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Quark-Hadron Mixed Phase

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JINT, Dubna, 141980 Russia

Abstract. The concept of a mixed quark-hadron phase is considered. Dynamics of heavy-ion collisins in a large energy range is outlined. Various possible probes of the mixed phase manifestation as well as modern status of experimental signals are discussed.

I am grateful to my colleagues A.Khvorostukhin, V.Skokov and A.Shanenko for fruitful collaboration.

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Quark-Hadron Mixed Phase

V. Toneev
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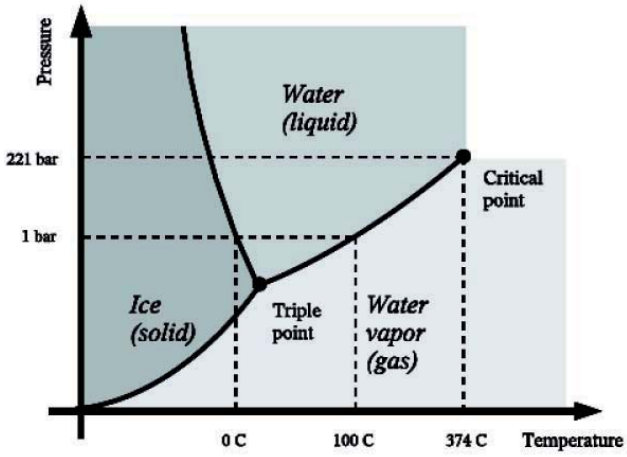
- Introductory remarks
- Mixed-phase concept
- A glance at collision dynamics
- Status of signals for a phase transition
- Concluding remarks

Dubna International Advanced School of Theoretical Physics,
Helmholtz International Summer School, July 14-26, 2008

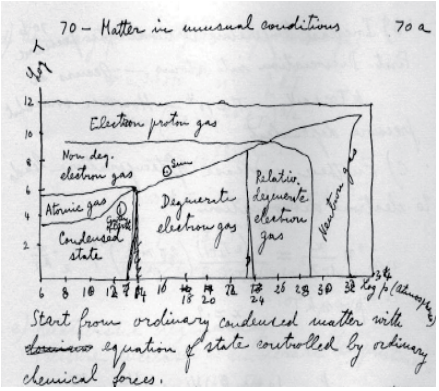
Introductory remarks



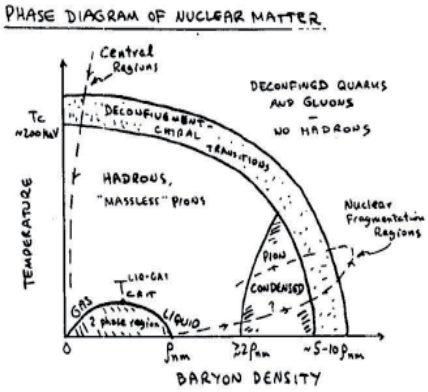
Phase diagrams

E.Fermi: Notes on themodymamics and statistics, 1953

G.Baym: Nucl. Phys. A**418** (1984) 433c

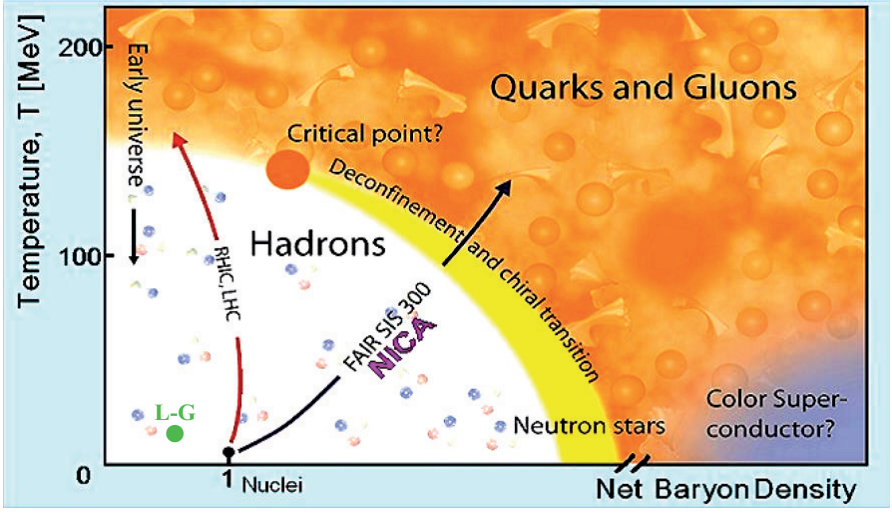


Condensed matter



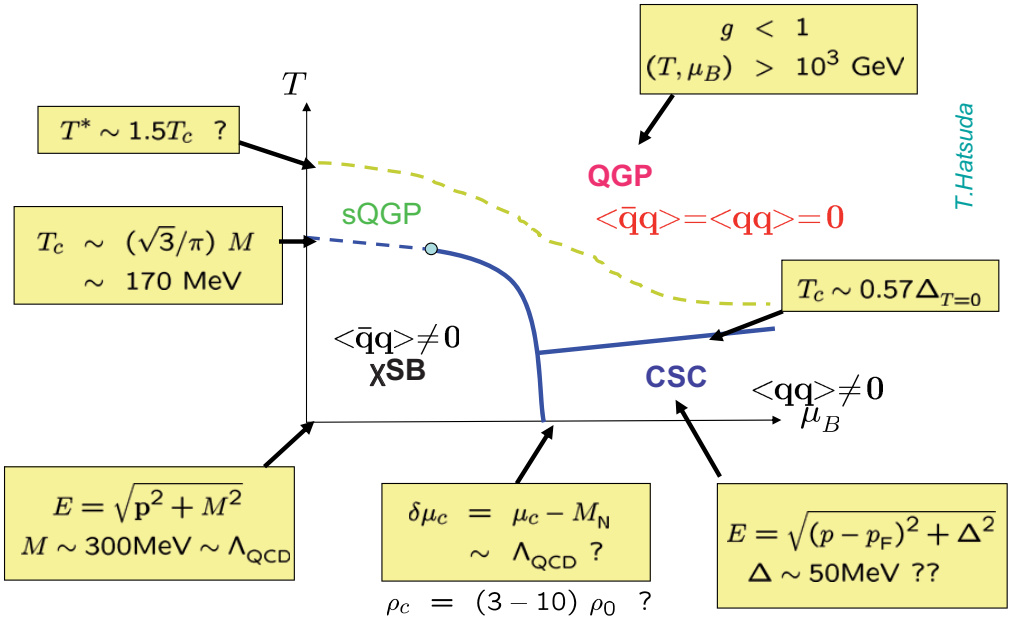
Nuclear matter

Phase diagram – artist's view



Phases of strongly interacting nuclear matter

Phase diagram for QCD matter (scales)



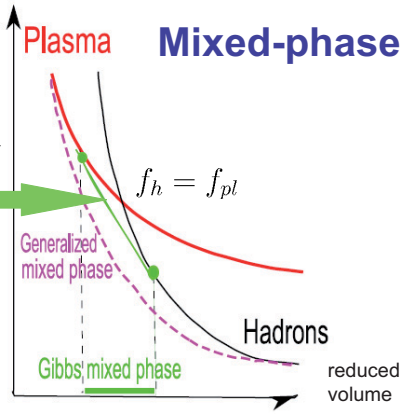
Mixed-phase concept

Mixed-phase concept

reduced free energy

$$f = \frac{F}{N} = -\frac{T \ln \mathcal{Z}}{N}$$

Maxwell construction



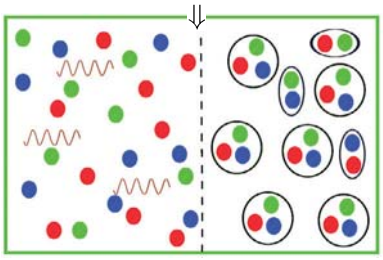
$$\left(\frac{\partial P}{\partial V}\right)_{N,T} < 0$$

$$P = -\left(\frac{\partial F}{\partial V}\right)_{N,T}$$

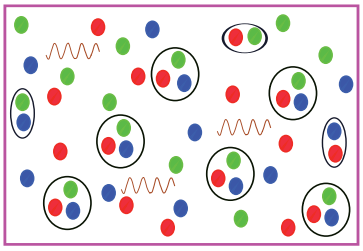
$$\frac{\partial^2 F}{\partial V^2} > 0$$

$$v = \frac{V}{N} = 1/\rho$$

The Gibbs mixed phase (spatially separated)



Generalized mixed phase (homogeneous)



Deconfinement transition (an illustrative example)

for the massless Stefan-Boltzmann gas

$$P = g_i \left(\frac{\pi^2}{90}\right) T^4$$

$$\varepsilon = g_i \left(\frac{\pi^2}{30}\right) T^4 = 3P$$

g_i – degeneracy factor

$$c_s^2 = P/\varepsilon = 1/3$$

Two-phase model: pions ⟺ quarks+gluons (pl)

Pion gas

$$g_\pi = 3$$

$$P_\pi = 3\left(\frac{\pi^2}{90}\right) T^4$$

$$\varepsilon_\pi = 3\left(\frac{\pi^2}{90}\right) T^4 = 3P_\pi$$

$$g_{pl} = 2\left(\frac{N_c N_f}{4} + N_c^2 - 1\right)$$

$$P_{pl} = 37\left(\frac{\pi^2}{90}\right) T^4 - B$$

$$\varepsilon_{pl} = 37\left(\frac{\pi^2}{30}\right) T^4 + B = 3P_{pl} + 4B$$

$N_f=2$, $N_c=3$

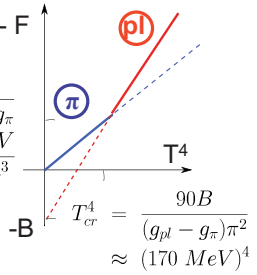
Plasma

Gibbs conditions fot the first order phase transition

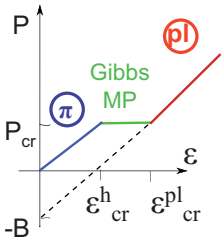
$P_\pi = P_{pl} = P_{cr}$

$T_\pi = T_{pl} = T_{cr}$

$$P_{cr} = \frac{B}{g_{pl} - g_\pi} \approx 35 \frac{MeV}{fm^3}$$

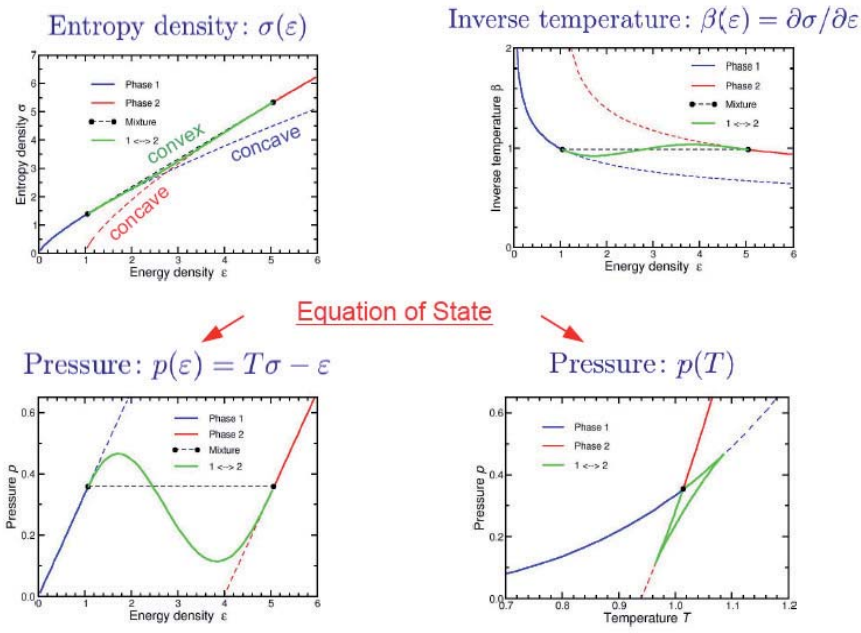


$$T_{cr}^4 = \frac{90B}{(g_{pl} - g_\pi)\pi^2} \approx (170\ MeV)^4$$



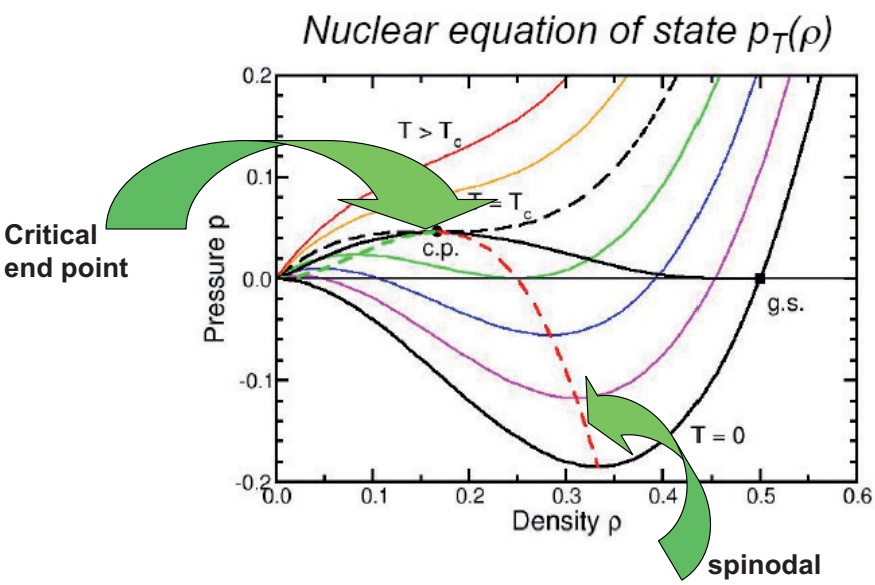
Latent heat ~1.5 GeV/fm^3

Simplest example (no conserved charge)

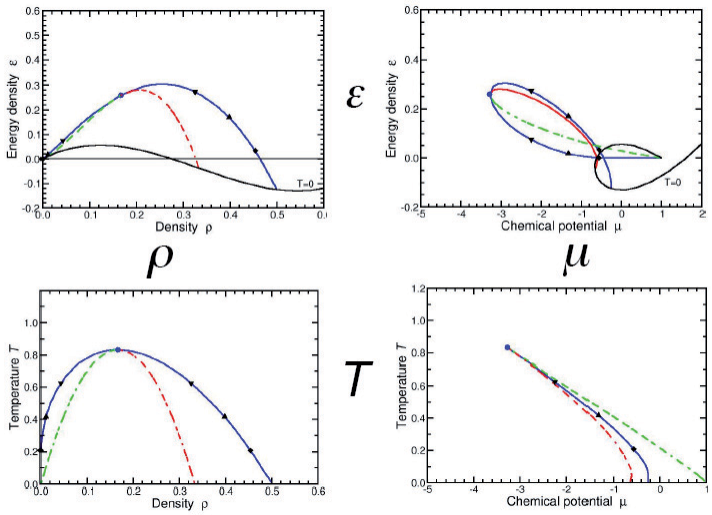


J.Randrup, Nucl.Phys. A752 (2005) 384c

Familiar example (one conserved charge)



