EPJ Web of Conferences **7**, 01007 (2010)
DOI:10.1051/epjconf/20100701007

Relativistic BCS-BEC Crossover at Quark Level

Lianyi He, Shijun Mao, and Pengfei Zhuang

Physics Department, Tsinghua University, Beijing 100084, China

Abstract. The non-relativistic G_0G formalism of BCS-BEC crossover at finite temperature is extended to relativistic fermion systems. The theory recovers the BCS mean field approximation at zero temperature and the non-relativistic results in a proper limit. For massive fermions, when the coupling strength increases, there exist two crossovers from the weak coupling BCS superfluid to the non-relativistic BEC state and then to the relativistic BEC state. For color superconductivity at moderate baryon density, the matter is in the BCS-BEC crossover region, and the behavior of the pseudogap is quite similar to that found in high temperature superconductors.

Acknowledgements

The work is supported by the NSFC Grant 10735040 and the 973-project 2006CB921404 and 2007CB815000.

References

1. A.J.Leggett, *Modern trends in the theory of condensed matter*, Springer-Verlag, Berlin (1980)
2. P.Nozieres and S.Schmitt-Rink, J.Low.Temp.Phys. **59**, 195 (1985)
3. Q.Chen, J.Stajic, S.Tan and K.Levin, Phys.Rept. **412**, 1 (2005)
4. Y. Nishida and H. Abuki, Phys.Rev. **D72**, 096004, (2005)
5. J.Deng, A.Schmitt and Q.Wang, Phys.Rev. **D78**, 034014 (2008)
6. L.He and P.Zhuang, Phys.Rev. **D76**, 056003 (2007); Phys.Rev. **D75**, 096004 (2007); Phys.Rev. **D75**, 096003 (2007)

Relativistic BCS-BEC Crossover at Quark Level

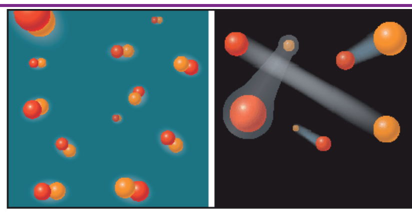
Pengfei Zhuang
Physics Department, Tsinghua University, Beijing 100084

July, 2008 Summer School on Dense Matter and HI Dubna 1

1) Motivation

July, 2008 Summer School on Dense Matter and HI Dubna 2

BCS-BEC



BEC of molecules ⟵ ***BCS fermionic superfluid***

BCS *(Barden, Cooper and Schrieffer, 1957):* ***normal superconductivity***

weak coupling, large pair size, k-space pairing, overlaping Cooper pairs

BEC *(Bose-Einstein-Condensation, 1924/1925):*

strong coupling, small pair size, r-space pairing, ideal gas of bosons,

first realization in dilute atomic gas with bosons in 1995.

BCS-BEC crossover *(Eagles, Leggett, 1969, 1980):*

BCS wave function at T=0 can be generalized to arbitrary attraction: a smooth crossover from BCS to BEC!

BEC

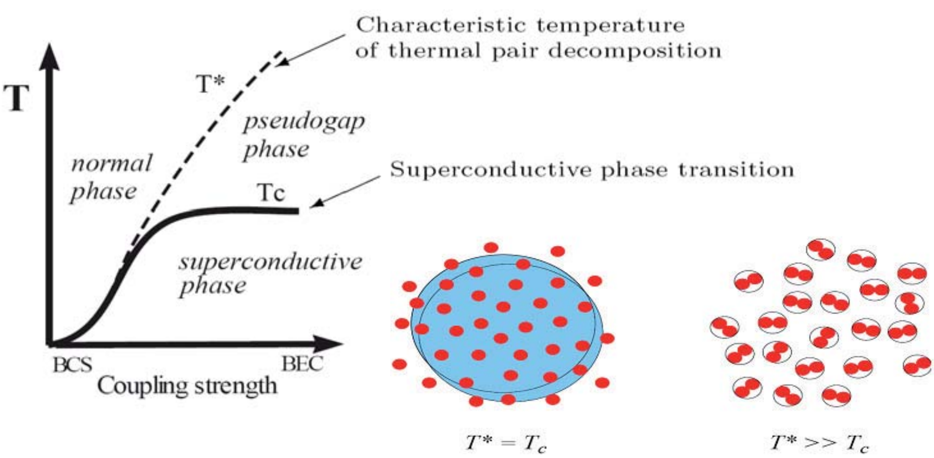
$$\Psi_0 = \exp\left(N_B^{1/2}\sum_{\mathbf{k}} \phi_{\mathbf{k}} c^+_{\mathbf{k}} c^+_{-\mathbf{k}}\right)|0\rangle$$

$$v_k = \frac{N_B^{1/2}\phi_k}{\left(1+N_B\phi_k^2\right)^{1/2}}$$

BCS

$$\Psi_0 = \prod_{\mathbf{k}}\left(u_{\mathbf{k}} + v_{\mathbf{k}} c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}}\right)|0\rangle$$

pairings



in BCS, T_c is determined by thermal excitation of fermions,

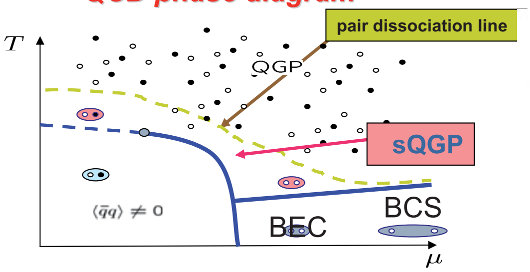
in BEC, T_c is controlled by thermal excitation of collective modes

BCS-BEC in QCD

in order to study the question of 'vacuum', we must turn to a different direction: we should investigate some 'bulk' phenomena by distributing high energy over a relatively large volume.

T. D. Lee, Rev. Mod. Phys. 47, 267(1975)

QCD phase diagram



rich QCD phase structure at high density, natural attractive interaction in QCD, possible BCS-BEC crossover ?

new phenomena in BCS-BEC crossover of QCD:
relativistic systems, anti-fermion contribution, rich inner structure (color, flavor), medium dependent mass, ……

theory of BCS-BEC crossover

**) Leggett mean field theory (Leggett, 1980)*
**)NSR scheme (Nozieres and Schmitt-Rink, 1985)*
extension of of BCS-BEC crossover theory at T=0 to T≠0 (above T_c)
Nishida and Abuki (2006,2007)
extension of non-relativistic NSR theory to relativistic systems, BCS-NBEC-RBEC crossover

$$\Omega_{fl} = \int \frac{d^4q}{(2\pi)^4} \ln\left[\frac{1}{G} - \chi(q)\right], \qquad \chi = \bigcirc \; G_0 G_0$$

**) G_0G scheme (Chen, Levin et al., 1998, 2000. 2005)*
asymmetric pair susceptibility $\chi = \bigcirc \; G_0 G$

extension of non-relativistic G_0G scheme to relativistic systems (He, Zhuang, 2006, 2007)

