Lectures on the Near-Side Ridge, Landau Hydrodynamics, and Heavy Quarkonia in High Energy Heavy-Ion Collisions

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Abstract. We give an introduction to three different topics that are of current interest in heavy-ion collisions. Particles associated with the near-side jet are found to exhibit a $\Delta\phi$ - $\Delta\eta$ correlation in the form of a ridge in the $\Delta\eta$ direction but a peak at $\Delta\phi \sim 0$. The experimental data support the description that the ridge particles are medium partons kicked by the jet. The measurement of the characteristics of the ridge provides a unique tool to probe the nature of the (jet parton)-(medium parton) collision and the momentum distribution of dense matter formed in the early stage of the heavy-ion collision. We find that the magnitude of the longitudinal momentum kick along the jet direction acquired by a medium parton in collision with the jet is about 1 GeV, and the early parton momentum distribution is in the form of a rapidity plateau with a thermal-type transverse momentum distribution. In the second lecture, we re-examine the validity of Landau hydrodynamics which provides a reasonable description of the space-time dynamics of the hot matter produced in high-energy heavy-ion collisions. We find that the rapidity distribution of produced particles should be more appropriately modified from Landau's result. Past successes of the Gaussian distribution in explaining experimental rapidity data can be understood, not because it is an approximation of the original Landau distribution, but because it is in fact a close representation of the modified distribution. In the final lecture, we give an introduction to the development of the potential model for quarkonia, using thermodynamical quantities obtained in lattice gauge calculations. We find that the potential model is consistent with the lattice gauge spectral function analysis, if the color-singlet heavy quark-antiquark potential is a linear combination of the color-singlet free energy F_1 and internal energy U_1 , with coefficients that depend on the equation of state. We find that the effects of dynamical quark modifies only slightly the stability of J/ψ and the quark drip line limits possible quarkonium states with light quarks to temperatures close to, and slightly greater than, the critical phase transition temperature.

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Near-Side Ridge and Early Parton Momentum Distribution

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- Introduction
 - ▶ the jet, near-side jet, away-side jet, and the ridge
 - ▶ how does the ridge arise
- The relation between the early parton momentum distribution & the ridge distribution
- Use the experimental ridge distribution to extract the early parton momentum distribution
- Conclusions
 - C.Y.Wong, Phy.Rev.C76,054908('07)
 - C.Y.Wong, Chin.Phys.Lett.25,3936('08)
 - C.Y.Wong, J.Phys.G35,104085('08)
 - C.Y.Wong, Phys. Rev.C78,064905('08)
 - C.Y.Wong, arXiv:0901.0726(PRC,in press)₁
 - C.Y.Wong, arXiv:0903.3879('09)

Why study the early parton momentum distribution?

- Early parton momentum distribution is an important quantity
- It provides information in early collision dynamics
- It provides initial value data for subsequent evolution to quark-gluon plasma
- Not much is known experimentally about early parton momentum distribution
- We propose to use the ridge distribution to extract early parton momentum distribution

Jet production

Jets are particles with high transverse momenta produced by nucleon-nucleon hard-scattering process.

3

Jet properties

► Jets are particles with high transverse momenta.

► Jets come as a back-to-back pair

- A jet is nearly degenerate with its fragments
- The jet fragments form a jet cone

4

The term “jet” are often used for different objects:

- jet parton (the totality of all jet fragments)
- trigger particle (one of the jet fragments)
- fragments in the jet cone (not including the trigger)

Ambiguity can be removed by judging from the context.

5

Near-side and away-side jets

Away-side jet is subject to strong absorption.
Absorption is strongest in the most-central collision

6

What is the ridge phenomenon?

- Occurrence of a near-side “jet”
- Associated particles are observed in coincidence with the jet
- The momentum and angles of these particles are measured:

$p_t(\text{particle})$

$\Delta\phi = \phi(\text{particle}) - \phi(\text{jet})$

$\Delta\eta = \eta(\text{particle}) - \eta(\text{jet})$

Find: $\Delta\phi$ - $\Delta\eta$ correlation

Probability distribution $P(\Delta\phi, \Delta\eta)$ in $\Delta\phi$ - $\Delta\eta$ is in the form of

- a “jet component”
- a “ridge component”.

7

Experimental information about the ridge

- (i) Ridge yield correlated with $N_{\text{participants}}$
- (ii) Ridge yield nearly independent of p_t , flavor, baryon, meson characters of the jet trigger
- (iii) $T_{\text{jet}} \gg T_{\text{ridge}} > T_{\text{inclusive}}$
- (iv) Ridge particles have $\Delta\phi \sim 0$
- (v) Ridge particles nearly uniform in $\Delta\eta$

8

Many Ridge Models

- C.Y.Wong, Phys.Rev.C76,054908('07);Chin.Phys.Lett.25,3936('08);Phys.Rev.C78,064905('08);J. Phys.G35,104085('08);arXiv:0901.0726(PRC,in press);arXiv:0903.3879.
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- S.Gavin, L.McLerran, G.Moschelle, arXiv:0806.4718
- A. Dumitru, F. Gelis, L. McLerran, and R. Venugopalan, Nucl. Phys. A810, 91 ('09)
- •••• many more

9

Questions

- How does the ridge phenomenon occur?
- What is the momentum distribution of the early medium partons?
- What is the dominant mechanism of jet momentum loss?

These questions are linked together and can be answered by the momentum kick model:

Ridge particles are medium partons kicked by the jet. The kicked partons carry direct information on the medium parton momentum distribution and the magnitude of the momentum kick. The momentum kick is related to the jet momentum loss.