Nuclear phase diagram in different representation (one conserved charge)

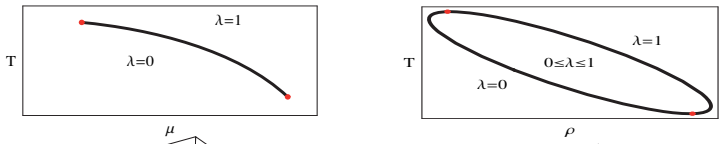


J.Randrup, CEOD, Darmstadt, July 9-13, 2007

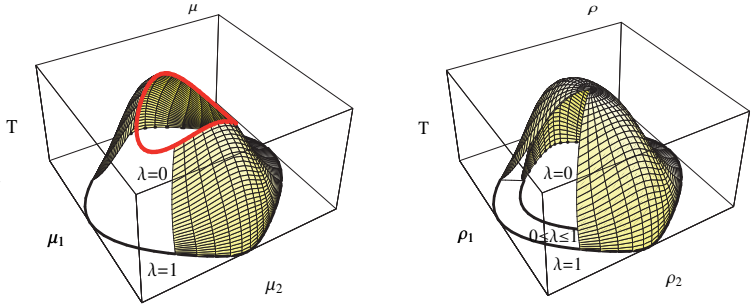
Two conserved charges

for an extensive thermodynamic quantity $A=\lambda A_2 + (1-\lambda) A_1$ $\lambda=V_2 / V$

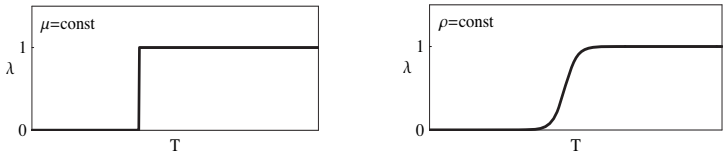
1 charge



2 charges



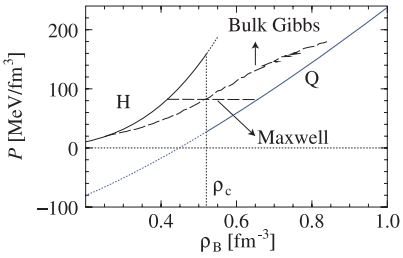
volume fraction

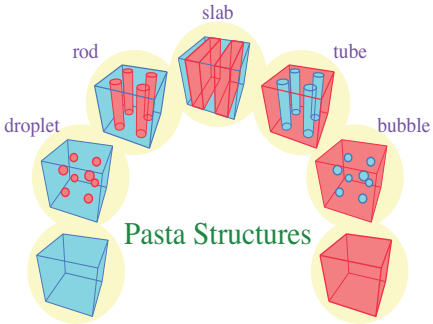


A.Sissakian, A.Sorin, V.Toneev, nucl-th/0608032

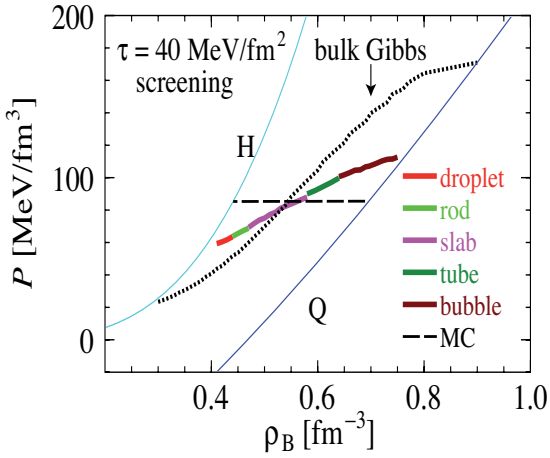
"Pasta" structures (finite size effects)

Compact stars, T=0





T.Maruyama et al., nucl-th/0605075



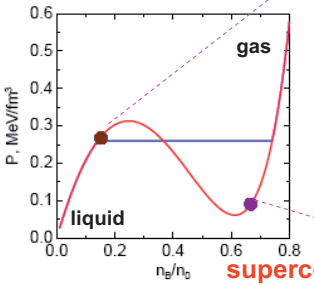
Charge density and particle density are non-uniform

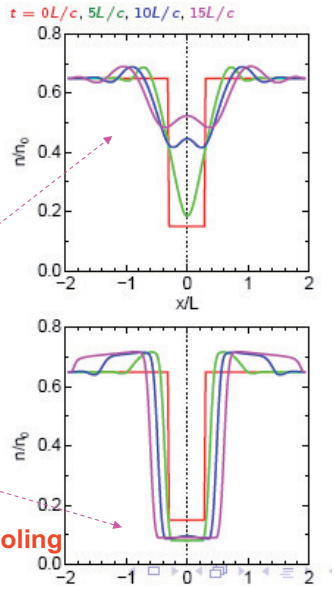
Dynamics of the first order phase transition

Evolution of a gas babble in a mixed phase. Van der Waals equation of state.

Navier-Stokes dynamics

Bubble resorption

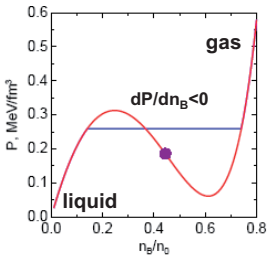


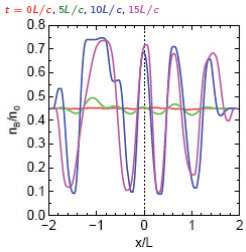


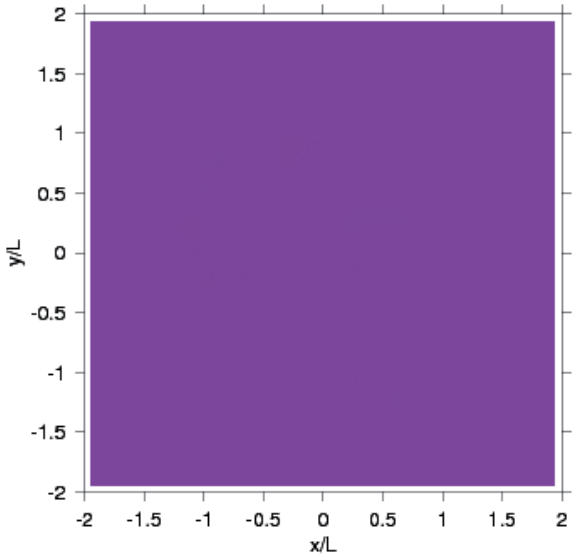
supercooling

V.Skokov, D.Voskresensky

Evolution of a mixed phase







evolving instabilities

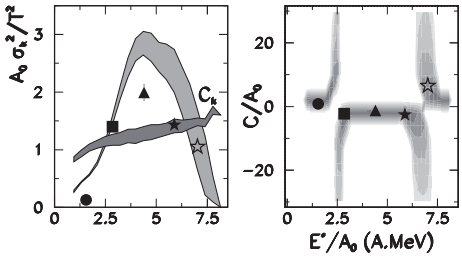
http://www-linux.gsi.de/~skokov/playme.gif

Heat capacity (nuclear matter)

$$T^{-1} = \frac{\partial S}{\partial E} \qquad C_i^{-1} = -T^2 \frac{\partial^2 S}{\partial E^2}$$

$$E_t = E_1 + E_2 \qquad T_1 = T_2 = T$$

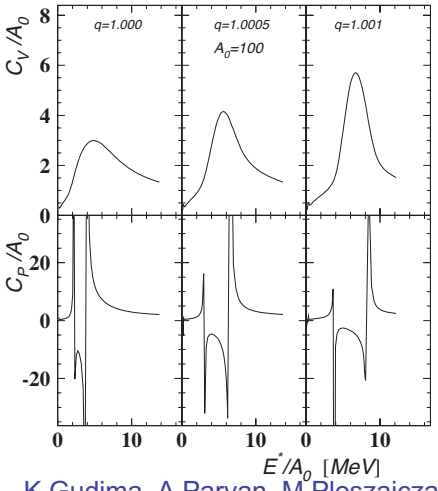
$$C_t \approx C_1 + C_2 = \frac{C_1^2}{C_1 - \sigma_1^2/T^2}$$



heat capacity is negative in the mixed phase

M.D'Agostino et al. NP **A734** (2004) 512

Statistical Tsallis model of multifragmentation



K.Gudima, A.Parvan, M.Ploszajczak
V.Toneev, PRL **85** (2000) 4691

Susceptibility

Nambu--Jona-Lasinio model

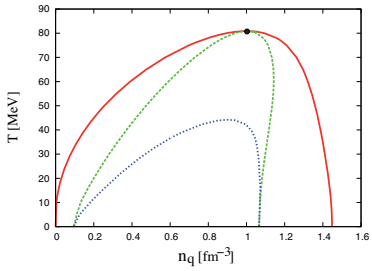
$$\chi_q = -\frac{1}{V}\frac{\partial^2 F}{\partial \mu_q^2} = \frac{1}{V}(< N^2 > - < N >^2)$$

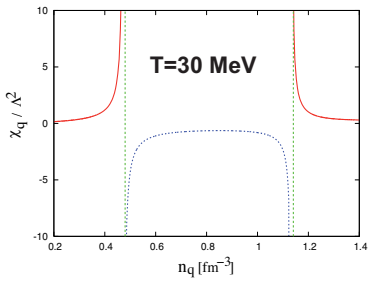
$$\left.\frac{\partial P}{\partial V}\right|_T = 0 \ : \ \text{isothermal}$$

$$\left.\frac{\partial P}{\partial V}\right|_S = 0 \ : \ \text{isentropic}$$

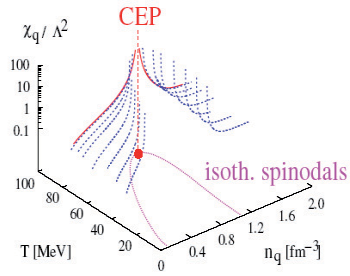
T_c=81 MeV

μ=330 MeV





poles, negative branch

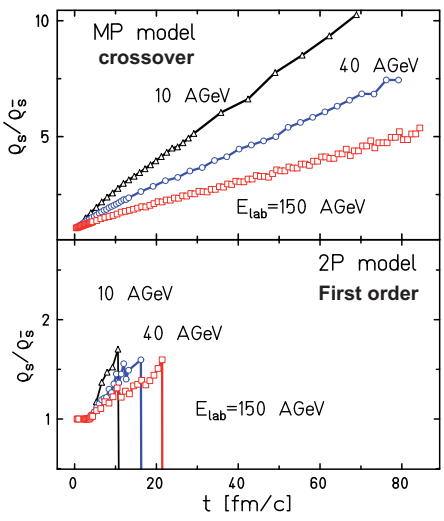


instability region shrinks toward the critical end point

K.Redlich, B.Friman, C.Sasaki, J. Phys. **G34** (2007) 437

Distillation in the mixed phase

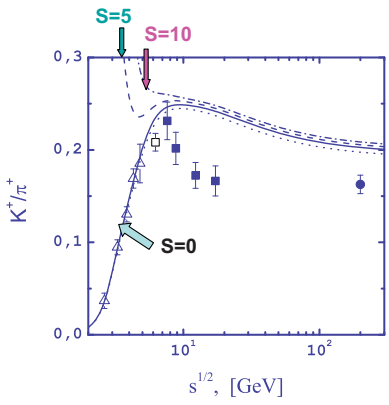
for unbound quarks in a fireball



strangeness separation

V.Toneev et al., Eur. Phys. J. **C32** (2004) 415.

K^+/π^+ excitation function, statistical equilibrium along the freeze-out line

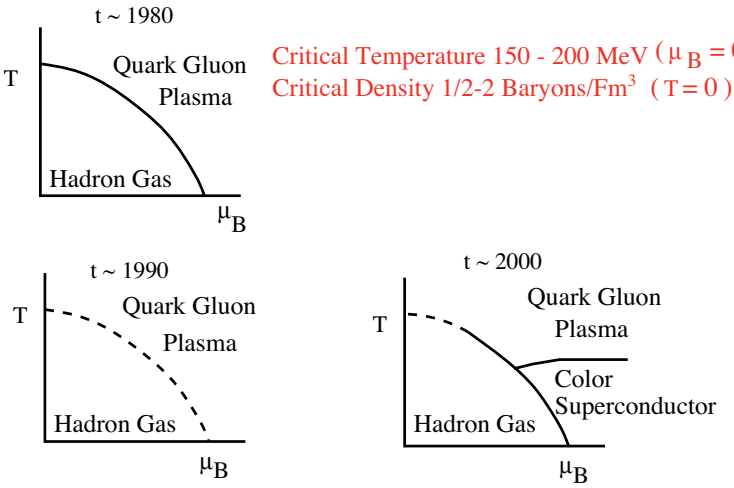


At CP the K^+/π^+ ratio should jump down on the S=0 curve

V.Toneev, A.Parvan, J. Phys. **G**: Nucl. Part. Phys. **31** (2005) 583

Lattice QCD calculations (history)

The Evolving QCD Phase Transition

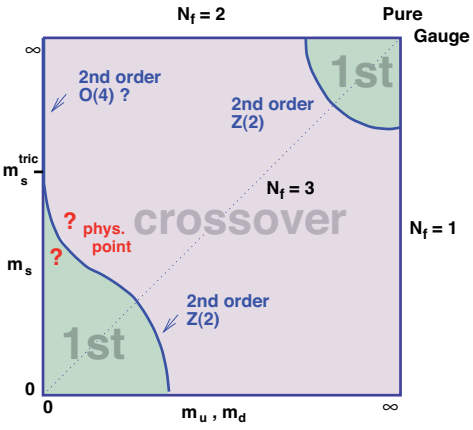


Critical Temperature 150 - 200 MeV ($\mu_B = 0$)
Critical Density 1/2-2 Baryons/Fm^3 (T = 0)

L. McLerran, Prama **60** (2003) 575

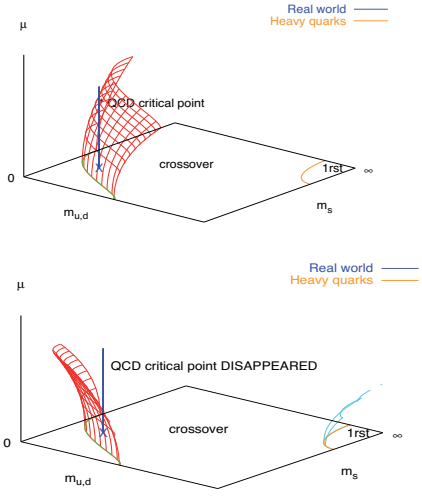
Lattice QCD predictions for the order of PT