**) Bose-fermion model (Friedderg, Lee, 1989, 1990)*
extension to relativistic systems (Deng, Wang, 2007)

Kitazawa, Rischke, Shovkovy, 2007, NJL+phase diagram
Brauner, 2008, collective excitations ……

2) Leggett Mean Field Theory at T = 0

A.J.Leggett, in Modern trends in the theory of condensed matter, Springer-Verlag (1980)

non-relativistic mean field theory

fermion number: n

Fermi momentum: $k_F, \quad n = k_F^3 / 3p^2$

Fermi energy: $e_F = k_F^2 / 2m$

$$L = \psi_\sigma^+ \left(i\partial_t - \frac{\nabla^2}{2m} - \sigma_3 h \right) \psi_\sigma + g\psi_\uparrow^+ \psi_\downarrow^+ \psi_\downarrow \psi_\uparrow$$

order parameter $\Delta = 2g \left\langle \psi_\downarrow^+ \psi_\downarrow \right\rangle = 2g \left\langle \psi_\uparrow^+ \psi_\uparrow \right\rangle$

thermodynamics in mean field approximation

$$\Omega = \frac{\Delta^2}{g} - \int \frac{d^3\vec{k}}{(2\pi)^3} \left[E_k - \xi_k - \frac{\Delta^2}{4\varepsilon_k} \right]$$

quasi-particle energy $E_k = \sqrt{x_k^2 + D^2}, \quad x_k = e_k - m, \quad e_k = k^2 / 2m$

renormalization to avoid the integration divergence

$$g \to a_s, \qquad \frac{m}{4\pi a_s} = \frac{1}{g} + \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\varepsilon_k}$$

$a_s < 0$ for attractive coupling, $a_s > 0$ for repulsive coupling

universality

gap equation $\frac{\partial\Omega}{\partial\Delta} = 0$

number conservation $n = -\frac{\partial\Omega}{\partial\mu}$

$$\Rightarrow \Delta(n), \quad \mu(n)$$

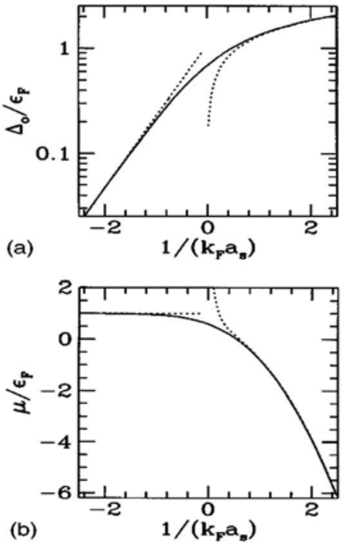
$$\begin{cases} -\frac{m}{4\pi a_s} = \int \frac{d^3k}{(2\pi)^3}\left(\frac{1}{2E_k} - \frac{1}{2\varepsilon_k}\right) \\ n = \int \frac{d^3k}{(2\pi)^3}\left(1 - \frac{\xi_k}{E_k}\right) \end{cases}$$

universality

$$\hat{\mu} = \mu/\varepsilon_F, \quad \tilde{\Delta} = \Delta/\varepsilon_F, \quad \eta = \frac{1}{k_F a_s}$$ **effective coupling**

$$\begin{cases} -\frac{\eta}{2} = \int_0^\infty dx x^2 \left(\frac{1}{\sqrt{(x^2-\hat{\mu})^2 + \tilde{\Delta}^2}} - \frac{1}{x^2}\right) \\ \frac{2}{3} = \int_0^\infty dx x^2 \left(1 - \frac{x^2-\hat{\mu}}{\sqrt{(x^2-\hat{\mu})^2 + \tilde{\Delta}^2}}\right) \end{cases} \Rightarrow \tilde{\Delta}(\eta), \quad \hat{\mu}(\eta)$$

non-relativistic BCS-BEC crossover



BCS limit

$$\eta \to -\infty, \quad \tilde{\Delta} = \frac{8}{e^2}e^{2\eta/\pi}, \quad \hat{\mu} = 1$$

BEC limit

$$\eta \to \infty, \quad \tilde{\Delta} = \sqrt{\frac{16\eta}{3\pi}}, \quad \hat{\mu} = -\eta^2$$

$$\mu = -\frac{\varepsilon_b}{2}, \quad \varepsilon_b = \frac{1}{m a_s^2}$$

$$n(p) = \frac{1}{e^{(\varepsilon-\mu)/T} - 1} \quad \Rightarrow \quad \mu \le 0$$

BCS-BEC crossover

$\eta < 0 \quad \to \quad \eta > 0,$

small $\Delta \quad \to \quad$ large Δ,

$\mu > 0 \quad \to \quad \mu < 0$

relativistic BCS-BEC crossover

Tsinghua University

$$E_k^{\pm} = \sqrt{\left(x_k^{\pm}\right)^2 + \Delta^2}, \quad x_k^{\pm} = \sqrt{k^2 + m^2} \pm \mu$$

$\mu - m$ **plays the role of non-relativistic chemical potential**

BCS-BEC crossover around

$$\mu - m = 0, \quad \mu = m$$

pair binding energy

$$e_b < 2m$$

$$\mu - m > -\frac{e_b}{2} \approx -m, \quad \mu \approx 0$$

at $\mu \to 0$ **fermion and anti-fermion degenerate, relativistic effect**

RBEC limit	RBEC-NBEC crossover	NBEC	NBEC-BCS crossover	BCS limit

0 ——— m ——— m →

relativistic mean field theory

He, Zhuang, 2007

Tsinghua University

extremely high T and high density: pQCD
finite T (zero density): lattice QCD
moderate T and density: models like 4-fermion interaction (NJL)

$$L = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi + \frac{g}{4}\left(\bar{\psi}i\gamma_5 C\bar{\psi}^T\right)\left(\psi^T iC\gamma_5\psi\right)$$

$C = i\gamma_0\gamma_2$ **charge conjunction matrix**

order parameter $\Delta = \frac{g}{2}\left\langle \psi^T iC\gamma_5\psi \right\rangle$

mean field thermodynamic potential

$$\Omega = \frac{\Delta^2}{g} - \int \frac{d^3\vec{k}}{(2\pi)^3}\left[E_k^+ + E_k^- - \xi_k^+ - \xi_k^-\right]$$

(arrows: E_k^+ — **antifermion**; E_k^- — **fermion**)

the most important thermodynamic contribution from the uncondensed pairs is from the Goldstone modes, $\varepsilon \sim T^4$, at T=0, the fluctuation contribution disappears, and MF is a good approximation at T=0.

broken universality

gap equation and number equation:

$$\begin{cases} \dfrac{1}{g} = \displaystyle\int \frac{d^3k}{(2\pi)^3}\left(\frac{1}{2E_k^-} - \frac{1}{2E_k^+}\right) \\ n = \displaystyle\int \frac{d^3k}{(2\pi)^3}\left[\left(1-\frac{\xi_k^-}{E_k^-}\right) - \left(1-\frac{\xi_k^+}{E_k^+}\right)\right] \end{cases}$$

renormalization to avoid the integration divergence

$$-\frac{1}{U} = \frac{1}{g} - \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\left(\frac{1}{\varepsilon_k - m} + \frac{1}{\varepsilon_k + m}\right), \qquad U = \frac{4\pi a_s}{m}$$

the ultraviolet divergence can not be completely removed, and a momentum cutoff Λ still exists in the theory.