10

Schematic picture of the momentum kick model

11

Ridge particles are medium partons kicked by the jet

- (i) Ridge yield correlated with $N_{\text{participants}}$
- (ii) Ridge yield nearly independent of p_t trigger, flavor, baryon, meson characters of the jet
- (iii) $T_{\text{jet}} \gg T_{\text{ridge}} \geq T_{\text{inclusive}}$
- (iv) $\Delta\phi \sim 0$ implies that the ridge particles acquire their azimuthally properties from the jet
- (v) jet-(medium parton) interactions are short-ranged because of non-perturbative screening

The most likely explanation:

ridge particles are medium partons kicked by the jet and they acquire a momentum kick q along the jet direction

12

Basic idea of the momentum kick model

Normalized initial medium parton momentum distribution $dF / d\vec{p}_i$

$$\text{before kick} = \frac{dF}{d\vec{p}_i}(\vec{p}_i), \quad \int d\vec{p}_i \frac{dF}{d\vec{p}_i}(\vec{p}_i) = 1$$

$$\text{after kick} = \left[\frac{dF}{d\vec{p}_i}(\vec{p}_i) \right]_{\vec{p}_i = \vec{p}_f - \vec{q}}$$

A single jet kicks $\langle N \rangle$ medium partons

$$\text{Ridge particle momentum distribution} = \langle N \rangle \left[\frac{dF}{d\vec{p}_i}(\vec{p}_i) \right]_{\vec{p}_i = \vec{p}_f - \vec{q}}$$

We observed only charge hadrons,,

$$\frac{dN_{ridge}^{AA}}{d\vec{p}_f}(\vec{p}_f) \text{ per trigger} = \frac{2}{3} \langle N \rangle \left[\frac{dF}{d\vec{p}_i}(\vec{p}_i) \right]_{\vec{p}_i = \vec{p}_f - \vec{q}}$$

A ridge particle is attenuated by a factor f_R

$$\frac{dN_{ridge}^{AA}}{N_{trig} d\vec{p}_f}(\vec{p}_f) = \frac{2}{3} f_R \langle N \rangle \left[\frac{dF}{d\vec{p}_i}(\vec{p}_i) \right]_{\vec{p}_i = \vec{p}_f - \vec{q}}$$

13

- Ridge yield per trigger depends on the number of kicked medium particles, $\langle N \rangle$.

14

Mathematical details

$$\text{trigger yield in pp} = \frac{dN^{pp}}{d\vec{p}_{trig}}(\vec{p}_{trig}) = \int d\vec{p}_j \frac{dN_j^{pp}}{d\vec{p}_j} \tilde{D}(\vec{p}_{trig}, \vec{p}_j)$$

We introduce $D(\vec{p}_{trig}, \vec{p}_j)$,

$$1 = \int d\vec{p}_j \frac{dN_j^{pp}}{d\vec{p}_j} D(\vec{p}_{trig}, \vec{p}_j)$$

$$D(\vec{p}_{trig}, \vec{p}_j) = \tilde{D}(\vec{p}_{trig}, \vec{p}_j) \left/ \frac{dN^{pp}}{d\vec{p}_{trig}}(\vec{p}_{trig}) \right.$$

jet source in an AA collision :

$$\frac{dN_j^{AA}}{d\vec{p}_j}(\vec{p}_j) = \sum_{i=1}^{N_{bin}} \frac{dN_j^{i(pp)}}{d\vec{p}_j}(\vec{p}_j)$$

$$N_{bin} = \int d\vec{p}_j \frac{dN_j^{AA}}{d\vec{p}_j}(\vec{p}_j) D(\vec{p}_{trig}, \vec{p}_j)$$

$$N_{trig} = \int d\vec{p}_j \frac{dN_j^{AA}}{d\vec{p}_j} \sum_{N=0}^{N_{max}} P(N) e^{-\zeta_a N} D(\vec{p}_{trig}, \vec{p}_j - \sum_{n=1}^N \vec{q}_n - \vec{\Delta}_r)$$

$$R_{AA} = \frac{1}{N_{bin}} \int d\vec{p}_j \frac{dN_j^{AA}}{d\vec{p}_j} \sum_{N=0}^{N_{max}} P(N) e^{-\zeta_a N} D(\vec{p}_{trig}, \vec{p}_j - \sum_{n=1}^N \vec{q}_n - \vec{\Delta}_r)$$

15

Simplification for N_{trig} and R_{AA}

$$N_{trig} = \int d\vec{p}_j \frac{dN_j^{AA}}{d\vec{p}_j} \sum_{N=0}^{N_{max}} P(N) e^{-\zeta_a N} D(\vec{p}_{trig}, \vec{p}_j - \sum_{n=1}^N \vec{q}_n - \vec{\Delta}_r)$$

$$R_{AA} = \frac{1}{N_{bin}} \int d\vec{p}_j \frac{dN_j^{AA}}{d\vec{p}_j} \sum_{N=0}^{N_{max}} P(N) e^{-\zeta_a N} D(\vec{p}_{trig}, \vec{p}_j - \sum_{n=1}^N \vec{q}_n - \vec{\Delta}_r)$$

The fragmentation function can be written approximately as

$$D(\vec{p}_{trig}, \vec{p}_j - \sum_{n=1}^N \vec{q}_n - \vec{\Delta}_r) = e^{-(\zeta_f + \zeta_r)N} D(\vec{p}_{trig}, \vec{p}_j)$$