E.Laermann, O.Phillipsen, Ann.Rev.Nucl.Part. Sci.,51 (2003) 163



μ_B=0

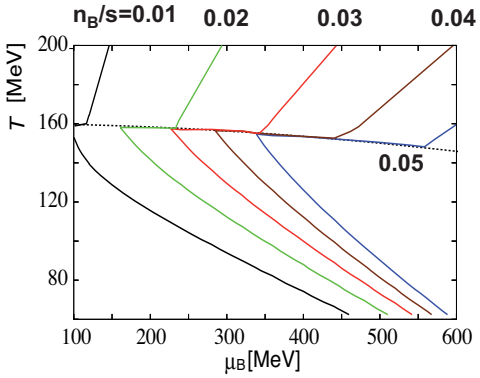
S.Kim et al. hep-lat/510069



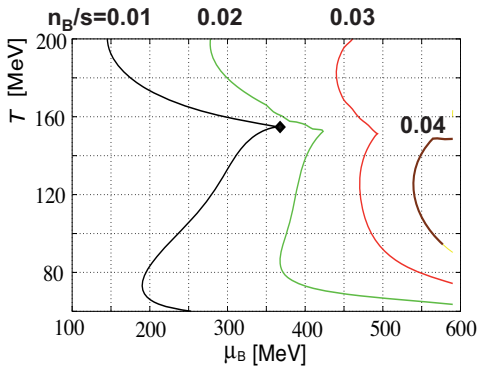
$\mu_B \neq 0$

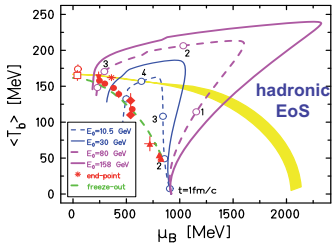
Time evolution near the QCD critical point

Two-phase EoS (first order)



Two-phase EoS with CP



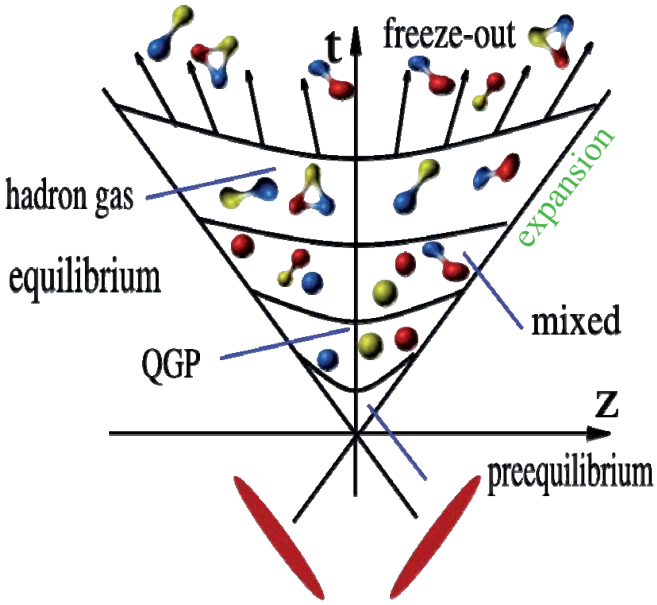


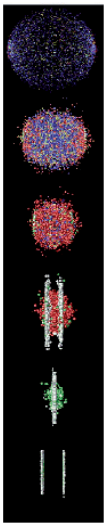
Critical point acts as an attracter of isentropic trajectories

C.Nonaka, M.Asakawa,
Phys. Rev. **C71** (2005) 044904

A glance at the collision dynamics

Evolution of nucleus-nucleus collisions





Nuclear interaction scales

d - repulsion NN force range

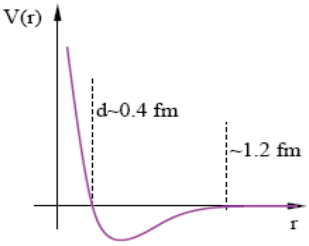
$\Lambda = \frac{1}{\sigma\rho}$ - (nucleon) mean free path

$\rho_0 \simeq 0.16\ fm^{-3}$ $\sigma \simeq 40\ mb \rightarrow \Lambda \sim$ 1.5 fm

Pauli principle

compression . . .

L - "macroscopic" length, 2-8 fm



units	d	Λ	L	d/Λ	$Kn = \Lambda/L$
air (10^{-8} cm)	1	10^5	10^8	10^{-3}	10^{-3}
liquid (10^{-8} cm)	1	2-10	10^8	0.1-0.5	10^{-7}
nuclei (0.4/1.2 fm)	1	1.5-2	2-8	0.2-0.6	1-0.2

$$\textit{kinetics} \Leftarrow \quad \boxed{d \ll \Lambda \ll L} \quad \Rightarrow \textit{hydrodynamics}$$

For nuclear case (intermediate energies) : $d < \Lambda < L$

Relativistic Boltzmann equation

$$\boxed{(p_\mu \partial^\mu)\ f_i(x,p_i) = \sum_j C^{rel}(x,p_i) + \sum_r R_{r\to i}}$$

$$= -f_i(x,p_i)\sum_j \int d\omega_j f_j(x,p_j) Q_{ij}\sigma^{ij} + \sum_{kj}\int d\omega_k\, d\omega_j \Phi(p_j p_k \mid x, p_i, \tau_f)$$

$$+ \sum_r \int d\omega_{k'} d\omega_r\ f_r(x,p_r)\ \Gamma^{r\to i+k'}\ \delta(p_r - p_i - k')$$

hadron production rate

$$\Phi(p_j p_k \mid x, p_i, \tau_f) = \int dx'\ \underbrace{f_k(x',p_i) f_j(x',p_j) Q_{ij}\sigma^{ij}}_{\text{collision rate}}\ \underbrace{\phi(x' \mid x, p_i, \tau_f)}_{\text{transition prob.}}$$

with $d\omega = d^3p/E,\ \ Q_{ij} = (p_i p_j)^2 - p_i^2 p_j^2 = \mid v_i - v_j \mid E_i E_j$

transition probability for a finite formation time

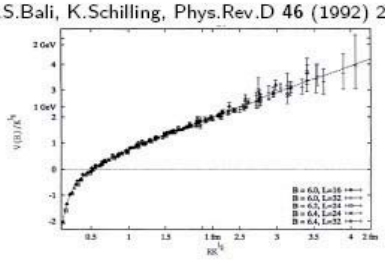
$$\phi(x' \mid x, p, \tau_f) = \frac{1}{\sigma}\frac{d\sigma}{d\omega}\ \theta(t - t' - \tau_f)\ \delta^{(3)}(\vec{x} - \vec{x}' - \frac{\vec{p}}{E}(t - t'))\ F(\tau_f)$$

- ⋆ multiple particle production ⇒ coupled set of equations for stable hadrons and resonances $\{h_i\}$; new flavors
- ⋆ finite formation time $\theta(t - \tau_f)$, $\tau_f = (E/m)\tau_f^0$ with $\tau_f^0 \sim 1$ fm; ⇒ memory (retarded) effect (non-Markovian process)

- new degrees of freedom (QCD) : quark/gluons , strings, formation of color rope

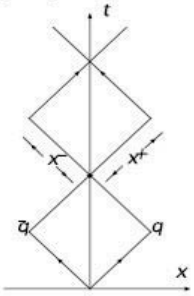
⋆ Hadron as a string

G.S.Bali, K.Schilling, Phys.Rev.D **46** (1992) 2636



$$V_{q\bar{q}} = -\frac{\alpha_{eff}}{r} + \kappa r$$

yo-yo mode



$$H_{yo-yo} = \mid p_1 \mid + \mid p_2 \mid + \kappa \mid x_1 - x_2 \mid$$

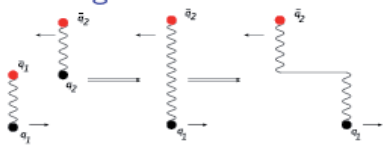
$$\frac{dp_{1,2}}{dt} = \pm\kappa\ ,\ \frac{dx_{1,2}}{dt} = \pm 1$$

$$x^+ = \frac{p^+}{\kappa} = \frac{E+p}{\kappa}\ ,\ x^- = \frac{p^-}{\kappa} = \frac{E-p}{\kappa},\ S = \frac{p^+p^-}{\kappa^2} = \frac{E^2-p^2}{\kappa^2} = \frac{m^2}{\kappa^2}$$

- Particularities of space-time evolution

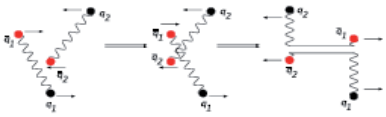
CLASSICAL STRING THEORY

★ string fusion



time ⇒

★ string rearrangement

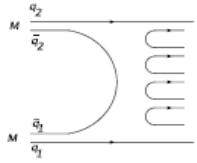


★ leading particle effect

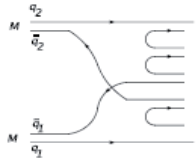
★ color rope formation

DUAL TOPOL. MODEL

★ planar diagram

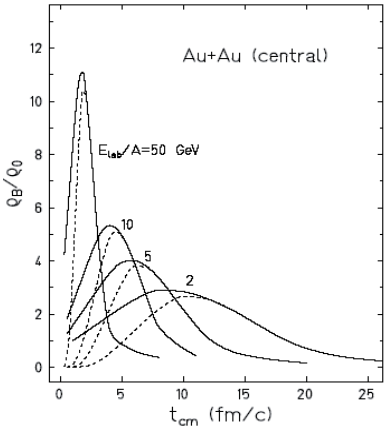


★ cylindrical diagram



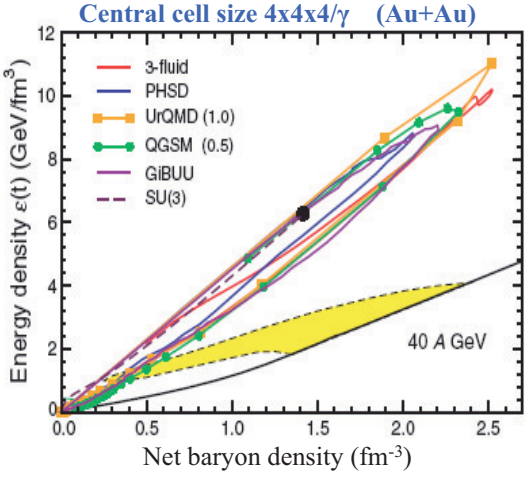
Kinetic results

QGSM model



I.C.Arsene et al., Phys. Rev. C75 (2007) 034902

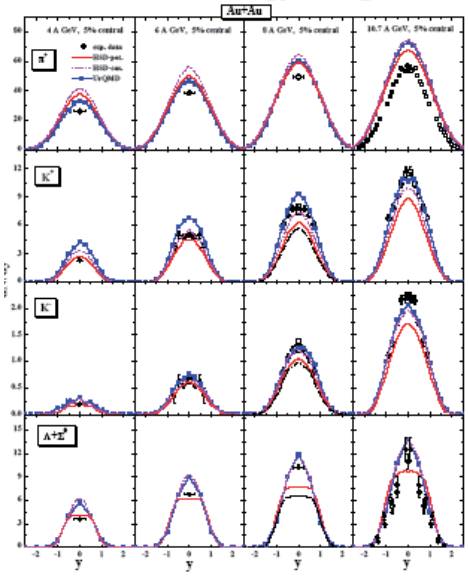
Central cell size 4x4x4/γ (Au+Au)



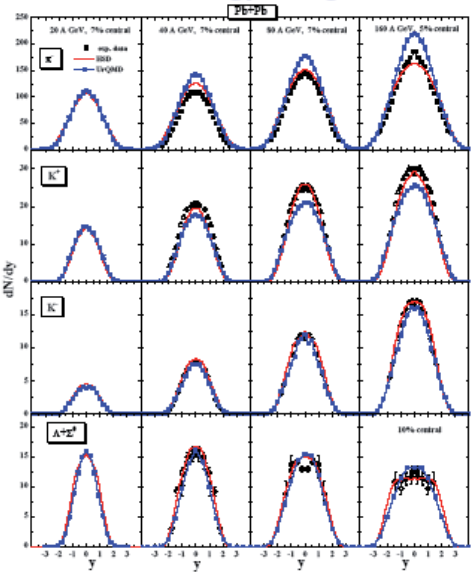
All dynamical models (w/o PT) demonstrate similar results

Rapidity distributions

AGS energy range



SPS energy range



H.Weber et al., Phys.Rev. **C67** (2003) 014904

Hydrodynamic approach

- Conservation laws (Gauss theorem) $\Rightarrow$ Fluid dynamics

$$\partial_\mu \; J_i^\mu \;=\; 0 \quad \text{net charge } i \text{ conservation} \qquad \boxed{4}$$

$$\partial_\mu \; T^{\mu\nu} \;=\; 0 \quad \text{energy momentum conservation} \qquad \boxed{10}$$

- Tensor decomposition of the charge current J^μ and energy-momentum tensor $T^{\mu\nu}$ with respect to 4-velocity u^μ

$$J_i^\mu \;=\; n_i u^\mu + (g_\nu^\mu - u^\mu u_\nu)\; J_i^\nu + \ldots$$

$$T^{\mu\nu} \;=\; \varepsilon\; u^\mu u^\nu - P\,(g^{\mu\nu} - u^\mu u^\nu) + \; q^\mu u^\nu + q^\nu u^\mu + \pi^{\mu\nu} + \ldots$$

Navier-Stockes

$$\text{with} \quad J_i^\mu \;=\; \int \frac{d^3p}{p_0}\; p^\mu \;\left[f_i(x,p) - \bar{f}_i(x,p)\right]$$

$$T^{\mu\nu} \;=\; \int \frac{d^3p}{p_0}\; p^\mu p^\nu \left[f(x,p) + \bar{f}(x,p)\right]$$

$$f_i(x,p) \;=\; \frac{g_i}{(2\pi)^3}\; \exp\left[((u_\mu p^\mu(x) - \mu_i(x))/T(x)) \pm 1\right]^{-1}$$

Gradient expansion

⋆ Perfect hydro in local thermodynamical equilibrium

$$\begin{aligned} J_i^\mu &= n_i u^\mu \\ T^{\mu\nu} &= \underbrace{\varepsilon\ u^\mu u^\nu - P\ (g^{\mu\nu} - u^\mu u^\nu)}_{\text{perfect hydro}} \quad \boxed{6} \end{aligned}$$

$$\boxed{+\ \text{EoS} \quad \varepsilon(p)}$$

⋆ First order dissipative corrections (viscosity, heat capacity)

$\Rightarrow$ acausality

Navier-Stokes

⋆ Second order corrections $\Rightarrow$ + 14 Grad equations

Israel-Stuart

Spatial-temporal variation of the macro fields has to be SMALL

Relation between kinetics and hydrodynamics

- The non-relativistic case (for nucleons)

$$\int d^3p \begin{bmatrix} \rho \\ \vec{v} \\ \epsilon \end{bmatrix} \frac{d}{dt} f(\vec{p}, \vec{x}, t) = \int d^3p \begin{bmatrix} 1 \\ \vec{p}/m_N \\ p^2/2m_N \end{bmatrix} \mathrm{I}\,(\vec{p}, \vec{x}, t)$$

Boltzmann equation + local equilibrium hypothesis

$$\begin{aligned} \vec{v} &= \vec{u} + \vec{c}\ , \quad \text{hydro } \vec{u} = <\vec{v}> \ , \quad \text{thermo } <\vec{c}> = 0 \\ \rho < c_i c_k > &= P\ \delta_{ik} + \Pi_{ik}\ , \quad \rho < c^2 c_k > = Q_k \end{aligned}$$

$$\frac{\partial \rho}{\partial t} - \frac{\partial}{\partial x_k} \rho u_k = 0$$

$$\frac{\partial \rho u_i}{\partial t} - \frac{\partial}{\partial x_k} \rho u_i u_k = \{ \frac{\partial}{\partial x_k} \Pi_{ik} - \frac{\partial}{\partial x_i} P \}$$

$$\frac{\partial \epsilon}{\partial t} - \frac{\partial}{\partial x_k} \epsilon u_k = \{ \frac{\partial}{\partial x_k} \Pi_{ik} u_i - \frac{\partial}{\partial x_i} P u_k - \frac{\partial}{\partial x_k} Q_k \}$$

$\rho, \vec{u}, \epsilon, P$

$\boxed{\text{EOS}}$

hydro: $\Lambda \ll L$; $Re = \frac{inertial}{viscous} \simeq \frac{M}{\Lambda/L} \simeq 4 - 10$ with $M = v/c_s$ and $c_s = \sqrt{\partial P/\partial \rho|_s} \approx 0.2$

turbulent regime $Re \simeq 10^2 - 10^3$

(near a phase transition ?)