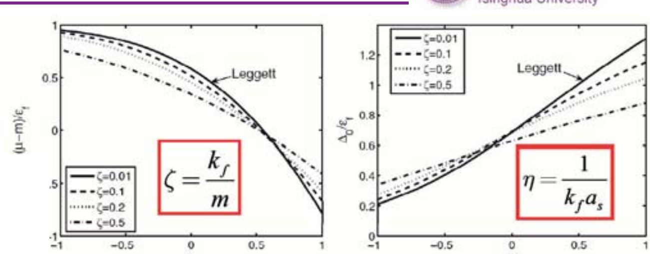
broken universality:

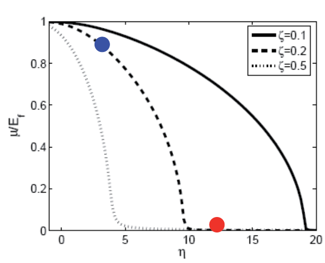
$$\begin{cases} -\dfrac{\pi}{2}\eta = \displaystyle\int_0^z dx x^2 \left[\left(\frac{1}{E_x^-} - \frac{1}{\varepsilon_x - 2\zeta^{-2}}\right) + \left(\frac{1}{E_x^+} - \frac{1}{\varepsilon_x + 2\zeta^{-2}}\right)\right] \\ \dfrac{2}{3} = \displaystyle\int_0^z dx x^2 \left[\left(1-\frac{\xi_x^-}{E_x^-}\right) - \left(1-\frac{\xi_x^+}{E_x^+}\right)\right] \end{cases}$$

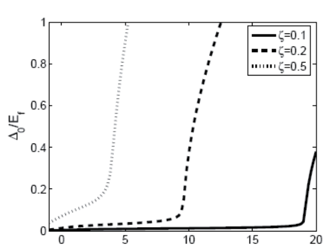
$$\eta = \frac{1}{k_F a_s}, \qquad \zeta = \frac{k_F}{m}$$

explicit density dependence !

non-relativistic limit







non-relativistic limit:

1) $\zeta = k_F/m << 1$ **non-relativistic kinetics**
2) $|\mu - m| << m$ **negligible anti-fermions**

$m(\eta,\zeta) = m \Rightarrow \eta_c^1(\zeta)$ • **NBEC**

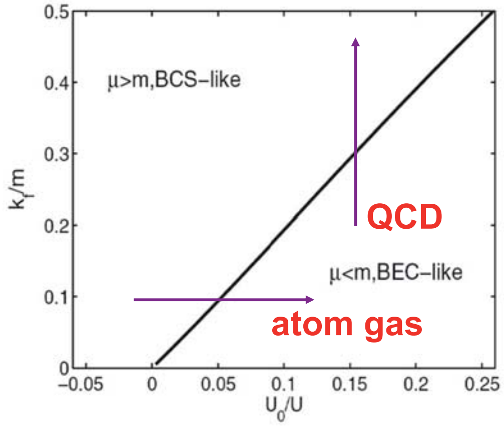
$m(\eta,\zeta) \to 0 \Rightarrow \eta_c^2(\zeta)$ • **RBEC**

density induced crossover

Tsinghua University

In non-relativistic case, only one dimensionless variable, $\eta = 1/k_F a_s$, changing the density of the system can not induce a BCS-BEC crossover.

However, in relativistic case, the extra density dependence $z = k_F/m$ may induce a BCS-BEC crossover.



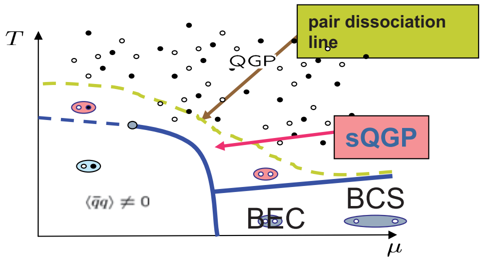
Tsinghua University

3) Fluctuations at $T \neq 0$

introduction

● the *Landau mean field theory* is a good approximation only at T=0 where there is no thermal excitation, *one has to go beyond the mean field at finite temperature.*

● *an urgent question in relativistic heavy ion collisions*



to understand the sQGP phase with possible bound states of quarks and gluons, one has to go beyond the mean field !

● *going beyond mean field self-consistently is very difficult*

NSR Theory (G_0G_0 Scheme) above T_c: Nishida and Abuki (2005),
Bose-Fermi Model: Deng and Wang (2006),
G_0G Scheme below T_c: He and Zhuang (2007), ……

July, 2008 Summer School on Dense Matter and HI Dubna 17

mean field fermion propagator

$$L=\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu}-m\right)\psi+\frac{g}{4}\left(\bar{\psi}i\gamma_5 C\bar{\psi}^T\right)\left(\psi^T iC\gamma_5\psi\right)$$

gap and number equations at finite T

$$\begin{cases}\dfrac{1}{g}=\displaystyle\int\frac{d^3k}{(2\pi)^3}\left[\frac{1-2f(E_k^-)}{2E_k^-}+\frac{1-2f(E_k^+)}{2E_k^-}\right]\\ n=\displaystyle\int\frac{d^3k}{(2\pi)^3}\left[\left(1-\frac{\xi_k^-}{E_k^-}\right)\left(1-2f(E_k^-)\right)-\left(1-\frac{\xi_k^+}{E_k^+}\right)\left(1-2f(E_k^+)\right)\right]\end{cases}$$

fermion propagator with diagonal and off-diagonal elements in Nambu-Gorkov space

$$S(k,\mu)=\begin{pmatrix}G(k,\mu) & F(k,\mu)\\ F(k,-\mu) & G(k,-\mu)\end{pmatrix}\qquad \begin{aligned}G^{-1}(k,\mu)&=G_0^{-1}(k,\mu)-\Sigma_{mf}(k)\\ F(k,\mu)&=-G(k,\mu)i\gamma_5\Delta G_0(k,-\mu)\end{aligned}$$

bare propagator and condensate induced self-energy

$$G_0^{-1}(k,\mu)=(k_0+\mu)\gamma_0-\vec{\gamma}\cdot\vec{k}-m \qquad \Sigma_{mf}(k)=-\Delta^2 G_0(-k,\mu)$$

gap and number equations in terms of the fermion propagator

$$\Delta\left[1+i\frac{g}{2}\sum_k Tr\left[G(k,\mu)G_0(-k,\mu)\right]\right]=0, \qquad n=-i\sum_k Tr\left[\gamma_0 G(k,\mu)\right]$$