Then

$$N_{trig} = \int d\vec{p}_j \frac{dN_j^{AA}}{d\vec{p}_j} D(\vec{p}_{trig}, \vec{p}_j) \sum_{N=0}^{N_{max}} P(N) e^{-(\zeta_a + \zeta_f + \zeta_r)N}$$

$$N_{trig} = N_{bin} \sum_{N=0}^{N_{max}} P(N) e^{-\zeta N}; \quad \zeta = \zeta_a + \zeta_f + \zeta_r.$$

$$R_{AA} = \sum_{N=0}^{N_{max}} P(N) e^{-\zeta N}$$

16

How to calculate the ridge yield per trigger?

$$\frac{dN_{ridge}^{AA}}{d\vec{p}} = \int d\vec{p}_j \frac{dN_j^{AA}}{d\vec{p}_j} \sum_{N=0}^{N_{max}} P(N) e^{-\zeta_a N} D(\vec{p}_{trig}, \vec{p}_j - \sum_{n=1}^N \vec{q}_n - \vec{\Delta}_r) \times \frac{2}{3} \sum_{n=1}^N \left\{ f_{Rn} \frac{dF_n}{d\vec{p}}(\vec{q}_n) \right\}$$

Result implied by using the average ridge yeild per kick

$$\frac{dN_{ridge}^{AA}}{d\vec{p}} = \frac{2}{3} f_R \frac{dF}{d\vec{p}}(\vec{q}) \times \int d\vec{p}_j \frac{dN_j^{AA}}{d\vec{p}_j} \sum_{N=0}^{N_{max}} NP(N) e^{-\zeta_a N} D(\vec{p}_{trig}, \vec{p}_j - \sum_{n=1}^N \vec{q}_n - \vec{\Delta}_r)$$

$$\frac{dN_{ridge}^{AA}}{N_{trig} d\vec{p}} = \frac{2}{3} f_R \langle N \rangle \frac{dF}{d\vec{p}}(\vec{q})$$

$$\langle N \rangle = \frac{\int d\vec{p}_j \frac{dN_j^{AA}}{d\vec{p}_j} \sum_{N=0}^{N_{max}} NP(N) e^{-\zeta_a N} D(\vec{p}_{trig}, \vec{p}_j - \sum_{n=1}^N \vec{q}_n - \vec{\Delta}_r)}{\int d\vec{p}_j \frac{dN_j^{AA}}{d\vec{p}_j} \sum_{N=0}^{N_{max}} P(N) e^{-\zeta_a N} D(\vec{p}_{trig}, \vec{p}_j - \sum_{n=1}^N \vec{q}_n - \vec{\Delta}_r)}$$

17

Relation between the ridge distribution and the early parton momentum distribution

The normalized ridge particle distribution in the collider frame :

$$\frac{dF}{d\eta d\phi p_t dp_t} = \left[\frac{dF}{dy_i d\phi_i p_{ti} dp_{ti}} \frac{E}{E_i} \right]_{\vec{p}_i = \vec{p}_f - q\vec{e}_{jet}} \sqrt{1 - \frac{m^2}{m_t^2 \cosh^2 y}}$$

- The kicked final partons subsequently materialize as hadrons by parton-hadron duality
- The ridge particle distribution depends on the initial parton momentum distribution and the momentum kick q.

18

Parametrization of initial parton momentum distribution

We parametrize the normalized initial parton distribution by

$$\frac{dF}{dy d\phi p_t dp_t} = A_i (1-x)^a \frac{\exp\left\{-\left(\sqrt{m^2 + p_t^2} - m\right)/T\right\}}{\sqrt{m_d^2 + p_t^2}}$$

$$x = \frac{(p_0 + p_z)_{parton}}{(p_0 + p_z)_{parent}} = \frac{\sqrt{m^2 + p_t^2}}{m} \exp\{|\gamma| - \gamma_b\} \leq 1$$

A_i is a normalization constant such that $\int d\vec{p} dF / d\vec{p} = 1$.

The parameters are :

$$a, T, m_d, m, \text{ and } \gamma_b$$

We assume parton - hadron duality and take $m = m_{parton} = m_{pion}$

$$\gamma_b = \gamma_{beam\ proton}$$

19

Ridge yield is a maximum at $\Delta\phi \sim 0$

For the same magnitude of $|\vec{p}_{tf}|$
but different $\Delta\phi$, the magnitude of
 $|\vec{p}_{ti}|$ is smallest when $\Delta\phi = 0$.

The smaller the magnitude of initial transverse momentum

$|\vec{p}_{ti}|$, the greater the probability of its occurrence.

Therefore, the ridge particle yield is a maximum at $\Delta\phi \approx 0$.

20

The width in $\Delta\phi$ depends on the magnitude of q .

21

The ridge shape in $\Delta\eta$

initial parton $dN/dy \sim (1-x)^a$

Boost invariant distribution corresponds to $a=0$.

The shape in $\Delta\eta$ around $\Delta\eta=0$ depends weakly on a .

The shape in $\Delta\eta$ around large $\Delta\eta$ depends strongly on a .

22

Experimental measurement contains both jet and ridge components

- We need to get a good description of the “jet” component
 - The jet component is produced mainly by fragmentation outside the medium. It is the same as in pp collisions for high pt jet.
 - We assume that the jet component is an attenuated pp
- (jet component in AA) = f_j (jet component in pp)
- We need to parameterize pp jet data.