Main ingredients of hydro approach

Non-equilibrium and initial state

- **to postulate an initial state** (Landau, Bjorken, …)
- **to calculate it in kinetic models** (QGSM, UrQMD)
- **to introduce many-fluid dynamics** $f(x,p) = \sum_j^M f_j^{eq}(x,p)$

A single fluid may consist of several particle spaces.
Different fluids may be of the same particle spaces.

3-fluid hydro:
Yu.B.Ivanov,
V.N.Russkikh,
V.D.Toneev
Phys.Rev **C73**
(2006) 044904

Equation of state (hadronic EoS)

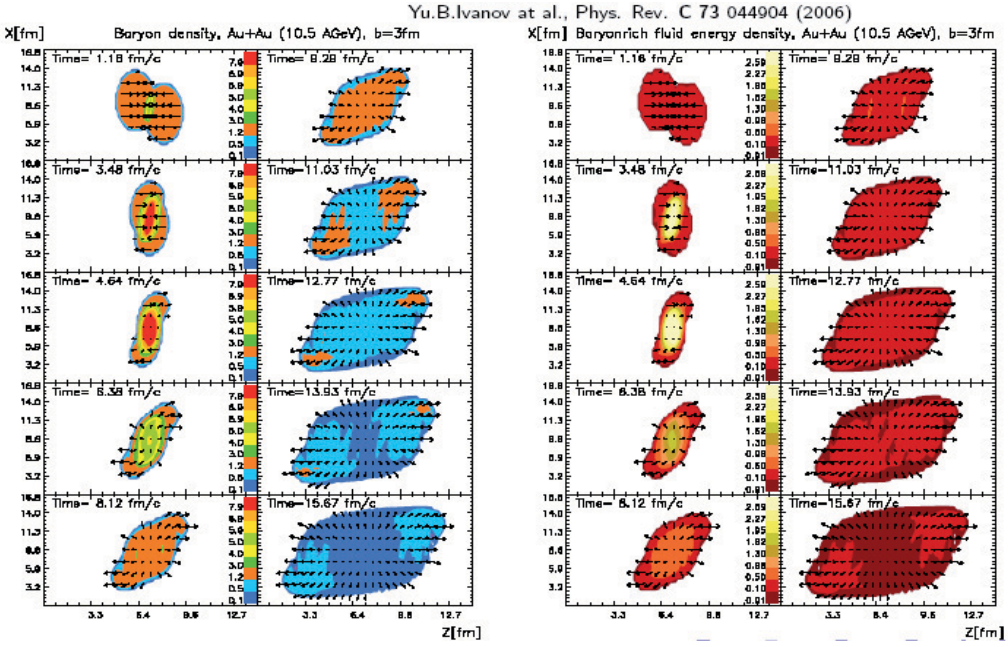
$$\varepsilon(n_B,T) = \varepsilon_{gas}(n_B,T) + W(n_B),$$
$$P(n_B,T) = P_{gas}(n_B,T) + n_B\frac{dW(n_B)}{dn_B} - W(n_B)$$

nuclear potential $W(n_B) = n_B m_N\left[-b\left(\frac{n_B}{n_0}\right) + c\left(\frac{n_B}{n_0}\right)^{\gamma+1}\right]$

Freeze-out procedure (locally $\varepsilon > \varepsilon_f$)

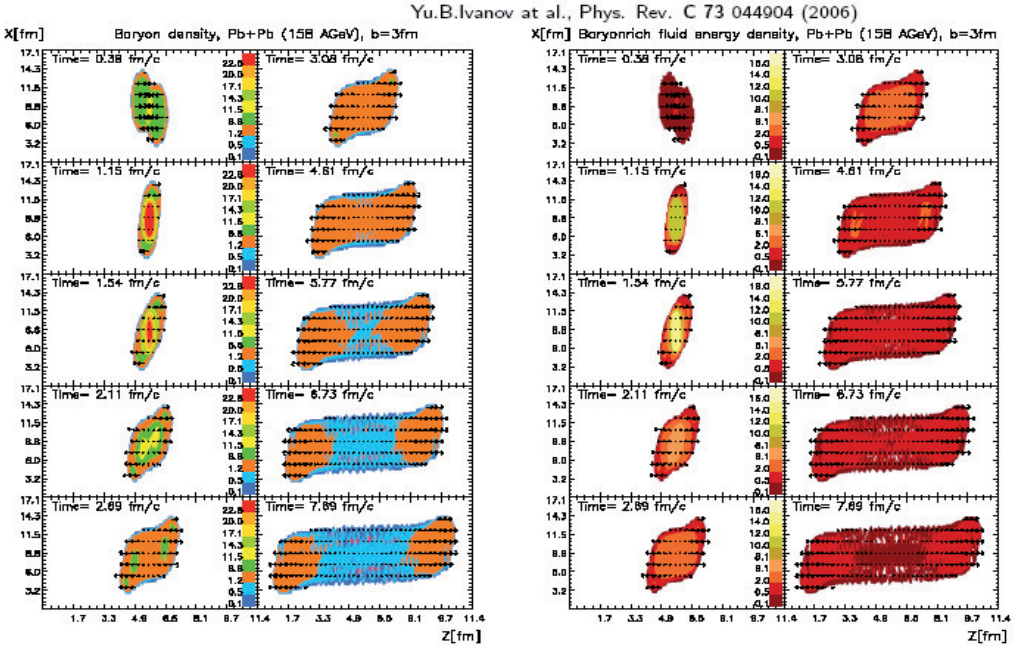
3-fluid hydrodynamic model

Yu.B.Ivanov at al., Phys. Rev. C 73 044904 (2006)



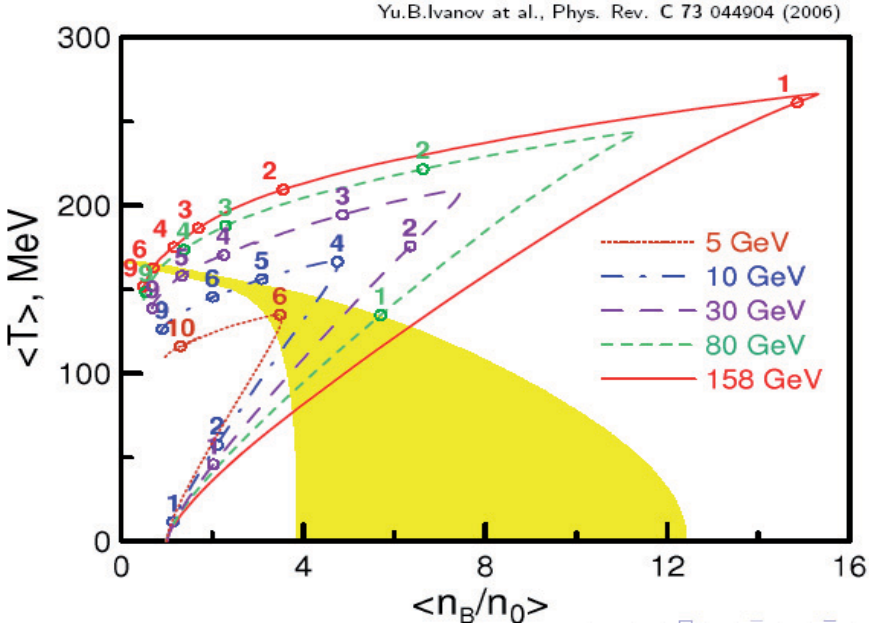
3-fluid hydrodynamic model, cont'd

Yu.B.Ivanov at al., Phys. Rev. C 73 044904 (2006)



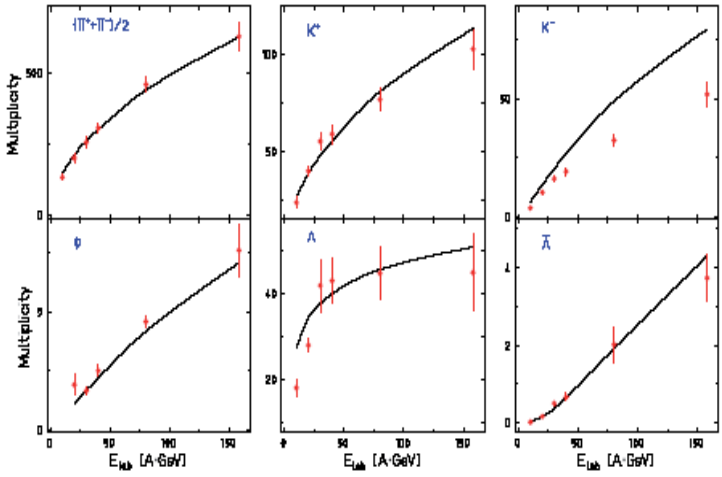
Phase diagram (hadronic EoS)

Yu.B.Ivanov at al., Phys. Rev. C 73 044904 (2006)



Mean particle multiplicity

★ Freeze out (local $\varepsilon < \varepsilon_f$) $\Rightarrow$ observable

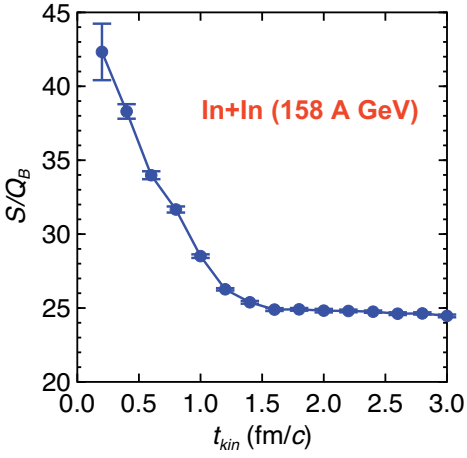


Hybrid model:

♣ **initial state of nucleus-nucleus interaction**

Entropy evolution and a fireball formation are calculated within the transport code QGSM which defines an initial state for subsequent hydrodynamic stage

for participant (central collisions)

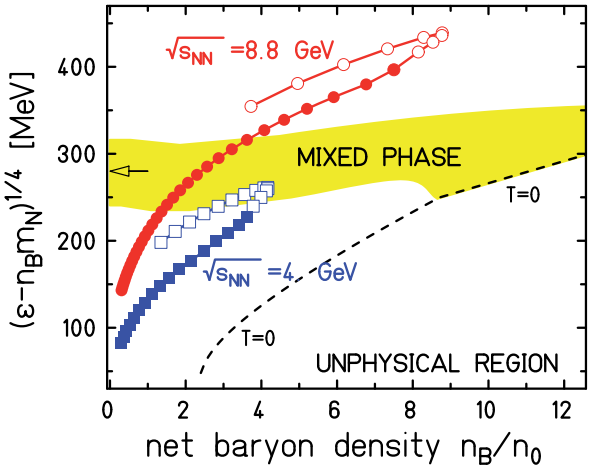


V.Skokov, V.Toneev,
Phys.Rev. **C73** (2006) 021902

Hybrid model, cont'd

♣ **Hydrodynamic expansion stage**

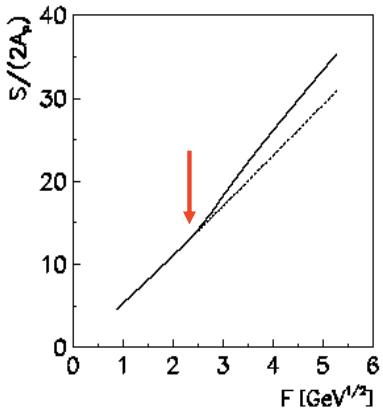
Isentropic expansion of the formed fireball is described by the 3D relativistic hydrodynamics with the above calculated energy and baryon densities as well as the velocity profile. The statistical quark-hadron equation of state with crossover phase transition is used.