Matsubara frequency summation $\sum_k = iT\sum_n\int\frac{d^3k}{(2\pi)^3},\quad k_0\to i\omega_n,\quad \omega_n=\begin{cases}2n\pi T, & bosons\\ (2n+1)\pi T, & fermions\end{cases}$

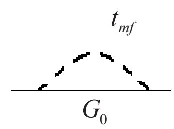
July, 2008 Summer School on Dense Matter and HI Dubna 18

mean field theory in G_0G scheme

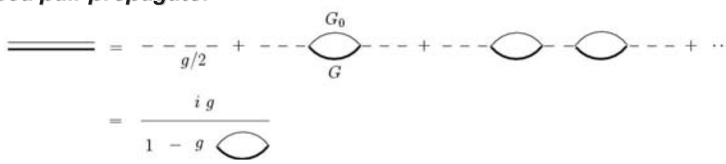
introducing a condensed-pair propagator

$$t_{mf}(q) = i\frac{\Delta^2}{T}\delta(q)$$

$$\longrightarrow \quad \Sigma_{mf}(k) = \sum_q t_{mf}(q)G_0(q-k,\mu) =$$



defining an uncondensed pair propagator



the name G_0G scheme,
G is the mean field
fermion propagator

$$t(q) = \frac{ig}{1-g\chi(q)} \qquad \chi(q) = -\frac{i}{2}\sum_k Tr\left[G(k,\mu)G_0(q-k,\mu)\right]$$

gap equation in condensed phase is determined by uncondensed pairs

$$t^{-1}(0) = 0 \longrightarrow \Delta(T,\mu)$$

problem:
there is no feedback of the uncondensed pairs on the fermion self-energy

going beyond mean field in G_0G scheme

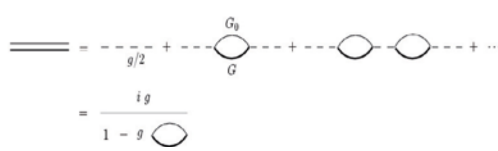
He, Zhuang, 2007

full pair propagator

$$t(q) = t_{mf}(q) + t_{pg}(q)$$

$$t_{mf}(q) = i\frac{\Delta^2}{T}\delta(q)$$

$$t_{pg}(q) = \frac{ig}{1-g\chi(q)} =$$

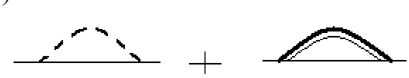


with full susceptibility and full propagator

$$\chi(q) = -\frac{i}{2}\sum_k Tr\left[G(k,\mu)G_0(q-k,\mu)\right] \qquad G^{-1}(k,\mu) = G_0^{-1}(k,\mu) - \Sigma(k)$$

full fermion self-energy $\Sigma(k) = \sum_q t(q)G_0(q-k,\mu)$

$$= \Sigma_{mf}(k) + \Sigma_{pg}(k) =$$



fermions and pairs are coupled to each other

new gap equation $t_{pg}^{-1}(0) = 0 \longrightarrow$ ***a new order parameter $\Delta(T,\mu)$ which is different from the mean field one***

all the formulas look the same as the mean field ones,
but we do not know the expression of the full fermion propagator G.

approximation in condensed phase

Tsinghua University

1) $t_{pg}^{-1}(0)=0 \longrightarrow t_{pg}(q)$ **peaks at** $q=0$

$$\Sigma_{pg}(k)=\sum_{q\neq 0}t_{pg}(q)G_0(q-k,\mu)\simeq -\left(-\sum_{q\neq 0}t_{pg}(q)\right)G_0(-k,\mu)=-\Delta_{pg}^2G_0(-k,\mu)$$

$$\Delta_{pg}^2=-\sum_{q\neq 0}t_{pg}(q)$$

the pseudogap is related to the uncondensed pairs and does not change the symmetry !

full self-energy $\Sigma(k)=\Sigma_{mf}(k)+\Sigma_{pg}(k)\simeq -\left(\Delta^2+\Delta_{pg}^2\right)G_0(-k,\mu)$

gap equation $t_{pg}^{-1}(0)=0 \longrightarrow$ ***mean field gap equation with*** $\Delta\to\sqrt{\Delta^2+\Delta_{pg}^2}$

2) $t_{pg}^{-1}(0)=0 \longrightarrow$ **expansion around** $q=0$

$$t_{pg}(q)=\frac{ig}{1-g\chi(q)}=\frac{-i}{\chi(q)-\chi(0)}\simeq\frac{-i}{Z_1q_0+Z_2q_0^2-\xi^2\vec q^2}$$

$$Z_1=\left.\frac{\partial\chi}{\partial q_0}\right|_{q_0=0},\quad Z_2=\left.\frac{1}{2}\frac{\partial^2\chi}{\partial q_0^2}\right|_{q_0=0},\quad \xi_2=-\left.\frac{1}{2}\frac{\partial^2\chi}{\partial \vec q^2}\right|_{q=0}$$

$$\begin{cases}\text{mean field gap equation with } \Delta\to\sqrt{\Delta^2+\Delta_{pg}^2}\\ \Delta_{pg}^2=\dfrac{1}{Z_2}\displaystyle\int\frac{d^3\vec q}{(2\pi)^3}\frac{f_b(\omega_q-\nu)+f_b(\omega_q+\nu)}{2\omega_q}\end{cases}\longrightarrow \Delta(T,\mu),\quad \Delta_{pg}(T,\mu)$$

$$\nu=Z_1/2Z_2,\qquad \omega_q=\sqrt{\xi^2\vec q^2/Z_2+\nu^2}$$

thermodynamics

Tsinghua University

$$\Omega=\Omega_{mf}+\Omega_{fl}$$

$$\Omega_{mf}=\frac{\Delta^2}{g}+\int\frac{d^3k}{(2\pi)^3}\left[\left(\xi_k^++\xi_k^--E_k^+-E_k^-\right)-\frac{1}{\beta}\ln\left(\left(1+e^{-\beta E_k^-}\right)\left(1+e^{-\beta E_k^+}\right)\right)\right]$$

$$\Omega_{fl}=\sum_q\ln\left[1-g\chi(q)\right]\simeq\frac{1}{\beta}\int\frac{d^3q}{(2\pi)^3}\ln\left(\left(1-e^{-\beta\omega_q^-}\right)\left(1-e^{-\beta\omega_q^+}\right)\right)$$

$$=\begin{cases}\dfrac{1}{\beta}\displaystyle\int\frac{d^3q}{(2\pi)^3}\ln\left(1-e^{-\beta q^2/2m_B}\right) & Z_1>>Z_2,\ \textit{non-relativistic boson gas}\\ \dfrac{2}{\beta}\displaystyle\int\frac{d^3q}{(2\pi)^3}\ln\left(1-e^{-\beta cq}\right) & Z_2>>Z_1,\ \textit{relativistic boson gas}\end{cases}$$

