=2,4,8$ MeV

Free and Internal Symmetry Energy

with (solid) and without (dashed) clusters

S. Typel, Talk 07

Symmetry energy

Approximations to the symmetry energy

a-d - Chen, Ko, Li '05
e - Kowalski et.al '06

S. Kubis,
Neutron stars with
non-homogeneous
core,
Talk 26.2.08, Ladek

Influence of cluster formation on the equation of state

T=10 MeV

Chemical
potential of
symmetric
nuclear matter.

Inclusion of
cluster formation
shifts down the
chemical potential.

The region of
thermodynamical
instability is reduced.

G.R., A. Grigo (2003)

Composition of symmetric nuclear matter

Low-density EoS and astrophysics

- H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi,
Progr. Theor. Phys. **100**, 1013 (1998); Nucl. Phys. **A637** 435 (1998).
- G. Röpke, A. Grigo, K. Sumiyoshi, and Hong Shen,
in: Superdense QCD Matter and Compact Stars, Ed. D. Blaschke and
A. Sedrakian, NATO Science Series, Springer, Dordrecht (2006),
pp. 75 - 91;
Physics of Particles and Nuclei Letters **2**, 275 (2005).
- J.M.Lattimer and F. D. Swesty,
Nucl. Phys. **A 535**, 331 (2001).
- C. J. Horowitz and A. Schwenk,
Nucl. Phys. **A 776**, 55 (2006).

Outline

- Schroedinger equation with medium corrections:
Self-energy and Pauli blocking
- Composition of the nuclear gas:
Generalized Beth-Uhlenbeck equation
- Quantum condensates:
Pairing and quartetting
- Alpha clustering in $4n$ nuclei
- Composition and the EoS of symmetric matter
- Symmetry energy in the low-density region

Summary

- The low-density limit of the nuclear matter EoS can be rigorously treated and has to be considered as benchmark.
- An extended quasiparticle approach can be given for single nucleon states and nuclei. In a first approximation, self-energy and Pauli blocking is included. An interpolation between low and high densities is possible.
- Compared with the standard quasiparticle approach, significant changes arise in the low-density limit due to clustering. Examples are Bose-Einstein condensation, and the behavior of the symmetry energy.

Thanks for attention