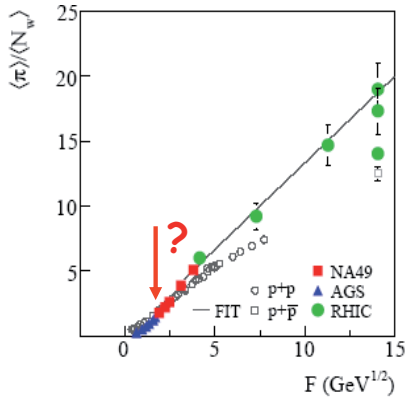


Status of signals of a phase transition in HIC

Onset of the deconfinement

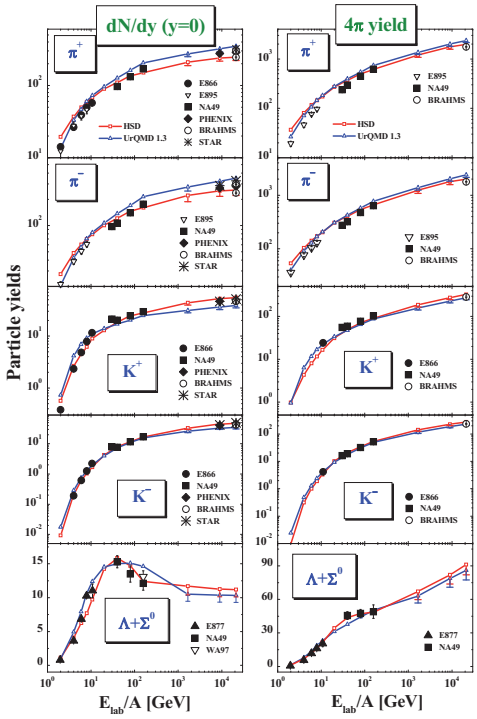
entropy $S \sim n_\pi$





$F=(\sqrt{s_{NN}}-2m_N)^{3/4} / (s_{NN})^{1/4} \approx 2.25\ GeV^{1/2}$
($E_{beam} \approx 30$ AGeV)

M.Gazdzicki, arXiv:0712.3001



Excitation functions of $\pi^\pm$, $K^\pm$, $(\Lambda+\Sigma^0)$ yields

Pion yield is overestimated by HSD below AGS energy and by UrQMD at the top AGS and above (deviations are < 20%).

Reasonable description of strangeness by both models.

E.Bratkovskaya

Strangeness enhancement

- **Strangeness production in a hadronic world (at low energy):**

  N+N –> N+Λ+K requires $\Delta E = 2M_N-(M_K+M_\Lambda+M_N) = 670$ MeV

  π+N –> Λ+K $\Delta E = (M_\pi+M_N)-(M_K+M_\Lambda) = 535$ MeV

- **Strangeness production in a QGP:**

  bare mass of strange quark $m_S \sim 150$ MeV

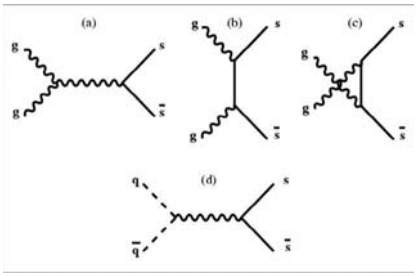
  => s-sbar pair production

  by q-qbar annihilation q+qbar –>s+sbar

  needs only ΔE=300 MeV

  => s-sbar pair can be also produced by

  gluon fusion g+g –>s+sbar



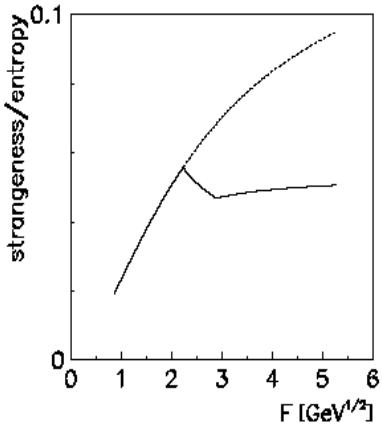
➔ **Strong enhancement of strangeness production in a QGP !**

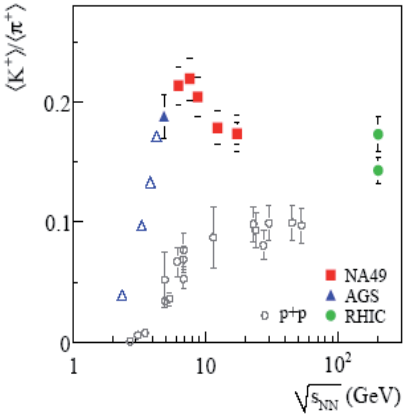
Rafelski-Müller: Phys. Rev. Lett. 48 (1982) 1066

=> strangeness enhancement increases with strangeness content –

stronger effect for multi-strange hadrons Ξ(uss), Ω(sss)

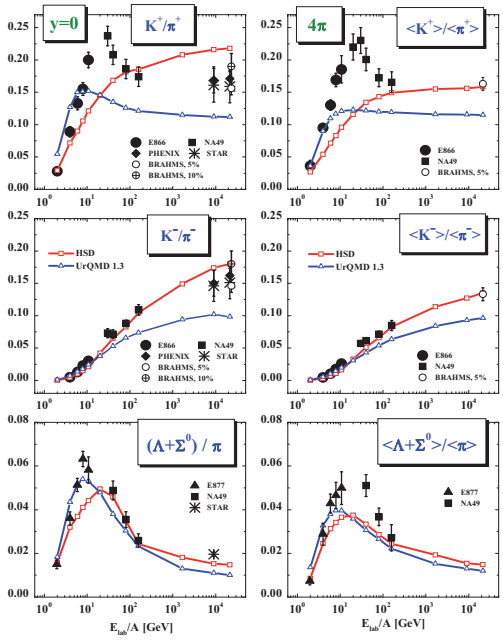
Strange-to-nonstrange particle ratio





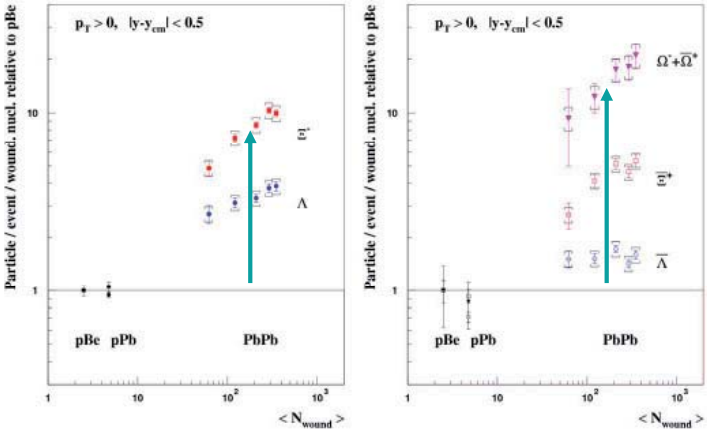
M.Gazdzicki, arXiv:0712.3001

Excitation functions of K^+/π^+, K^-/π^- and $(\Lambda+\Sigma^0)/\pi$ ratios



Experimental K^+/π^+ ratios show a peak at ~30 A GeV ("horn") which is not reproduced by hadronic transport models HSD and UrQMD

Hyperon enhancements at 160 *A* GeV/*c*



[ ] systematic errors:

10% for Λ, Ξ

15% for Ω

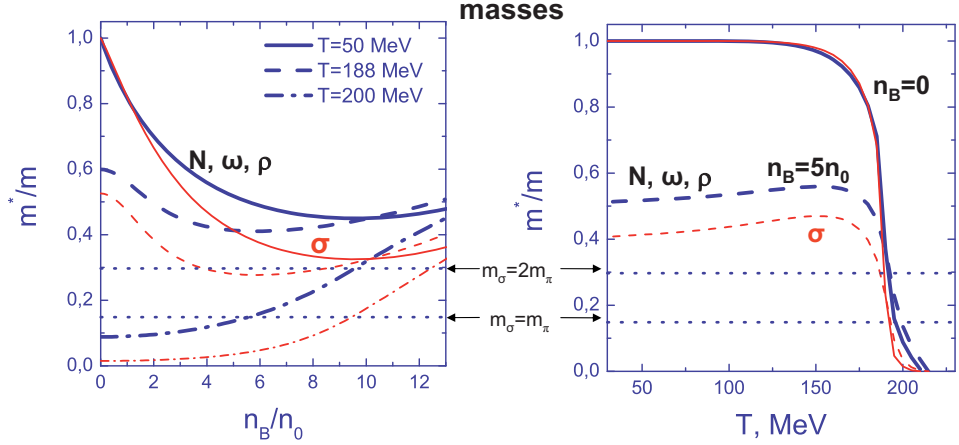
Most peripheral class:

$\langle N_{wound} \rangle = 62 \pm 4$

Significant centrality dependence of strangeness enhancements for all hyperons except for $\bar{\Lambda}$

In-medium effect in RMF

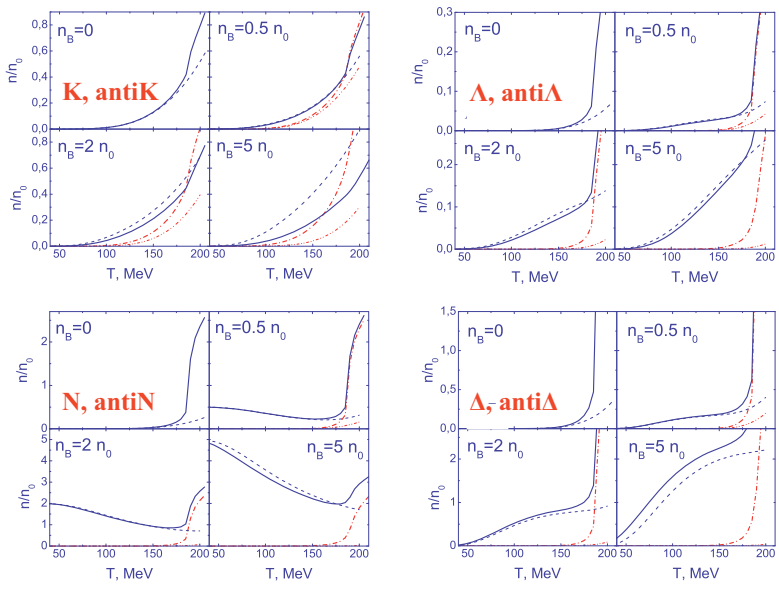
Baryon density and temperature dependence of hadron effective masses



Relativistic mean-field model with scaled hadron masses and couplings simulates a partial restoration of the chiral symmetry

A.Khvorostukhin, V.Toneev, D.Voskresensky,
Nucl. Phys. **A791** (2007) 180

In-medium (anti)particle number densities vs. T, n_B



SHMC:
Solid => particles

Dashed - dotted => antiparticles

Ideal Gas:
Dashed => particles

Dashed – double - dotted => antiparticles

Strong increase of the hadron yield at T above ~160 MeV

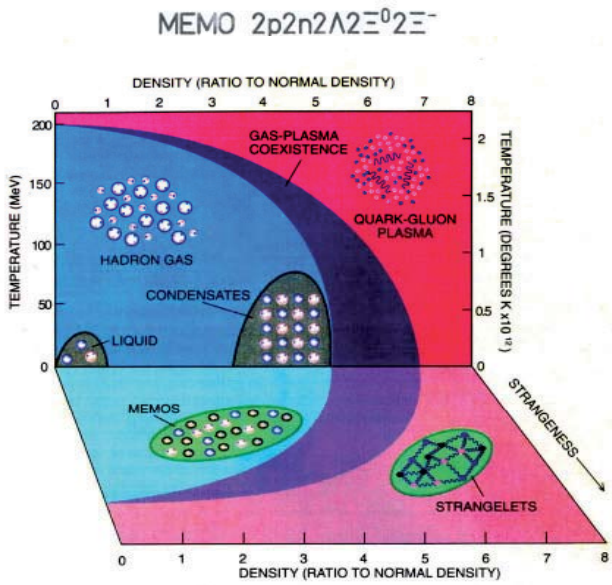
Metastable Exotic Multistrange Object (W.Greiner)

each baryon sits in the $1s_{1/2}$-s

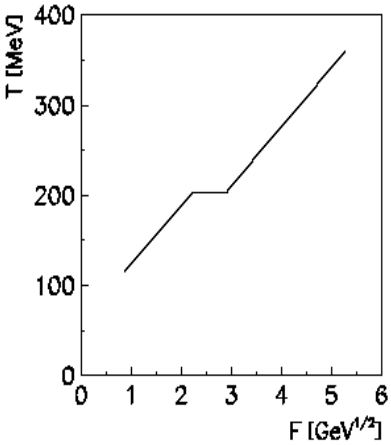
PAULI BLOCKED!

→ METASTABLE!

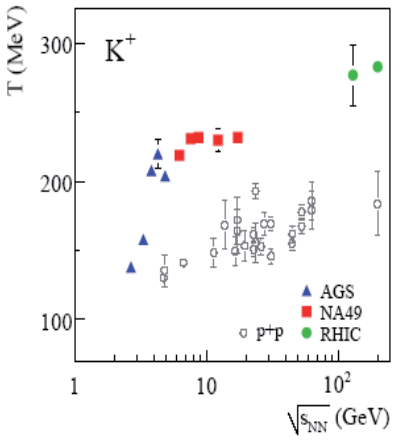
Properties:
Strangeness content S/A=1
Charge z=σ
Density ρ≈4ρ



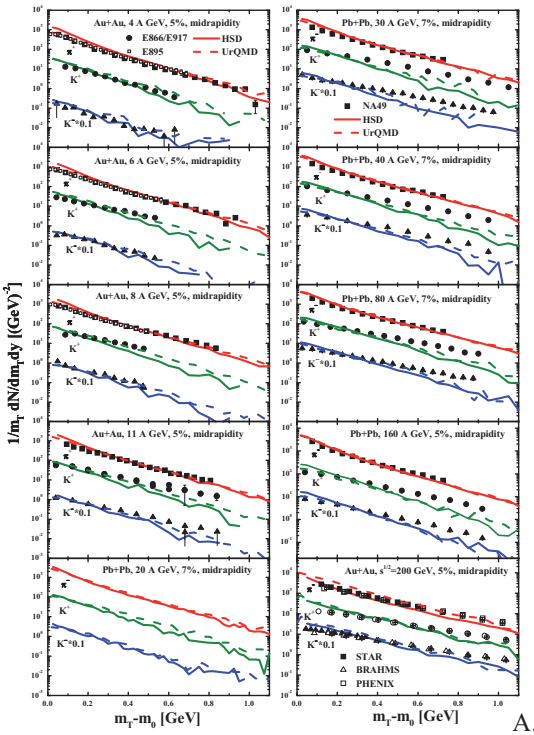
Transverse temperature



$$dN/m_T dm_T \sim exp(-m_T/T)$$



M.Gazdzicki, arXiv:0712.3001



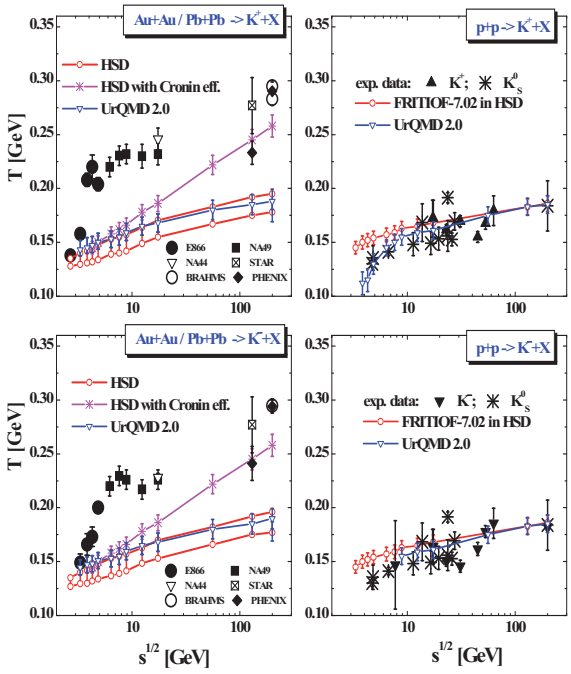
m_T spectra for central Au+Au collisions from AGS to RHIC energies

Pions slopes are only slightly underestimated by hadronic transport models HSD and UrQMD

Kaon slopes are too small above 5 A GeV !