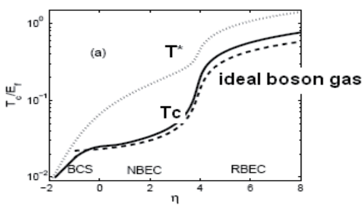
number of bosons

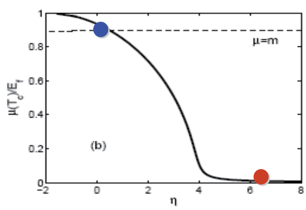
$$n_B=\frac{n}{2}-\int\frac{d^3k}{(2\pi)^3}\left[f\left(\xi_k^-\right)-f\left(\xi_k^+\right)\right]=Z_1\left(\Delta^2+\Delta_{pg}^2\right)=n_B^{mf}+n_B^{fl}$$

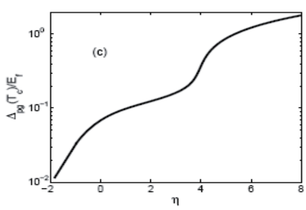
fraction of condensed pairs

$$P_c=\frac{n_B^{mf}}{n/2}=\frac{Z_1\Delta^2}{n/2}$$

BCS-NBEC-RBEC crossover







T_c : critical temperature

$T < T_c$: $\Delta \neq 0$, condensed phase

$T > T_c$: $\Delta = 0$, normal or pseudogap phase

T^* : pair dissociation temperature

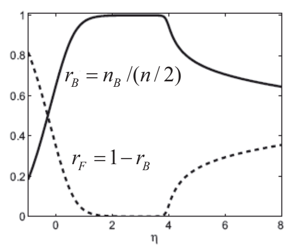
$T_c < T < T^*$: $\Delta_{pg} \neq 0$, pseudogap phase

$T > T^*$: $\Delta_{pg} = 0$ normal phase

BCS: $\eta < 0,\ \mu > m$ **no pairs**

NBEC: $0 < \eta < m/k_F,\ 0 < \mu < m$ **heavy pairs, no anti-pairs**

RBEC: $\eta > m/k_F,\ \mu \sim 0$ **light pairs, almost the same number of pairs and anti-pairs**



number fractions at T_c , in RBEC, T_c is large enough and there is a strong competition between condensation and dissociation.

discussion on G_0G

can the symmetry be restored in the pseudogap phase?

fermion propagator including fluctuations (to the order of Δ^2/Λ^2) :

$$S^{-1}(k) = \begin{pmatrix} G_0^{-1}(k,\mu) - \Sigma_{pg}(k,\mu) & i\gamma_5\Delta \\ i\gamma_5\Delta & G_0^{-1}(k,\mu) - \Sigma_{pg}(k,-\mu) \end{pmatrix}$$

the pseudogap appears in the diagonal elements of the propagator and does not break the symmetry of the system.

Kandanoff and Martin:
the scheme $\chi \sim GG$ can not give a correct symmetry restoration picture and the specific heat $C_V \sim T^2$ is wrong.

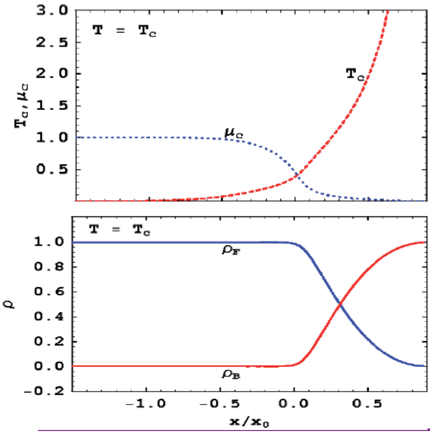
BCS-BEC in Bose-fermion model

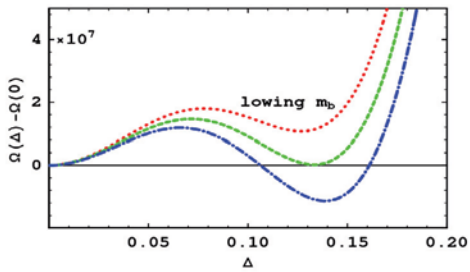
Deng, Wang, 2007

$$\mathcal{L}_0 = \mathcal{L}_{f0} + \mathcal{L}_{b0} + \mathcal{L}_{I0}\,,$$
$$\mathcal{L}_{f0} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi\,,$$
$$\mathcal{L}_{b0} = \partial_\mu\varphi^*\partial^\mu\varphi - m_b^2\varphi^*\varphi\,,$$
$$\mathcal{L}_{I0} = g(\varphi\bar{\psi}_C i\gamma_5\psi + \varphi^*\bar{\psi}i\gamma_5\psi_C)$$

$$\Omega = -\sum_{e=\pm}\int\frac{d^3k}{(2\pi)^3}\left\{\epsilon_k^e + 2T\ln\left[1+\exp\left(-\frac{\epsilon_k^e}{T}\right)\right]\right\} + \frac{(m_b^2-\mu_b^2)\Delta^2}{4g^2} + \frac{1}{2}\sum_{e=\pm}\int\frac{d^3k}{(2\pi)^3}\left\{\omega_k^e + 2T\ln\left[1-\exp\left(-\frac{\omega_k^e}{T}\right)\right]\right\}$$

mean field thermodynamics $\Delta = 2g\phi\,,$





fluctuation changes the phase transition to be first-order.

4) Application to QCD: Color Superconductivity and Pion Superfluid

motivation

*** QCD phase transitions like chiral symmetry restoration, color superconductivity, and pion superfluid happen in non-perturbative temperature and density region, the coupling is strong.**

*** relativistic BCS-BEC crossover is controlled by $\mu - m$, the BEC-BCS crossover would happen when the light quark mass changes in the QCD medium.**

*** effective models at hadron level can only describe BEC state, they can not describe BEC-BCS crossover. One of the models that enables us to describe both quarks and mesons and diquarks is the NJL model at quark level.**

$$L_{NJL} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m_0 + \mu\gamma_0\right) + G_S\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\tau_i\gamma_5\psi\right)^2\right) + G_D\left(\bar{\psi}^C_{i\alpha}\varepsilon^{ij}\varepsilon^{\alpha\beta\gamma}i\gamma_5\psi_{j\beta}\right)\left(\bar{\psi}_{i\alpha}\varepsilon^{ij}\varepsilon^{\alpha\beta\gamma}i\gamma_5\psi^C_{j\beta}\right)$$

disadvantage: no confinement

*** there is no problem to do lattice simulation for real QCD at finite isospin density**

color superconductivity

order parameters of spontaneous chiral and color symmetry breaking

$$\sigma = \langle\bar{\psi}\psi\rangle$$

$$\Delta = \Delta^3 = \left\langle\bar{\psi}^C_{i\alpha}\varepsilon^{ij}\varepsilon^{\alpha\beta 3}i\gamma_5\psi_{j\beta}\right\rangle$$ **color breaking from SU(3) to SU(2)**

leading order of $1/N_c$ for quarks, and next to leading order for mesons & diquarks

quark propagator in 12D Nambu-Gorkov space

$$\Psi = \begin{pmatrix}\psi_{u1}\\ \psi^C_{d2}\\ \psi_{d2}\\ \psi^C_{u1}\\ \psi_{d1}\\ \psi^C_{u2}\\ \psi_{u2}\\ \psi^C_{d1}\\ \psi_{u3}\\ \psi^C_{u3}\\ \psi_{d3}\\ \psi^C_{d3}\end{pmatrix}$$

$$\bar{\Psi} = \left(\bar{\psi}_{u1}\ \ \bar{\psi}^C_{d2}\ \ \bar{\psi}_{d2}\ \ \bar{\psi}^C_{u1}\ \ \bar{\psi}_{d1}\ \ \bar{\psi}^C_{u2}\ \ \bar{\psi}_{u2}\ \ \bar{\psi}^C_{d1}\ \ \bar{\psi}_{u3}\ \ \bar{\psi}^C_{u3}\ \ \bar{\psi}_{d3}\ \ \bar{\psi}^C_{d3}\right)$$