23

The observed distribution in the momentum kick model

$$\left. \frac{dN_{ch}}{d\eta d\phi p_t dp_t} \right|_{AA, total}^{observed} = \frac{2}{3} f_R \langle N \rangle \left. \frac{dF}{d\eta_f d\phi_f p_{tf} dp_{tf}} \right|_{AA, ridge} + f_j \left. \frac{dN_{jet}}{d\eta_f d\phi_f p_{tf} dp_{tf}} \right|_{pp, jet}$$

f_R and f_j are the attenuation factors

$$\text{we take } f_j = \int_0^\lambda \exp\{-x / \lambda\} dx / \int_0^\lambda dx = \frac{1 - e^{-1}}{1} = 0.632$$

We need the pp near-side jet data

24

The pp near-side jet data can be described by

$$\left. \frac{dN_{jet}}{dy d\phi p_t dp_t} \right|_{pp} = N_{jet} A_{jet} \exp \left\{ -\sqrt{m^2 + p_t^2} / T_{jet} \right\} \exp \left\{ -\frac{((\Delta\phi)^2 + (\Delta\eta)^2) / 2\sigma^2}{2\pi\sigma^2} \right\}$$

$$N_{jet} = 0.75, \quad A_{jet} = \frac{\exp\{m/T_{jet}\}}{T_{jet}(m + T_{jet})} \text{ is a normalization constant,}$$

$$T_{jet} = 0.55 \text{ GeV,}$$

$$\sigma = \sigma_0 \frac{m_a}{\sqrt{m_a^2 + p_t^2}}, \quad \sigma_0 = 0.5, \quad m_a = 1.1 \text{ GeV}$$

25

pp near-side jet data (open blue circles)

Data from
PRL95,152301(05) & J. Phy. G34, S679 (07)

26

The initial parton momentum distribution

$$\frac{dF}{dyd\phi p_t dp_t} = A_i (1-x)^a \frac{\exp\left\{-\left(\sqrt{m^2 + p_t^2} - m\right)/T\right\}}{\sqrt{m_d^2 + p_t^2}}$$

$$x = \frac{\sqrt{m^2 + p_t^2}}{m_b} \exp\{|\gamma| - \gamma_b\} \leq 1$$

Assuming $m_b = m_\pi = m$, $\gamma_b = \gamma_{beam}$, we find

$$q = 1.0 \text{ GeV}, \quad \frac{2}{3} f_R \langle N \rangle = 2.5, \quad a = 0.5, \quad T = 0.5 \text{ GeV}, \quad m_d = 1 \text{ GeV}.$$

27

AA near-side data (black solid points) described well by the momentum kick model around $\Delta\eta \sim 0$

Data from STAR Collaboration
PRL95,152301(05) & J. Phy. G34, S679 (07)

28

AA near-side data (black solid points) described well by the momentum kick model around $|\Delta\eta|\sim 3.3$

Data from STAR Col.
 F. Wang et al.
 arXiv:0707.0815 ('07)

29

Momentum kick model gives the correct prediction

30

Shape of early parton momentum distribution

31

Possible evolution scenario of medium partons

32

Conclusions

- The ridge particles can be described as medium partons kicked by the jet, and they carry information on the early parton momentum distribution and the momentum kick.
- The parton momentum distribution at the moment of jet-parton collision is relatively flat in rapidity with a thermal-like transverse momentum distribution and sharp kinematic boundaries.
- The magnitude of the momentum kick gained by the parton is 1 GeV, which is also the momentum loss by the jet in a jet-parton collision.

33

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Landau Hydrodynamics

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- Introduction
- Landau hydrodynamics
 - predictions on total N
 - prediction on differential dN/dy
 - space-time dynamics
- Modification of Landau's dN/dy
- Comparison with experiment
- Conclusions

C.Y. Wong, arXiv:0809.0517('08)
C.Y.Wong, Phys.Rev.C78,054902('08)

1

Why study Landau hydrodynamics?

- L.D. Landau, Izv. Akad. Nauk SSSR, 17, 51 (1953)
- Belenkij and L.D.Landau,Usp.Fiz.Nauk. 56, 309 (1955)
- It gives a good description of experimental data
- It gives a simple description of the space-time dynamics of the dense hot matter produced in heavy-ion collisions
- Dense hot matter evolution is needed in many problems
- It is very simple

2

Recent revived interest in Landau hydrodynamics

- BRAHM dN/dy data agree with Landau hydrodynamics

$$\frac{dN}{dy} = Ks^{1/4} \frac{1}{\sqrt{2\pi L}} \exp\left\{-\frac{y^2}{2L}\right\}$$

$$L = \ln\left(\frac{\sqrt{s_{NN}}}{2m_p}\right)$$

L is the logarithm of the
Lorentz contraction factor

Murray, J. Phys. G30, S667 (2004)

3

Landau hydrodynamics give the correct multiplicity in AA collisions

4

Landau hydrodynamics exhibits limiting fragmentation

Steinberg, arxiv:nucl-ex/0702019

Unanswered questions:

- Is Landau's formula for y or η ?
- Landau rapidity distribution is actually

$$\frac{dN}{dy} = K s^{1/4} A_{norm} \exp\left\{\sqrt{L^2 - y^2}\right\}$$

The Gaussian Landau distribution is only an approximation

- Does the original Landau distribution agree with data?