A.Mishra et al.,Phys. Rev. C69 (2004) 015202

Inverse T slopes of K^+ and K^- spectra



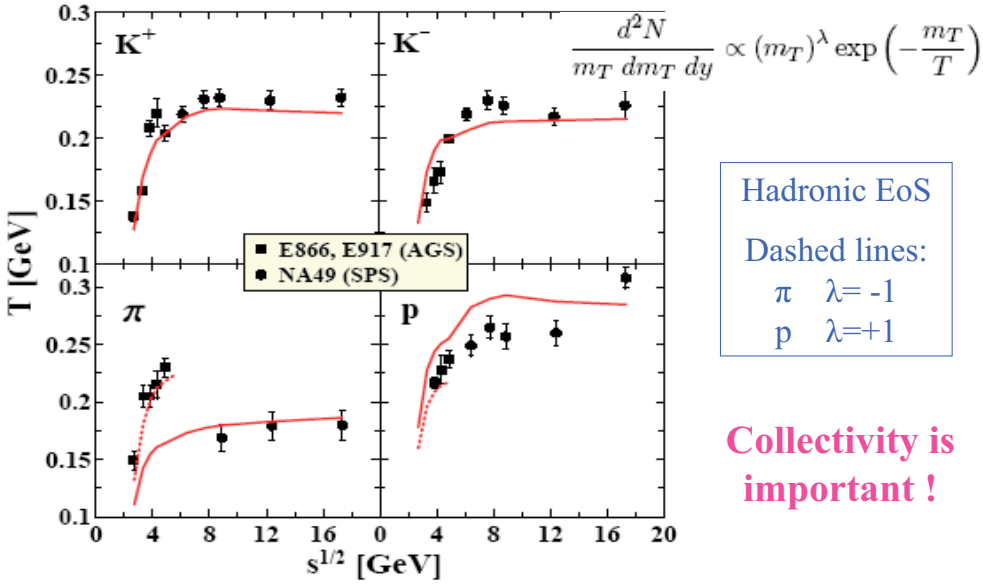
Phys.Rev. **C69** (2004) 015202
PRL **92** (2004) 013302

In HSD and UrQMD hadronic rescattering has only a small impact on the slope of kaon spectra.

Cronin effect – initial semi-hard gluon radiation – leads to the substantial broadening of the m_T spectra at RHIC, however, has a very small effect at low energies.

The hadron-string picture fails ? → New degrees of freedom

3-fluid hydrodynamic model, central Au+Au



Hadronic EoS

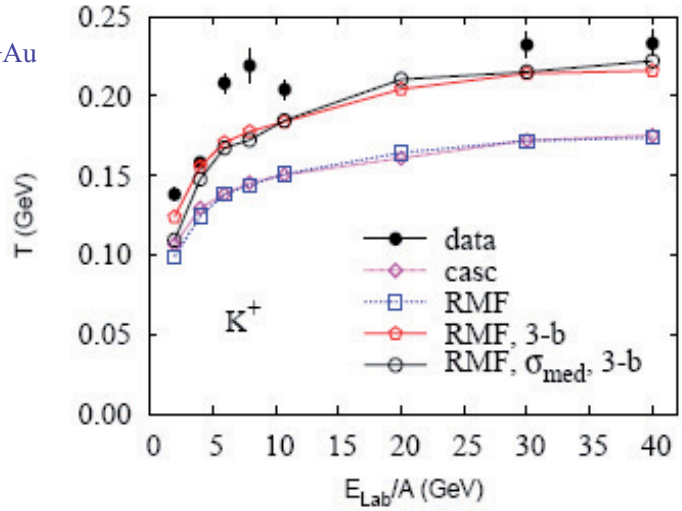
Dashed lines:
π λ= -1
p λ=+1

Collectivity is important !

Yu.Ivanov, V.Russkikh, nucl-th/0607070

Transport model with 3-body collisions

Central Au+Au



Inclusion of 3-body collisions is important

A.Larionov et al., Phys.Rev. **C76** (2007) 044909

Remark: Blast-wave description of spectra

Model : thermalization plus hydro-dynamical transverse flow description

$$\frac{d^2 N_j}{m_T dy dm_T} = \int_0^{R_G} A_j m_T \cdot K_1\left(\frac{m_t \cosh\rho}{T}\right) \cdot I_0\left(\frac{p_t \sinh\rho}{T}\right) r dr$$

$$\rho(r) = \tanh^{-1} \beta_\perp(r)$$

$$\beta_\perp(r) = \beta_S \left[\frac{r}{R_G}\right]^n \quad r \le R_G$$

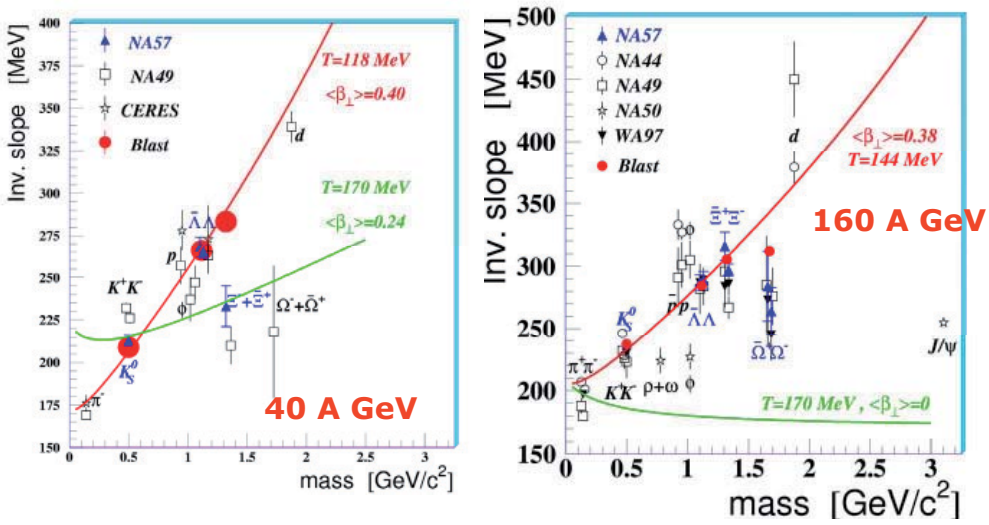
Uniform particle density

$$< \beta_\perp > = \frac{2}{2+n} \beta_S$$

E Schnedermann, J Sollfrank and U Heinz, Phys. Rev. **C48** (1993) 2462

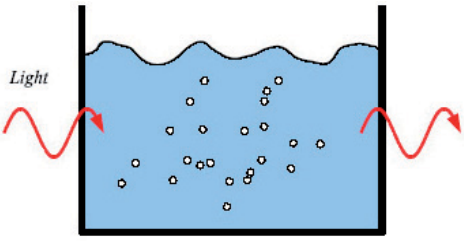
Parameters: $<\beta_T>$, T

Blast wave description of the inverse slope T values at 40 and 160 A GeV

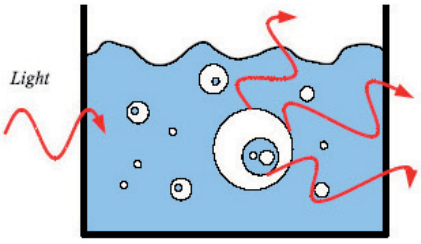


- **indication of an early freeze-out of the Ξ, Ω**

Fluctuations



below the critical point

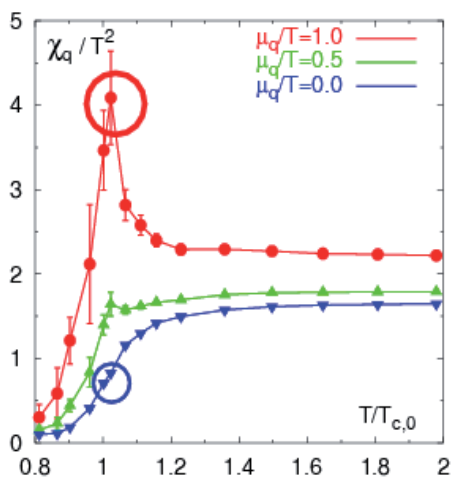


at the critical point

- in the vicinity of the critical point: all length scales are equally important.
- fluctuation scales become arbitrarily large → measurable.
- at the critical point: light is strongly scattered.

Fluctuations

Lattice QCD predictions: Fluctuations of the quark number density (susceptibility) at $\mu_B > 0$ (C.Allton et al., PR D68 (2003) 014507)



$$\frac{\chi_q}{T^2} = \left[\frac{\partial^2}{\partial(\mu_q / T)^2} \frac{P}{T^4} \right]_{T_{fixed}}$$

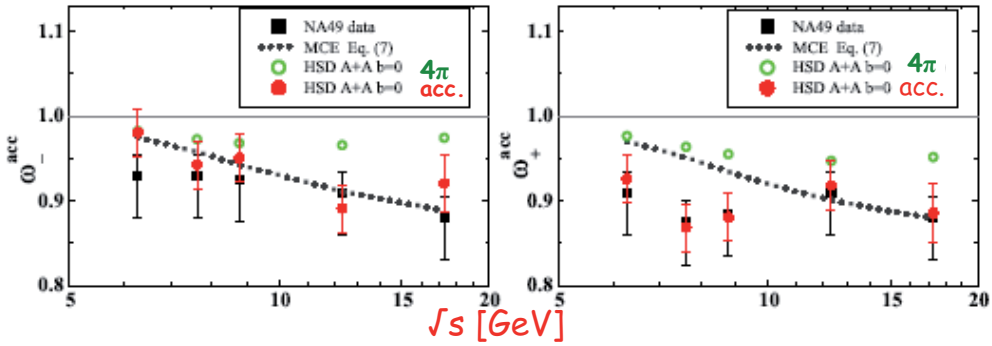
χ_q (quark number density fluctuations) will diverge at the critical end point

Experimental observation:

- Baryon number fluctuations
- Charge number fluctuations

Multiplicity fluctuations: experimental status

$$\omega_i = \frac{< N_i >^2 - < N_i^2 >}{< N_i >}$$



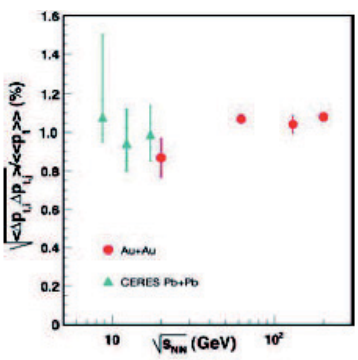
Strong dependence on acceptance

V.Konchakovski et al., nucl-th/0703052

Event-by-event fluctuations, cont'd

p_T fluctuation

$$\sigma(p_T)_{dyn} = \sqrt{< \Delta p_{Ti} \Delta p_{Tj} >} / \bar{p_T}$$

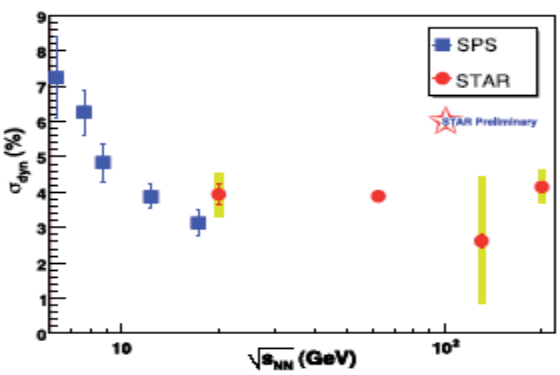


No hint at CEP phenomena

C.Pruneau et. al., NP **A774** (2006) 661

The relative deviation of the K/π fluctuation width from the mixed event background width

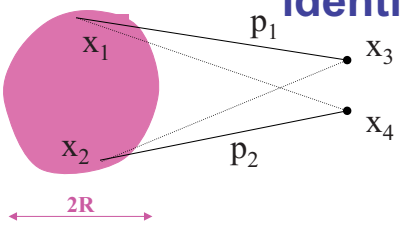
$$\sigma_{dyn} = \sqrt{\sigma^2_{data} + \sigma^2_{mix}}$$

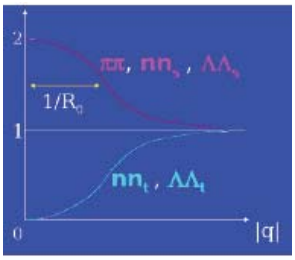


Exhibits a steep rise towards lower energies (√s=6.2 GeV)

S.Das et al., SQM06

Hanbury-Brown and Twiss correlations of identical particles





$q = p_1 - p_2$, $\Delta x = x_1 - x_2$

$$\mathcal{A}_{12} = \frac{1}{\sqrt{2}}\left[e^{ip_1x_1 + ip_2x_2} + (-1)^s e^{ip_1x_2 + ip_2x_1}\right], \quad s \text{ - spin of a pair}$$

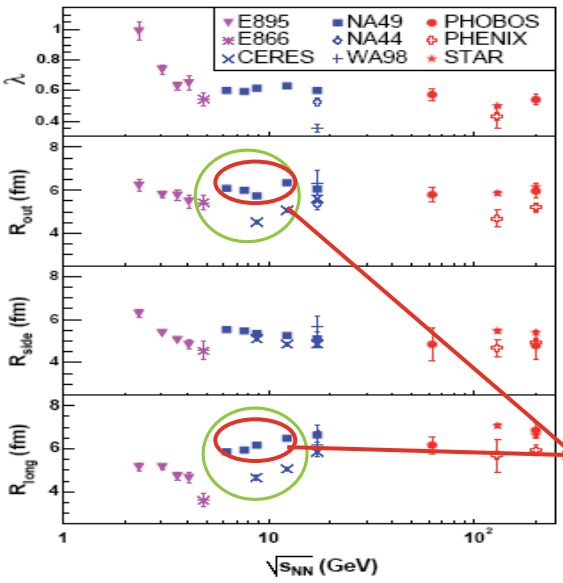
$$|\mathcal{A}_{12}|^2 = 1 + (-1)^s \cos\left[q(x_1 - x_2)\right]$$

$$C_2(q) = \int d^4x_1\, d^4x_2\, |\mathcal{A}_{12}|^2\, \rho(x,y)$$

Correlation function $\quad C_2 = 1 + (-1)^S < Cos\ q\Delta x >$

$$\rightarrow 1 + \lambda \exp(-R^2_{long}q^2_{long} - R^2_{side}q^2_{side} - R^2_{out}q^2_{out} - 2R^2_{out}q_{out}q_{long})$$

HBT correlations: experimental status of ($\pi^-\pi^-$)

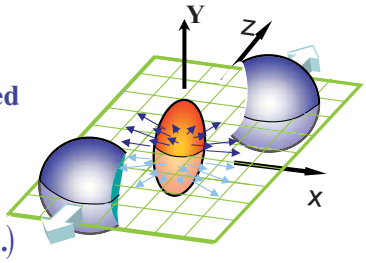


NA49 and CERES data are contradictory

Increase of the lifetime ⇒ Softening of the equation of state due to phase transition?