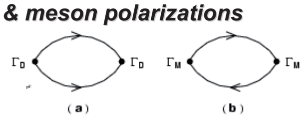
$$S = \begin{pmatrix} S_A &&&&& \\ & S_B &&&& \\ && S_C &&& \\ &&& S_D && \\ &&&& S_E & \\ &&&&& S_F \end{pmatrix}$$

$$S_I = \begin{pmatrix} G_I^+ & \Xi_I^- \\ \Xi_I^+ & G_I^- \end{pmatrix} \quad I = A,B,C,D,E,F$$

$$E_k = \sqrt{\vec{k}^2 + M_q^2} \qquad M_q = m_0 - 2G_S\sigma$$

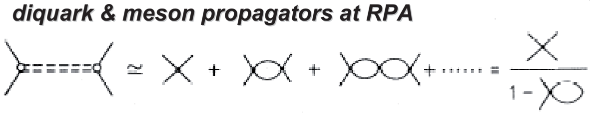
$$E_\Delta^\pm = \sqrt{\left(E_k \pm \mu\right)^2 + \left(2G_D\Delta\right)^2}$$

diquark & meson polarizations



Π_D Π_M

diquark & meson propagators at RPA



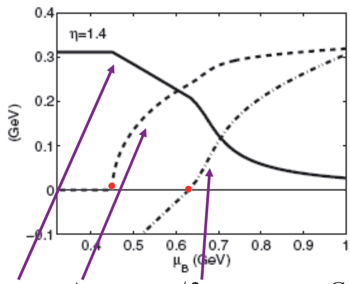
BCS-BEC and color neutrality

He, Zhuang, 2007

Tsinghua University

gap equations for chiral and diquark condensates at T=0

$$\begin{cases} m - m_0 = 8G_s m \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_p}\left[\frac{E_k-\mu_B/3}{E_\Delta^-} + \frac{E_k+\mu_B/3}{E_\Delta^+} + \Theta\left(E_k - \mu_B/3\right)\right] \\ \Delta = 8G_d \Delta \int \frac{d^3k}{(2\pi)^3}\left[\frac{1}{E_\Delta^-} + \frac{1}{E_\Delta^+}\right] \end{cases}$$

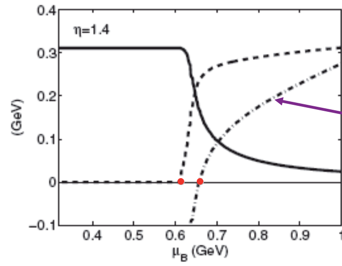


$m \quad \Delta \quad \mu_B/3 - m \quad \eta = G_d/G_s$

there exists a BCS-BEC crossover

to guarantee color neutrality, we introduce color chemical potential:

$$\mu_r = \mu_g = \mu_B/3 + \mu_8/3, \qquad \mu_b = \mu_B/3 - 2\mu_8/3$$



$\mu_r - m$

color neutrality speeds up the chiral restoration and reduces the BEC region

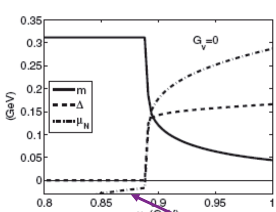
vector meson coupling and magnetic instability

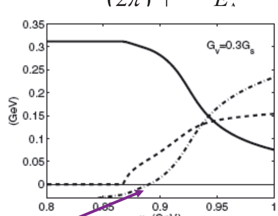
Tsinghua University

vector-meson coupling $L_V = -G_V\left[\left(\bar\psi\gamma_\mu\psi\right)^2 + \left(\bar\psi\gamma_\mu\gamma_5\tau\psi\right)^2\right]$

vector condensate $\rho_V = 2G_V\langle\bar\psi\gamma_0\psi\rangle$

gap equation $\rho_V = 8G_V \int \frac{d^3k}{(2\pi)^3}\left[\frac{E_k+\mu_B/3}{E_\Delta^+} - \frac{E_k-\mu_B/3}{E_\Delta^-} + \Theta\left(-E_k+\mu_B/3\right)\right]$



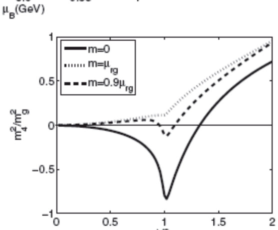


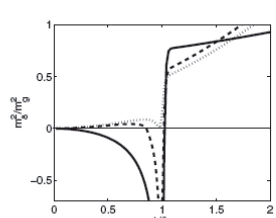
$\eta = 1$

vector meson coupling slows down the chiral symmetry restoration and enlarges the BEC region.

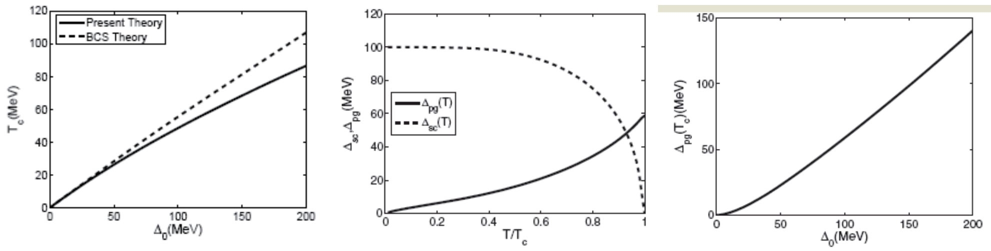
$\mu_r - m$

Meissner masses of some gluons are negative for the BCS Gapless CSC, but the magnetic instability is cured in BEC region.





beyond men field



$\Delta_0 = \Delta(T=0)$ ***is determined by the coupling and chemical potential***