We need to answer these questions in Landau hydrodynamics

6

Initial configuration

7

Landau Hydrodynamics

- Local thermal equilibrium
- Particle number is proportional to entropy
- Entropy is conserved in Landau hydrodynamics
- First stage, independent
 - (i) 1-D longitudinal expansion
 - (ii) transverse expansion
- Second stage of rapidity freeze-out
when the transverse displacement is equal
to the transverse dimension of the system

$$x(t_m) = a ; \quad t_m = \text{rapidity freeze-out time}$$

$$\frac{dN}{dy} = K s^{1/4} A_{norm} \exp\left\{\sqrt{L^2 - y^2}\right\} \quad K \approx 2.$$

8

Conservation of Entropy in Landau hydrodynamics

- Mean-free path is small, low viscosity
- The only means to destroy entropy conservation is shock wave.
- Landau hydrodynamics is applied after the completion of the compression stage. It deals only with the expansion of the dense matter after shock-wave compression

9

Landau hydrodynamics predictions on total N_{ch}

- ⊗ Hydrodynamical flow is isentropic (entropy conserving).
- ⊗ Entropy S is proportional to particle number N
- ⊗ Conservation of entropy means that we can get the total number of particles from the initial entropy.

$$\text{Initial total energy} = E = \left(\sqrt{s_{NN}} / 2 \right) 2A = \sqrt{s_{NN}} A$$

$$\text{Initial volume} = V = \frac{4\pi}{3} r_0^3 A / \gamma; \quad \gamma = \frac{\sqrt{s_{NN}}}{m_p}$$

$$\text{Energy density} = \varepsilon = \frac{E}{V} = \gamma \sqrt{s_{NN}} / \frac{4\pi}{3} r_0^3$$

$$\text{Entr opy density} = \sigma = c\varepsilon^{3/4} = c \left(\gamma \sqrt{s_{NN}} / \frac{4\pi}{3} r_0^3 \right)^{3/4}$$

$$\text{Total entropy} = \sigma V = c \left(\gamma \sqrt{s_{NN}} / \frac{4\pi}{3} r_0^3 \right)^{3/4} \frac{4\pi}{3} r_0^3 A / \gamma \\ \propto \left(\sqrt{s_{NN}} \right)^{1/2} A$$

$$\text{Total number of particles} = N \propto \left(\sqrt{s_{NN}} \right)^{1/2} A$$

$$N_{ch} / (N_{part} / 2) = K \left(\sqrt{s_{NN}} \right)^{1/2} \quad K \approx 2 \quad \text{for} \quad \sqrt{s_{NN}} \text{ in GeV.}$$

10

Landau prediction on $N_{ch} / (N_{part} / 2)$ agrees with experiment

11

1-D longitudinal expansion

Hydrodynamic equations

$$\frac{\partial T^{00}}{\partial t} + \frac{\partial T^{01}}{\partial z} = 0$$

$$\frac{\partial T^{01}}{\partial t} + \frac{\partial T^{11}}{\partial z} = 0$$

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}$$

We use simple equation of state

$$p = \varepsilon / 3$$

$$\text{Introduce } u^0 = \cosh y \quad u^1 = \sinh y$$

$$t_+ = t + z \quad t_- = t - z$$

Introduce logarithm of $t_\pm$

$$y_\pm = \ln(t_\pm / \Delta)$$

12

1-D longitudinal expansion solution

Hydrodynamic equations becomes

$$\frac{\partial \varepsilon}{\partial t_-} + 2 \frac{\partial(\varepsilon e^{-2y})}{\partial t_-} = 0$$

$$2 \frac{\partial(\varepsilon e^{+2y})}{\partial t_+} + \frac{\partial \varepsilon}{\partial t_-} = 0$$

Simple approximate solution

$$\begin{cases} \varepsilon = \varepsilon_0 \exp\left\{-\frac{4}{3}(y_+ + y_- - \sqrt{y_+ y_-})\right\} \\ y = (y_+ - y_-) / 2 \end{cases}$$

We can prove that this is an approximate solution by direct substitution.

Advantage: simplicity

Minor disadvantages :

- (i) only for the bulk part of fluid, not the edge part
- (ii) limited choices of initial conditions

13

Solution of 1-D longitudinal expansion

14

Transverse expansion during the first stage

Hydrodynamic equation
$$\frac{\partial T^{02}}{\partial t} + \frac{\partial T^{02}}{\partial x} = 0$$

$$T^{02} = (\varepsilon + p)u^0 u^2 = \frac{4}{3}u^0 u^0 v_x; \quad u^2 = u^0 v_x$$

$$T^{22} = (\varepsilon + p)u^2 u^2 - pg^{22} \approx g$$

Hydrodynamic equation becomes

$$\frac{4}{3}\varepsilon u^0 u^0 \frac{\partial v_x}{\partial t} = -\frac{\partial p}{\partial x} = -\frac{1}{3}\frac{\partial \varepsilon}{\partial x}$$

Transverse displacement is

$$x(t) = \frac{1}{2} \frac{\partial v_x}{\partial t} t^2; \quad \frac{\partial v_x}{\partial t} = \frac{2x(t)}{t^2}$$

Hydrodynamic equation is

$$\frac{4}{3}\varepsilon u^0 u^0 \frac{2x(t)}{t^2} = \frac{1}{3} \frac{\varepsilon}{a}$$

$$x(t) = \frac{t^2}{4a u^0 u^0} = \frac{t^2}{4a \cosh^2 y}$$

15

Landau condition for rapidity freeze-out

Angle (and rapidity) will not change after the transverse displacement is equal to the transverse dimension.

$$x(t) = \frac{t^2}{4a \cosh^2 y} = a = \text{transverse diameter}$$

Rapidity freeze-out occurs at different time t for different rapidity (and z).