Directed flow v_1 & elliptic flow v_2

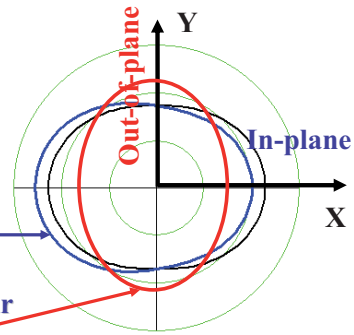
Non central Au+Au collisions :
interaction between constituents leads to a pressure gradient => spatial asymmetry is converted to an asymmetry in momentum space => collective flow



$$\frac{dN}{dy p_T dp_T d\varphi} = \frac{dN}{dy p_T dp_T}\frac{1}{2\pi}\left(1+2v_1\cos(\varphi)+2v_2\cos(2\varphi)+\ldots\right)$$

$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle$ – directed flow

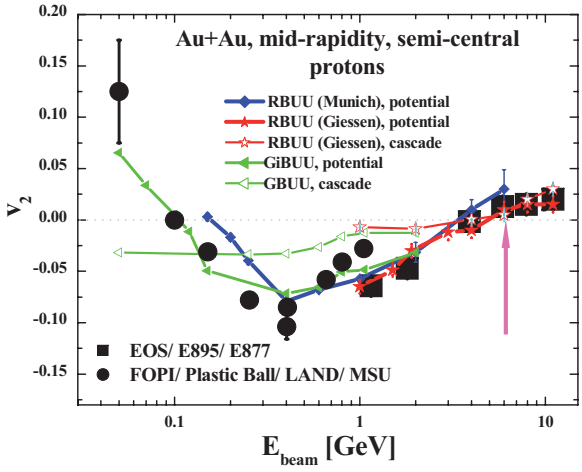
$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$ – elliptic flow



$V_2 > 0$ indicates in-plane emission of particles

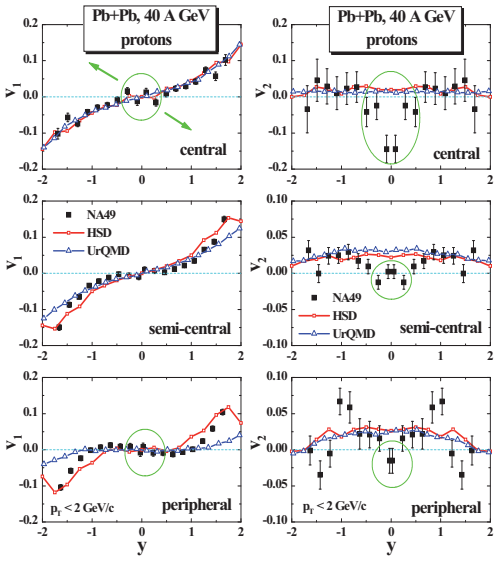
$V_2 < 0$ corresponds to a squeeze-out perpendicular to the reaction plane (out-of-plane emission)

Collective flow: v_2 excitation function



- Proton v_2 at low energy shows sensitivity to the nucleon potential.
- Cascade codes fail to describe the exp. data.
- AGS energies: transition from squeeze-out to in-plane elliptic flow

v_1 and v_2 flows for Pb+Pb at 40 AGeV



Small wiggle in v_1 at midrapidity is not described by HSD and UrQMD

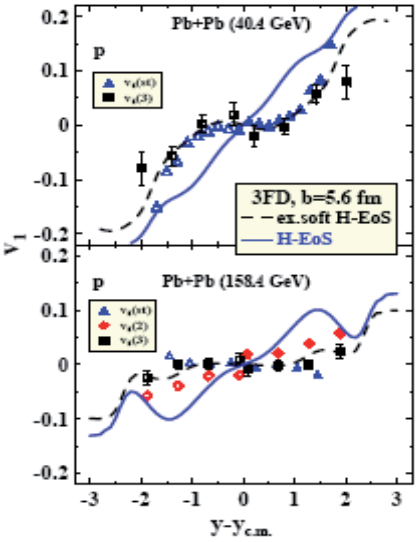
Too large elliptic flow v_2 at midrapidity from HSD and UrQMD for all centralities !

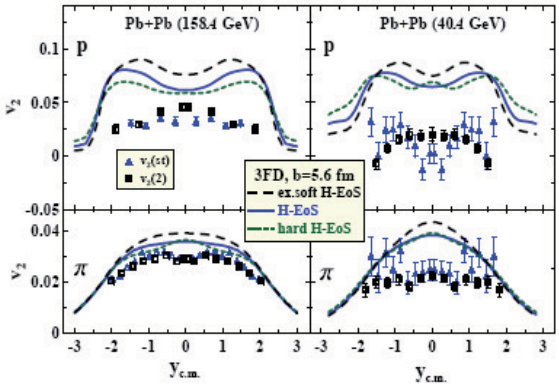
Experiment (NA49): breakdown of elliptic v_2 flow at midrapidity !

Signature for a first order phase transition

H.Stoecker et al., JPG 31 (2005) S929

3 fluid hydro with hadronic EoS

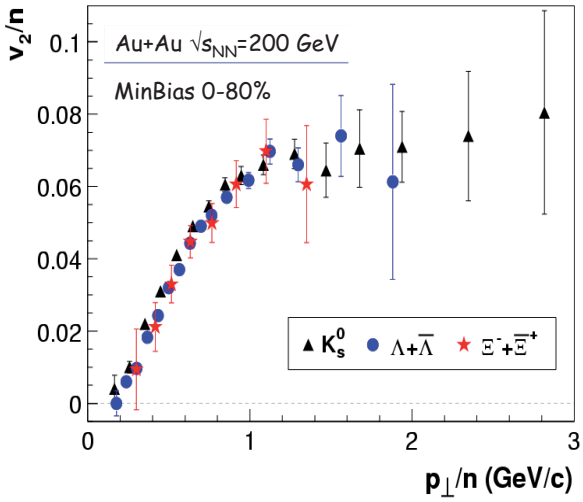




Softening of EoS is needed

Y.Ivanov, V.Russkikh, arXiv:0710.3708

Flow scaling at partonic level



- The complex behaviour of v_2 can be « simply » explained at partonic level

$$v_2^p(p_t) = \frac{v_2^B(3p_t)}{3}$$

$$v_2^p(p_t) = \frac{v_2^M(2p_t)}{2}$$

$$v_2^p(p_t) = \frac{v_2^h(np_t)}{n}$$

... at intermediate p_T !

Idea of flow per constituent – Coalescence/Recombination

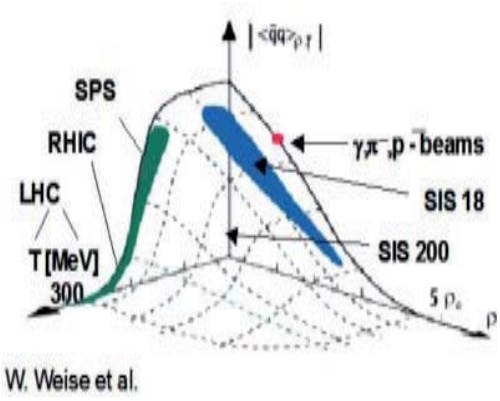
At RHIC energies elliptic flow develops at partonic level.

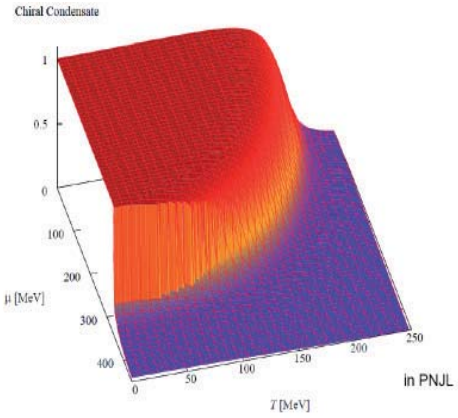
What about 40 AGeV ?

Chiral condensate and dileptons

$< \bar{q}q >_0 = (-230\ MeV)^3$

$m_V^2 \sim < \bar{q}q >$ (G.Brown, M.Rho)

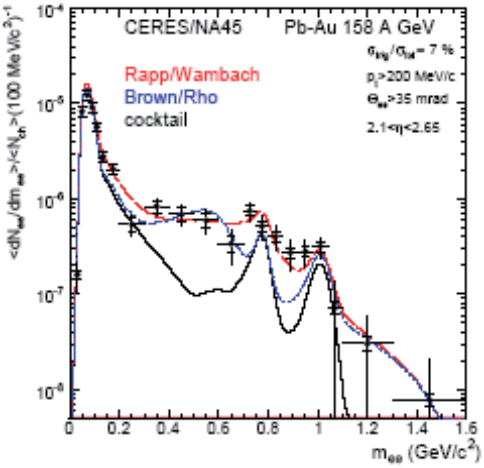




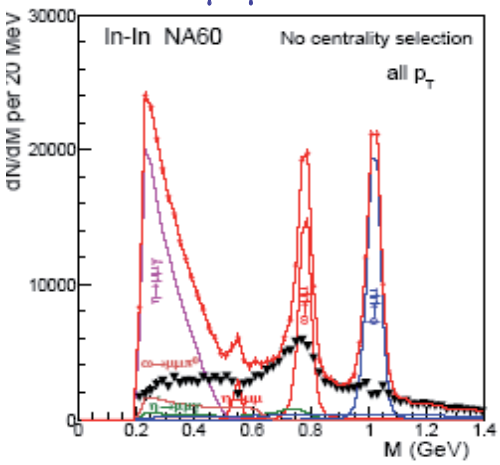
KFukushima, QM2008

Charactrerstics of dilepton experiments

$e^+ e^-$

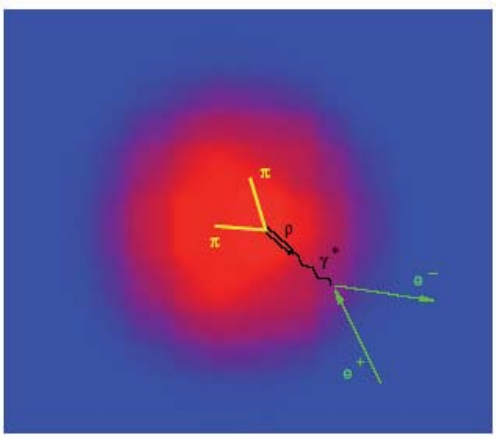


$\mu^+ \mu^-$



HADES: C+C ⟶ $e^+ e^-$ +X at 1 & 2 A GeV

Dilepton production



In-medium effect

J.Brown and M.Rho, Phys. Rev. Lett. **66** (1991) 2720; Phys. Rept. **269** (1996) 333

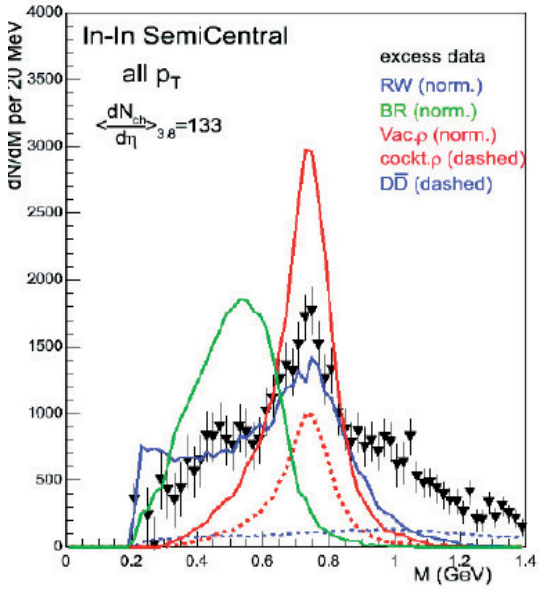
R.Rapp and J.Wambach, Adv. Nucl. Phys. **25** (2000) 1

$$\frac{d^4 N_{ee}}{dq^4} = -\int dx^4\ \mathcal{L}(M)\ \frac{\alpha^2}{\pi^3 q^2}\ \frac{Im\Pi_{em}(q, T(x), \mu_B(x))}{e^{q_0/T(x)} - 1}$$

$$q^2 = M^2 = q_0^2 - \vec{q}^{\,2}$$

ρ-meson spectral function

NA60 $\mu^+\mu^-$ data at 158 AGeV



High precision NA60 data allow to distinguish among in-medium models

Clear evidence for a broadening of the ρ-meson spectral function
(chiral symmetry restoration?)

NA60 Collaboration,
PRL **96** (2006) 162302

Dropping ρ-mass scenarios

In the one-loop approximation

electromagnetic current correlation function

$$Im\Pi_{em}(M) = \frac{m_\rho^4}{g^2}\frac{Im\Pi}{(M^2-m_\rho^2)^2+(Im\Pi)^2}$$

ρ-meson self-energy for free pions

$$Im\Pi = -\frac{g_{\rho\pi\pi}^2}{48\pi}\frac{(M^2-4m_\pi^2)^{3/2}}{M}$$

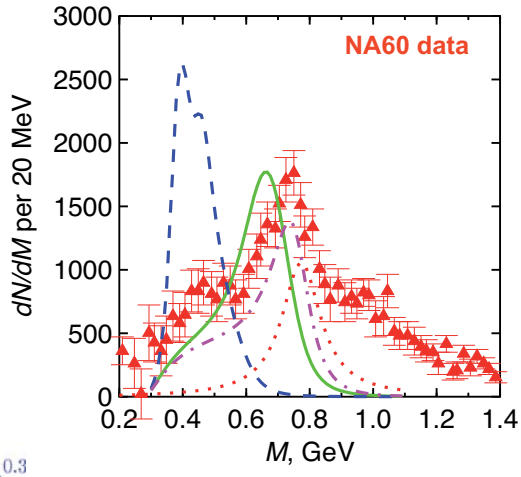
$$g_{\rho\pi\pi} = 6.05 \quad g = 5.03 \quad m_\rho = 770\ MeV$$

$$m_\rho \to m_\rho^\star, \qquad \frac{m_\rho^2}{g} = \frac{m_\rho^{\star 2}}{g^\star}$$

$$m_\rho^* = m_\rho(1-0.15\frac{n_B(x)}{n_0})\{1-[\frac{T(x)}{T_c}]^2\}^{0.3}$$

T.Hatsuda, S.H. Lee, PR **C46** (1992) R34

V.Skokov and V.Toneev, Phys. Rev. **C73** (2006) 021902



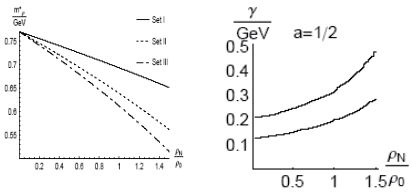
Dashed-dotted: no in-medium effect
Dotted: ρ-mesons from freeze-out

Remarks

♣ Brown-Rho scaling $\frac{m_\rho^\star}{m_\rho} = \left(\frac{<\bar{q}q>^\star}{<\bar{q}q>}\right)^{1/2}$

To be consistent with QCD sum rules, both the collision broadening and dropping ρ-mass should be taken into account (relation to the quark condensate ?)