$\Delta_0 = 100-200$ MeV $\leftrightarrow$ $\mu_q = 300-500$ MeV

- ***going beyond mean field reduces the critical temperature of color superconductivity***
- ***pairing effect is important around the critical temperature and dominates the symmetry restored phase***

pion superfluid

NJL with isospin symmetry breaking

$$L_{NJL} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m_0 + \mu\gamma_0\right)\psi + G\left(\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\tau_i\gamma_5\psi\right)^2\right)$$

quark chemical potentials

$$\mu = \begin{pmatrix} \mu_u & 0 \\ 0 & \mu_d \end{pmatrix} = \begin{pmatrix} \mu_B/3 + \mu_I/2 & 0 \\ 0 & \mu_B/3 - \mu_I/2 \end{pmatrix}$$

chiral and pion condensates with finite pair momentum

$$\sigma = \langle\bar{\psi}\psi\rangle = \sigma_u + \sigma_d, \quad \sigma_u = \langle\bar{u}u\rangle, \quad \sigma_d = \langle\bar{d}d\rangle$$

$$\pi_+ = \sqrt{2}\langle\bar{u}i\gamma_5 d\rangle = \frac{\pi}{\sqrt{2}}e^{2i\vec{q}\cdot\vec{x}}, \quad \pi_- = \sqrt{2}\langle\bar{d}i\gamma_5 u\rangle = \frac{\pi}{\sqrt{2}}e^{-2i\vec{q}\cdot\vec{x}}$$

quark propagator in MF

$$S^{-1}(p,\vec{q}) = \begin{pmatrix} \gamma^\mu p_\mu - \vec{\gamma}\cdot\vec{q} + \mu_u\gamma_0 - m & 2iG\pi\gamma_5 \\ 2iG\pi\gamma_5 & \gamma^\mu k_\mu + \vec{\gamma}\cdot\vec{q} + \mu_d\gamma_0 - m \end{pmatrix} \qquad m = m_0 - 2G\sigma$$

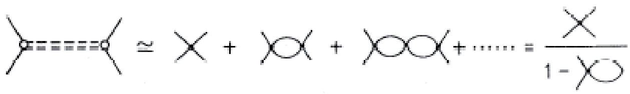
thermodynamic potential and gap equations:

$$\Omega = G(\sigma^2 + \pi^2) - \frac{T}{V}\text{Tr Ln } S^{-1}$$

$$\frac{\partial\Omega}{\partial\sigma_u} = 0, \ \frac{\partial^2\Omega}{\partial\sigma_u^2} \ge 0, \quad \frac{\partial\Omega}{\partial\sigma_d} = 0, \ \frac{\partial^2\Omega}{\partial\sigma_d^2} \ge 0, \quad \frac{\partial\Omega}{\partial\pi} = 0, \ \frac{\partial^2\Omega}{\partial\pi^2} \ge 0, \quad \frac{\partial\Omega}{\partial q} = 0, \ \frac{\partial^2\Omega}{\partial q^2} \ge 0$$

mesons in RPA

meson propagator D at RPA



considering all possible channels in the bubble summation

meson polarization functions

$$\Pi_{mn}(k) = i\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left(\Gamma_m^* S(p+k)\Gamma_n S(p)\right) \qquad \Gamma_m = \begin{cases} 1, & m=\sigma \\ i\tau_+\gamma_5, & m=\pi_+ \\ i\tau_-\gamma_5, & m=\pi_- \\ i\tau_3\gamma_5, & m=\pi_0 \end{cases}$$

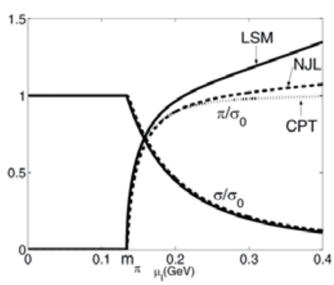
pole of the propagator determines meson masses M_m

$$\det\begin{pmatrix} 1-2G\Pi_{\sigma\sigma}(k) & -2G\Pi_{\sigma\pi_+}(k) & -2G\Pi_{\sigma\pi_-}(k) & -2G\Pi_{\sigma\pi_0}(k) \\ -2G\Pi_{\pi_+\sigma}(k) & 1-2G\Pi_{\pi_+\pi_+}(k) & -2G\Pi_{\pi_+\pi_-}(k) & -2G\Pi_{\pi_+\pi_0}(k) \\ -2G\Pi_{\pi_-\sigma}(k) & -2G\Pi_{\pi_-\pi_+}(k) & 1-2G\Pi_{\pi_-\pi_-}(k) & -2G\Pi_{\pi_-\pi_0}(k) \\ -2G\Pi_{\pi_0\sigma}(k) & -2G\Pi_{\pi_0\pi_+}(k) & -2G\Pi_{\pi_0\pi_-}(k) & 1-2G\Pi_{\pi_0\pi_0}(k) \end{pmatrix}_{k_0=M_m,\vec{k}=0} = 0$$

mixing among normal σ, π_+, π_- in pion superfluid phase,

the new eigen modes $\bar{\sigma}, \bar{\pi}_+, \bar{\pi}_-$ are linear combinations of σ, π_+, π_-

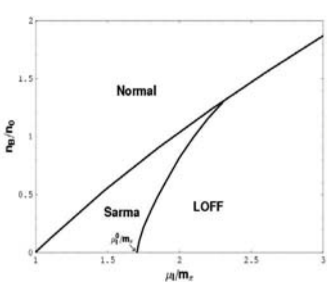
phase diagram of pion superfluid



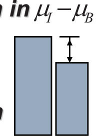
chiral and pion condensates at $T=\mu_B=\vec{q}=0$ in NJL, Linear Sigma Model and Chiral Perturbation Theory, there is no remarkable difference around the critical point.

analytic result:
critical isospin chemical potential for pion superfluidity is exactly the pion mass in the vacuum:

$$\mu_I^c = m_\pi$$



pion superfluidity phase diagram in $\mu_I - \mu_B$ plane at T=0



μ_I: average Fermi surface

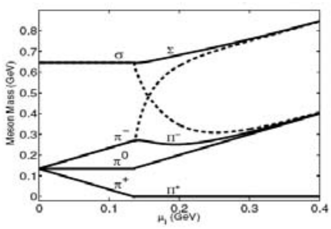
$\mu_B(n_B)$: Fermi surface mismatch

homogeneous (Sarma, $\vec{q}=0$) and inhomogeneous pion superfluid (LOFF, $\vec{q}\neq 0$)

magnetic instability of Sarma state at high average Fermi surface leads to the LOFF state