16

Rapidity distribution

Rapidity distribution at time t

$$dS = \sigma u^0 dz \quad z = t \sinh y / \cosh y$$

$$dS = \sigma u^0 t dy / \cosh^2 y = \sigma t dy / \cosh y$$

$$\sigma = c\epsilon^{3/4}$$

$$dS = c\epsilon_0^{3/4} \exp\left\{-(y_+ + y_- - \sqrt{y_+ y_-})\right\} t \frac{dy}{\cosh y}.$$

We need the rapidity distribution at $t = t_m(y)$

$$dS = c\epsilon_0^{3/4} \left[\exp\left\{-(y_+ + y_- - \sqrt{y_+ y_-})\right\} t \right]_{t=t_m(y)} \frac{dy}{\cosh y}$$

$$y_{\pm} = \ln\left(\frac{2a}{\Delta}\right) \pm y = y_b \pm y$$

Final entropy distribution

$$dS \propto \exp\left\{-\left(2y_b - \sqrt{y_b^2 - y^2}\right)\right\} dy$$

$$N \propto S$$

$$\text{Final rapidity distribution : } \frac{dN}{dy} \propto \exp\left\{\sqrt{y_b^2 - y^2}\right\}$$

17

Comparison with experiment

Modified distribution gives better agreement than Landau distribution

18

Comparison with experiment

Modified distribution gives better agreement than Landau distribution

19

Conclusions

- Landau's prediction on N_{ch} agrees with data
- The rapidity distribution in Landau hydrodynamics should be modified. The modified rapidity distribution gives better agreement with experimental data than the Landau distribution
- The quantitative agreement of Landau hydrodynamics supports its use in other problems of heavy ion collisions, such as J/ψ suppression, jet quenching, and ridge jet-medium interaction,....

20

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Quarkonia in Quark-Gluon Plasma

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- Introduction
- Potential model of quarkonia in QGP using lattice gauge thermodynamic quantities
- Stability of quarkonia in QGP
- Effect of dynamical quarks on quarkonia stability
- Quark drip lines for quarkonia with light quarks
- Conclusions

C.Y.Wong,E.S.Swanson,T.Barnes,Phys.Rev.C65,014903('02)

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C.Y.Wong,H.Crater,Phys.Rev.D75,034505('07)

Why study quarkonia in quark-gluon plasma ?

- Heavy quarkonia may be used as a plasma diagnostic tool
- Successes of the recombination model
- Successes of the thermal mode
 - quarkonia with light quarks may be stable in quark-gluon plasma, at least near T_c

Q-Qbar interaction in Quarkonia

- The screening between Q and Qbar in QGP is non-perturbative in nature and must be studied non-perturbatively.
- Need lattice gauge theory
- Need to understand the results from lattice gauge theory
- Need to study dynamical effects in screening, response and relaxation of the medium to Q-Qbar motion

What kind of framework in lattice gauge theory?

- Lattice is a subsystem in a much large thermal bath
- Subsystem is in constant contact with bath
- In the grand-canonical ensemble (which is used in the lattices gauge calculations), the subsystem particle numbers and energy of the subsystem are adjusted to maintain the same temperature as the bath

Types of lattice calculations

- Polyakov lines $\langle \text{tr}(L(r/2)L^+(-r/2)) \rangle$

- Meson correlators

$$\langle \psi_1(x_1) O \psi_2(x_2) \psi_1^+(x_1') O \psi_2^+(x_2') \rangle$$

Gauge invariant correlator

$$\langle \psi_1(\vec{0}, t) O \psi_2(\vec{0}, t) \psi_1^+(\vec{0}, 0) O \psi_2^+(\vec{0}, 0) \rangle$$

Why study heavy quarkonia in a hot medium?

- Two new surprising results from lattice gauge calculations
 - Lattice spectral function analyses in quenched QCD show that J/ψ is stable up to $1.6T_c$
 - Lattice static $Q\bar{Q}$ “potential” appears to be very strong between 1 and 2 T_c
 - Shuryak, Zahed, Brown, Lee, and Rho suggested that even light quarkonia may be bound in quark-gluon plasma
- We need
 - to confirm these lattice gauge results
 - to study effects of dynamical quarks on J/ψ stability
 - to assess the strength of the $Q\bar{Q}$ potential
 - to examine the stability of quarkonia

Lattice gauge spectral analyses in the quenched approximation show that the width of J/ψ remains narrow up to $T \leq 1.6 T_c$

M. Asakawa, T. Hatsuda, and Y. Nakahara, Nucl. Phys. A715, 863 (03)

S. Datta, F. Karsch, P. Petreczky, and I. Wetzorke, Phys. Rev. D69,094507(04)

The drastic change of the spectral function between $1.62\text{-}1.70T_c$ suggests the occurrence of spontaneous dissociation at $1.62\text{-}1.70T_c$.

Why mass of J/psi roughly constant of T?

- In Debye screening, the screening potential is

$$V(r) = -\frac{\alpha \exp\{-\mu r\}}{r} - \frac{\alpha}{r_D} = -\frac{\alpha \exp\{-\mu r\}}{r} - \alpha\mu$$

The mass of the system is constant to first order in μ

Questions:

- 1 What does the potential model say about the J/ ψ spontaneous dissociation temperature?
- 2 What are the effects of dynamical quarks?
- 3 What is the strength of interaction between a static quark and antiquark?

Kaczmarek et al. calculated the color-singlet F_1 and U_1 in the quenched approximation [hep-lat/0309121]

$$\langle \text{Tr} L(r) L^\dagger(0) \rangle = e^{-F_1(r,T)/T}$$

$$U_1(r,T) = F_1(r,T) - T \frac{\partial F_1(r,T)}{\partial T}$$

$$U_1(r,T) = F_1(r,T) + T S_1(r,T)$$

$F_1(r,T)$ was calculated in the Coulomb gauge

U_1 is much deeper than F_1 and can hold many more bound states

What is the $Q\bar{Q}$ potential?

Initial conjectures:

1. $F_1(r,T)$ is the $Q\bar{Q}$ potential

(Digal et.al '01, Wong '02, Blaschke et.al 05)

2. $U_1(r,T)$ is the $Q\bar{Q}$ potential

$$U_1(r,T) = F_1(r,T) + T S_1(r,T)$$

(Kaczmarek et al. '02, Shuryak et al '04)

To extract $Q - \overline{Q}$ potential
from lattice gauge F_1 and U_1 ?

- We need to understand the meaning of U_1 .
- We need to understand the meaning of $TS_1 = U_1 - F_1$.

O. Kaczmarek et al. hep-lat/0506019

Why such behavior?

Thermodynamical Quantities U_1, S_1, F_1

- Internal energy U_1 (kinetic plus potential energies)
- Entropy S_1 (the degree of different ordering)

$$S_1 = -\sum \{ n_i \ln n_i - (1 + n_i) \ln (1 + n_i) \}$$

TS_1 is the energy associated with entropy

- Free energy $F_1 = U_1 - TS_1$
(the internal energy minus the energy associated with entropy)

Thermodynamical Quantities

- Internal energy U (kinetic plus potential energies)
- Entropy S (the degree of different ordering)

$$S = \sum \{ n_i \ln n_i - (1 + n_i) \ln (1 + n_i) \}$$

TS is the energy associated with entropy

- Free energy $F = U - TS$
(the internal energy minus the energy associated with entropy)

Equilibrium condition

- $dU = T dS - P dV$
Define free energy $F = U - TS$, we get
 $dF = - SdT - PdV$
If the temperature is held fixed (thermal bath) and the volume does not change locally, then F is a constant.
- In a constant temperature environment, equilibrium is reached when $dF=0$,
i.e. the variation of F with respect to all degrees of freedom is zero in a thermal bath.

Entropy increases as Q and Qbar move away from each other

The number of gluons increases as Q and Qbar move away from each other because the first order of screening cancels but the second order change of gluon number always increases.

We want Q-Qbar interaction without changing the gluon internal energy.

We therefore need to subtract out the increases in gluon energy when Q and Qbar changes separations.

We find in this procedure that the $Q-\bar{Q}$ potential is

$$U_{Q\bar{Q}}^{(1)} = U_1 - U_g$$

By the first law of thermodynamics,

$$\frac{dU_g}{dV} = \varepsilon = T \frac{dS_g}{dV} - p \quad (S_g \text{ is the gluon entropy})$$

$$p = \frac{3p}{\varepsilon} \frac{dU_g}{3dV} = a(T) \frac{dU_g}{3dV}$$

$a(T) = 3p/\varepsilon$ is known from the equation of state

$$\text{So, } \frac{dU_g}{dV} = \frac{3}{3+a} T \frac{dS_g}{dV},$$

$$\text{and } U_g = \frac{3}{3+a} TS_g. \quad \text{But } TS_g = U_1 - F_1$$

$$\text{Hence, } U_{Q\bar{Q}}^{(1)} = U_1 - U_g = U_1 - \frac{3}{3+a} (U_1 - F_1)$$

$$U_{Q\bar{Q}}^{(1)} = \frac{3}{3+a} F_1 + \frac{a}{3+a} U_1$$

F₁ and U₁ fractions depend on T

Solve for Q- $\bar{Q}$ bound states

$$\left\{ -\frac{\nabla^2}{2\mu_{\text{red}}} + \begin{bmatrix} U_{00}^{(1)}(r, T) - U_{00}^{(1)}(r \rightarrow \infty, T) \\ F_1(r, T) - F_1(r \rightarrow \infty, T) \\ U_1(r, T) - U_1(r \rightarrow \infty, T) \end{bmatrix} \right\} \psi(r, T) = \varepsilon(T) \psi(r, T)$$

Spontaneous dissociation temperatures in quenched QCD

Heavy Quarkonium	Spectral Analysis	$U_{\varrho\bar{\varrho}}^{(1)}$ Potential	F_1 Potential	U_1 Potential
J/ψ	1.62-1.70 T_C	1.62 T_C	1.40 T_C	2.60 T_C
χ_c, ψ'	below 1.1 T_C	unbound	unbound	1.18 T_C
Y		4.10 T_C	3.50 T_C	$\sim 5.0 T_C$
χ_b	1.15-1.54 T_C	1.19 T_C	1.10 T_C	1.73 T_C

The comparison of the spontaneous dissociation temperatures in quenched QCD indicates that the $U_{\varrho\bar{\varrho}}^{(1)}$ potential,

$$U_{\varrho\bar{\varrho}}^{(1)} = \frac{3}{3+a} F_1 + \frac{a}{3+a} U_1,$$

may be the appropriate $Q\bar{Q}$ potential to study quarkonium stability.

We may also use the $U_{\varrho\bar{\varrho}}^{(1)}$ potential in full QCD to study quarkonium stability.