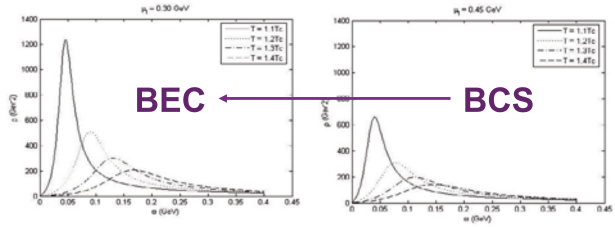
BCS-BEC crossover of pion superfluid

meson mass, Goldstone mode

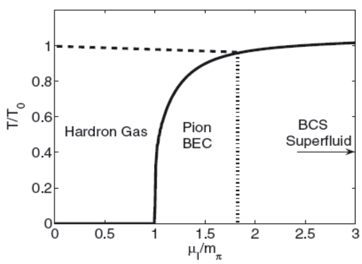


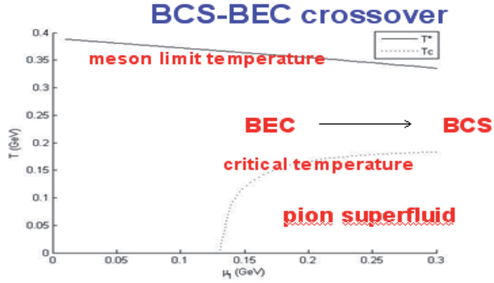
meson spectra function

$$\rho(\omega,\vec{k}) = -2\,\mathrm{Im}\,D(\omega,\vec{k})$$



phase diagram



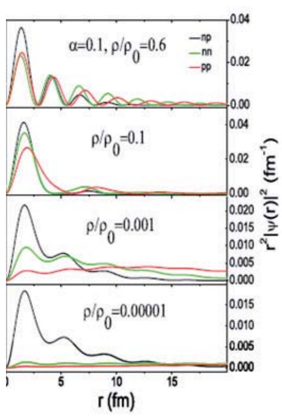


BCS-BEC crossover in asymmetric nuclear matter

★ *transition from BCS pairing to BEC in low-density asymmetric nuclear matter,* *U. Lombardo, P. Nozières:PRC64, 064314 (2001)*

★ *spatial structure of neutron Cooper pair in low density uniform matter,* *Masayuki Matsuo:PRC73, 044309 (2006)*

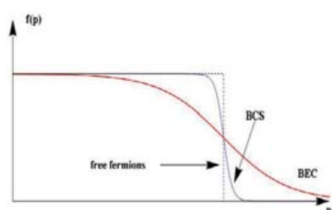
★ *BCS-BEC crossover of neutron pairs in symmetric and asymmetric nuclear matters, J. Margueron: arXiv:0710.4241*



asymmetric nuclear matter with both np *and* nn *and* pp *pairings* *Mao, Huang, Zhuang, 2008*

considering density dependent Paris potential and nucleon mass

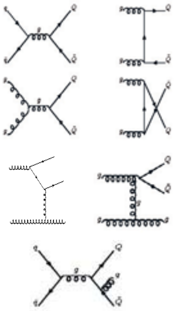
$$\psi(\vec{r}) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{r}} \varphi(\vec{p})$$

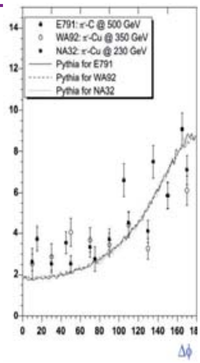


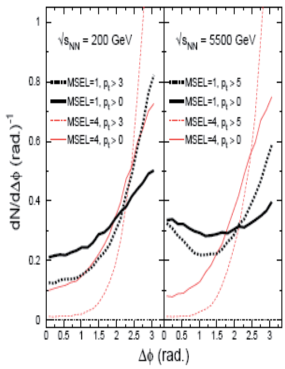
there exists a strong Friedel oscillation in BCS region, and it is washed away in BEC region.

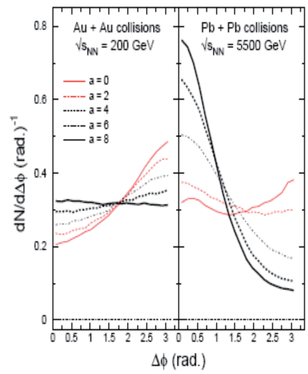
near side $D\bar{D}$ correlation as a signature of sQGP

Zhu, Xu, Zhuang, PRL100, 152301(2008)









* c quark motion in QGP:

$$d\vec{p}/dt = -\gamma(T)\vec{p} + \vec{\eta},$$

$$\gamma(T) = aT^2,$$

we take drag coefficient to be a parameter charactering the coupling strength

* QGP evolution:
ideal hydrodynamics

for strongly interacting quark-gluon plasma:

- at RHIC, the back-to-back correlation is washed out.
- at LHC, c quarks are fast thermalized, the strong flow push the D and Dbar to the near side!

large drag parameter is confirmed by R_AA and v_2 of non-photonic electrons (PHENEX, 2007; Moore and Teaney, 2005; Horowitz, Gyulassy, 2007).

July, 2008 Summer School on Dense Matter and HI Dubna 37

conclusions

* BCS-BEC crossover is a general phenomena from cold atom gas to quark matter.

* BCS-BEC crossover is closely related to QCD key problems: vacuum, color symmetry, chiral symmetry, isospin symmetry ……

* BCS-BEC crossover of color superconductivity and pion superfluid is not induced by simply increasing the coupling constant of the attractive interaction but by changing the corresponding charge number.

* There are potential applications in heavy ion collisions (at CSR/Lanzhou, FAIR/GSI and RHIC/BNL) and compact stars.

thanks for your patience

July, 2008 Summer School on Dense Matter and HI Dubna 38


International Conference on Strangeness in Quark Matter 2008
October 6 - 10, 2008 Beijing, China
E-mail: sqm2008@mail.tsinghua.edu.cn
http://qm.phys.tsinghua.edu.cn/SQM2008
Conference Topics
• Strange and Heavy Quark Production in Nuclear Collisions
• Strange and Heavy Quark Production in Elementary Processes
• Bulk Matter Phenomena Associated with Strange and Heavy Quarks
• Astrophysics of Strangeness
• Open Questions and New Developments
International Advisory Committee
Jörg Aichelin (Nantes)
Federico Antinori (Padova)
Tamás Biró (Budapest)
Peter Braun-Munzinger (GSI)
Jean Cleymans (Cape Town)
László Csernai (Bergen)
Timothy Hallman (BNL)
Huan Zhong Huang (UCLA)
Takeshi Kodama (Rio de Janeiro)
Carlos Lourenço (CERN)
Yu-Gang Ma (Shanghai)
Jes Madsen (Aarhus)
Yasuo Miake (Tsukuba)
Berndt Müller (Duke)
Grazyna Odyniec (LBNL)
Helmut Oeschler (Darmstadt)
Johann Rafelski (Arizona)
Hans Georg Ritter (LBNL)
Karel Safarik (CERN)
Jack Sandweiss (Yale)
George S. F. Stephans (MIT)
Horst Stöcker (Frankfurt)
Thomas Ullrich (BNL)
Nu Xu (LBNL)
William A. Zajc (Columbia)
Local Organizing Committee
Xiangzhou Cai (Shanghai)
Weiqin Chao (Beijing)
Liewen Chen (Shanghai)
Jianping Cheng (Beijing)
Jinghua Fu (Wuhan)
Yuanning Gao (Beijing)
Xiaomei Li (Beijing)
Zuotang Liang (Jinan)
Feng Liu (Co-chair, Wuhan)
Yuxin Liu (Beijing)
Qing Wang (Beijing)
Qun Wang (Hefei)
Hushan Xu (Lanzhou)
Daicui Zhou (Wuhan)
Pengfei Zhuang (Co-chair, Beijing)
Bingsong Zou (Beijing)
SQM 2008
清華大學
Tsinghua University
July,
ubna
39