Quenched QCD & Full QCD

- Quenched QCD is inadequate as it neglects the effects of dynamical quarks
- We need to study full QCD with dynamical quarks
- Kaczmarek et al. (PRD 71, 114510 '05) have obtained F_1 and U_1 in full QCD (with 2 flavors)
- Karsch et al. has obtained $a(T)$ (PLB 478, 447).
- We can use the potential model to study the stability of quarkonium in full QCD

$$F_1(r) = -\frac{4}{3} \frac{\alpha_s}{r} f(r) + C(T)[1 - f(r)]$$

Full QCD with two flavors
Kaczmarek et al ('05)

$$f(r) = \frac{1}{\exp\{(r - r_0(T))/d(T)\} + 1}, \quad \alpha_s = 0.3,$$

Spontaneous dissociation temperatures in quenched QCD & full QCD

Heavy Quarkonium	Quenched QCD $U_{\varrho\bar{\varrho}}^{(1)}$	2-flavor QCD $U_{\varrho\bar{\varrho}}^{(1)}$	Spectral Analysis Quenched QCD
J/ψ	1.62T _C	1.42T _C	~ 1.6 T _C
χ _c , ψ '	unbound	unbound	below 1.1 T _C
Y	4.10T _C	3.30 T _C	
χ _b	1.18T _C	1.22T _C	1.15-1.54

Recent lattice calculations in

J/ψ appears to have narrow width even up to $\sim 2T_c$ in full QCD

Quarkonia in quark-gluon plasma

The Q-Qbar potential extracted from lattice calculations can be used to examine the stability of quarkonia of different quark masses.

We can treat the quark mass as a variable and obtain the spontaneous dissociation temperature as a function of the reduced mass.

The quark drip line

- The quark drip line is the line in the (μ, T) space above which $Q\text{-}Q\text{bar}$ is unbound.
- It can also be characterized by the nature of the $Q\text{-}Q\text{bar}$ state: $1s$ drip line, $1p$ drip line, ..
- Given the $Q\text{-}Q\text{bar}$ potential, the drip line can be determined by locating the spontaneous dissociation temperature as a function of the reduced mass.

Quark drip lines in quark-gluon plasma in quenched QCD

Stability of quarkonia in quark-gluon plasma in full QCD

Dynamical quarks modifies the 1s drip line but not the 1p drip line.

Can the potential model be applied to light quarkonia?

- Because of the strong coupling, light quarks become quasiparticles and acquire masses
- The quasiparticle masses of quarks in quark-gluon plasma can be estimated by looking at the equation of state (Levai et al., Szabo et al, Iavanov et al.). They found that the quasi-particle masses of u, d, and s quarks are 0.3-0.4 GeV for $T_c < T < 2T_c$.

Light quark quarkonia

For light quarks with a mass of 300-400 MeV, quarkonia with light quarks can be stable up to $1.06T_c$.

Szabo et al. JHEP 0306, 008 ('03)
Results from Levai et al and
Ivanov et al are similar.

Conclusions

- The potential model is consistent with the lattice gauge spectral function analysis, if the $Q\text{-}\bar{Q}$ potential is a linear combination of F_1 and U_1 , with coefficients depending on the equation of state.
- The effects of the dynamical quarks modifies only slightly the stability of J/ψ . J/ψ dissociates spontaneously at about $1.62 T_c$ in quenched QCD and at $1.42 T_c$ in 2-flavor QCD.
- The interaction between a static quark and antiquark is such that the quark drip lines limit possible quarkonium states with light quarks to T close to T_c .

J/ψ-nucleon collisions

- J/ψ precursor collide with nucleons at high energy
- J/ψ-nucleon cross section depends on root-mean-square separation
- J/ψ colides with comovers at low energy

J/psi-nucleon high energy cross section

Zhang & Wong, PRC68,035211(03)

$$\sigma_{\text{tot}} = \frac{1}{s} \text{Im} A(s, t = 0)$$

$$A(s, t) = \sum_{ijk\ell} \int d\vec{b} e^{-i\vec{Q} \cdot \vec{b}} \langle \Psi_I(ij) \Psi_{II}(kl) | V_T(ik) V_T(jl) | \Psi_I(ij) \Psi_{II}(kl) \rangle$$

$$V_T(\vec{b}) = \int_{-\infty}^{\infty} dz V(\vec{r})$$

$$Q^2 = -t$$

Zhang & Wong, PRC68,035211(03)

Cross section dependence on color-octet admixture & c-cbar separation

For color-singlet C-Cbar, cross section increases with C-Cbar separation.

Low-energy π -J/ ψ cross section

Use the Barnes-Swanson quark-interchange model

Martins, Blaschke, Quack, PRC 51, 2723 ('95)
C.Y. Wong, E. Swanson, T. Branes, PRC65,014903('010)

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s} \frac{1}{|\vec{p}_A|^2} |M_{fi}|^2$$

Test the formulation in $\pi\pi$ scattering

PHYSICAL REVIEW C 65 014903

FIG. 4. Comparison of our theoretical $I=2\pi\pi$ phase shift (solid curve) with the data of Hoogland *et al.* [41].

Wong, Swanson, Barnes PRC 65 014903 (2001)

J/ψ dissociation by collision with gluon

$$\sigma_{\text{dis}}^{\text{El}}(E) = \frac{4\pi}{3} \alpha (2\mu E + 2\mu B) I^2 / \sqrt{2\mu E}$$

E = gluon energy, B = binding energy

$$I = \int dr u_{1\rho}(r) r u_{1\rho}(r)$$

J/ψ dissociation in collision with gluons

Wong, PRC72, 034906(05)

Inverse reaction

Wong, PRC72, 034906(05)