QCD sum rules



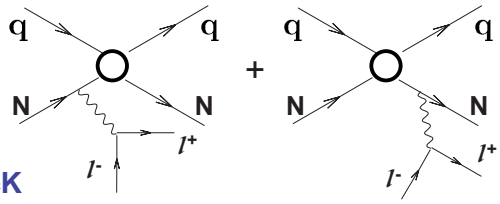
J.Ruppert, T.Renk and B.Muller, Phys. Rev. **C73** (2006) 034907

♣ **Dileptons carry a direct information on the ρ-meson spectral function only if the assumption on vector dominance is valid (**G.E.Brown and M.Rho, nucl-th/0509001; nucl-th/0509002**). This is not so in the Harada- Yamawaki vector manifestation of Hidden Local Symmetry – HLS (**M.Harada and K.Yamawaki, Phys. Rept. 381 (2003) 1**). In this case the electromagnetic coupling is governed by the parameter a of HLS theory which runs with temperature and density (**see G.E. Brown et al., arXiv:0804.3196**).**

♣ **Experimental subtraction of the hadronic cocktail: what about in-medium ω-mesons ? ρ-ω interference ?**

♣ Quark(anti-quark)-hadron bremsstrahlung

In the soft photon approximation (similarly to the pn bremsstrahlung)



$$\frac{d\sigma_{qN}^{l^+l^-}(s,M)}{dM^2} = \frac{\alpha^2}{3\pi^2}\,\frac{\bar{\sigma}(s)}{M^2}\,\ln\left[\frac{s^{1/2}-m_N-m_{\bar{q}}}{M}\right]\times K$$

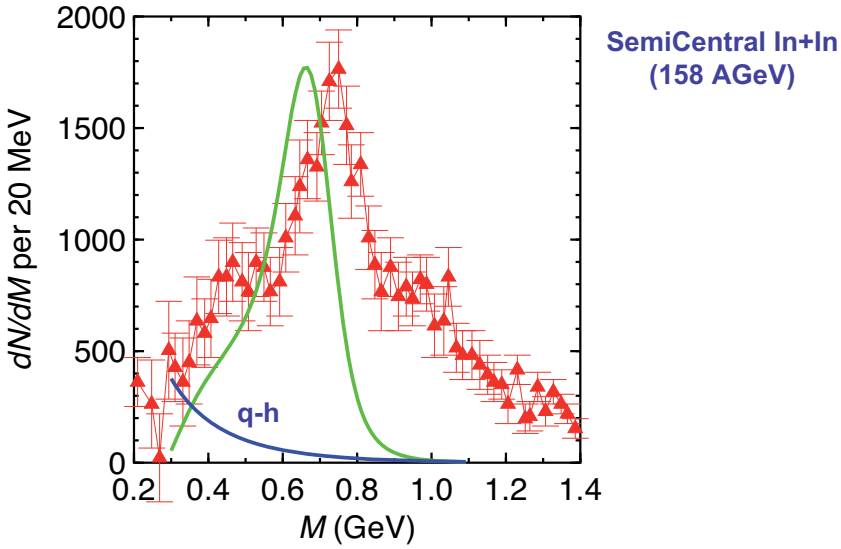
$$\bar{\sigma}(s) \approx 2\sigma_{el}^{qN}\left[\frac{s}{(m_N+m_{\bar{q}})^2}-1\right] \qquad \bar{\sigma}_{el}^{qN} = \left[\frac{18\ (mb\cdot GeV)\ m_N}{s-(m_N+m_{\bar{q}})^2}+10\ mb\right]\times\frac{1}{3}$$

Dilepton emission rate

$$\frac{dR_{\bar{q}N}^{l^+l^-}}{dM^2} = \int\frac{d^3k_{\bar{q}}}{(2\pi)^3}f(\mathbf{k}_{\bar{q}N})\int\frac{d^3k_N}{(2\pi)^3}f(\mathbf{k}_N)\,\frac{d\sigma_{\bar{q}N}^{l^+l^-}(s,M)}{dM^2}\,v_{rel}$$

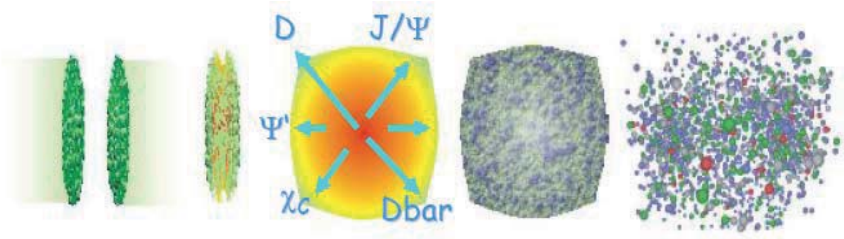
♣ **Intermadiate mass dileptons. Quark-antiquark annihilation from the plasma phase (**K.Dusling and I.Zahed, arXiv:0712.1982; hep-ph/0701253**) were considered but not from the q-h mixed phase !**

First estimate of the q-h bremsstrahlung (preliminary)



K=10 Further study is needed

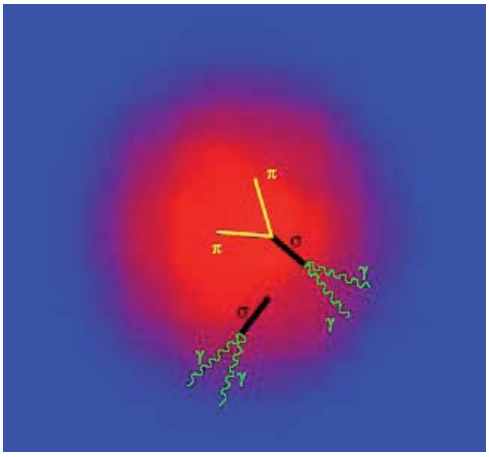
Charmonium in heavy-ion collisions



Charmonium creation versus absorption

'hidden' and 'open' charm will be discussed in other lectures

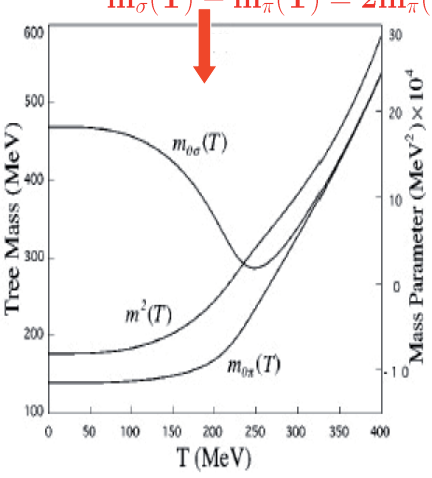
Production of two correlated photons



A sign of chiral symmetry restoration

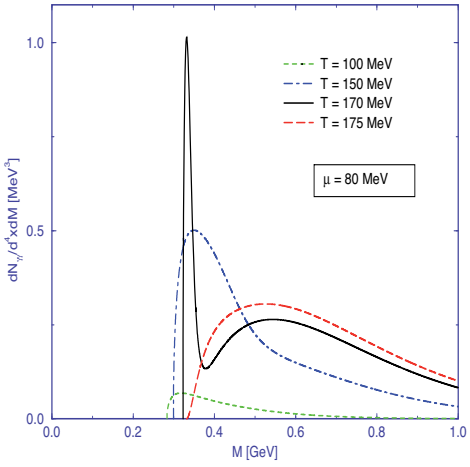
$\sigma \longrightarrow \pi\pi$ $\sigma \longrightarrow \gamma\gamma$

$$\mathbf{m}_\sigma(\mathbf{T}) - \mathbf{m}_\pi(\mathbf{T}) = 2\mathbf{m}_\pi(\mathbf{T})$$



S.Chiki, T.Hatsuda, PR D58 (1998) 076001

Invariant mass distribution for $\sigma \longrightarrow \gamma\gamma$



M.Volkov et al., PL B424 (1998) 235

$\pi^+\pi^- \Rightarrow \gamma\gamma$ in NJL model

$$\sigma^{\pi^+\pi^-\to\gamma\gamma}(s) = \sigma_{Born}(s) + \sigma_{interf}(s) + \sigma_{res}(s)$$

$$\begin{aligned}
\sigma_{Born}(s) &= 16\sigma_0 \left(2 - \kappa^2 - \frac{1-\kappa^4}{2\kappa} \ln\left[\frac{1+\kappa}{1-\kappa}\right]\right), \\
\sigma_{interf}(s) &= 4\sigma_0\, s\, \mathrm{Re}\left[\mathcal{A}_{\pi^+\pi^-\to\gamma\gamma}(s)\right] \frac{1-\kappa^2}{\kappa} \ln\left[\frac{1+\kappa}{1-\kappa}\right] \\
\sigma_{res}(s) &= \sigma_0\, s^2\, |\mathcal{A}_{\pi^+\pi^-\to\gamma\gamma}(s)|^2
\end{aligned}$$

where $\sigma_0 = \pi\alpha^2/4s\kappa$, $\kappa^2 = 1 - 4M_\pi{}^2(T,\mu)/s$, and

$$\begin{aligned}
\mathcal{A}_{\pi^+\pi^-\to\gamma\gamma} &= \frac{1}{(6\pi f_\pi(T,\mu))^2}\left[\frac{40m(T,\mu)}{M^2{}_\sigma(T,\mu) - s - i\sqrt{s}\,\Gamma_\sigma(s|T,\mu)}\right. \\
&\times \left. f_1(\mu,T) - f_2(\mu,T)\right]
\end{aligned}$$

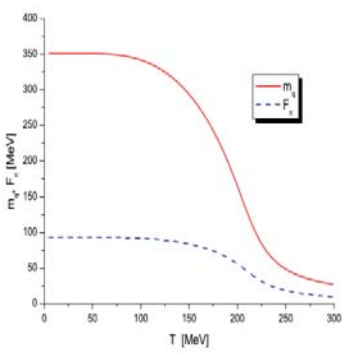
V.Yudichev, Round Table I, Dubna, 2005

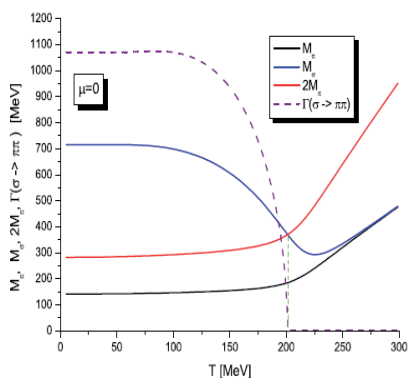
The gap equation

$$m(T,\mu) - m_0 = 8m(T,\mu)G\; I_1{}^\Lambda(m|T,\mu)$$

$$I_1{}^\Lambda(m|T,\mu) = \frac{N_c}{4\pi^2}\int^\Lambda \frac{k^2}{E(k)}\left(1 - n(k;T,\mu) - \bar{n}(k;T,\mu)\right)dk$$

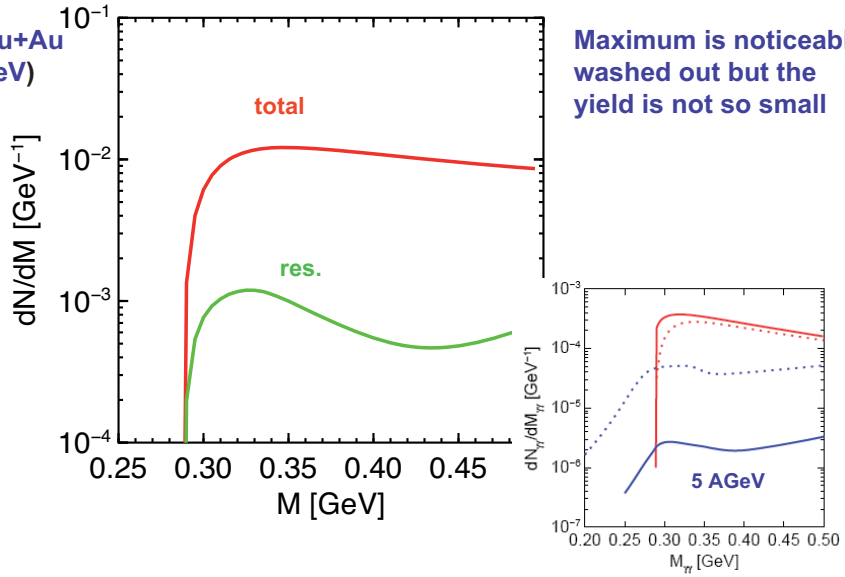
$T=\mu=0$	G [GeV^{-2}]	Λ [MeV]	m_0 [MeV]
Type I, $M_\pi, f_\pi, <\psi\bar\psi>$	11.7205	618.7	5.76
Type II, $M_\pi, f_\pi, \rho\to\pi\pi$	3.4105	1037.4	2.08





Pion annihilation into 2 photons (preliminary)

Central Au+Au (40 AGeV)

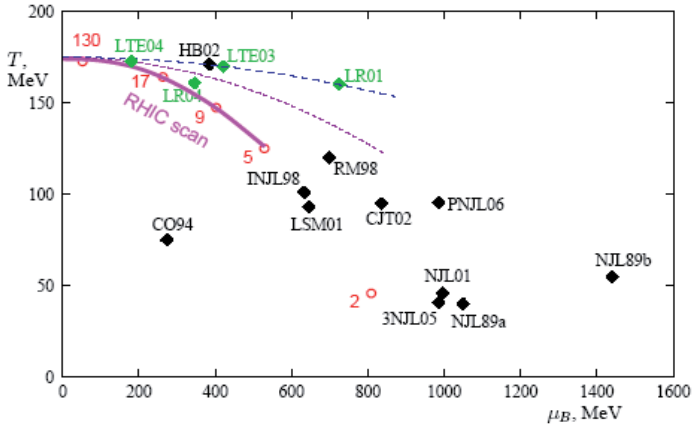


Maximum is noticeably washed out but the yield is not so small

V.Toneev and V.Skokov, Round Table I, Dubna (2005)

Locating the QCD critical point: theoretical predictions

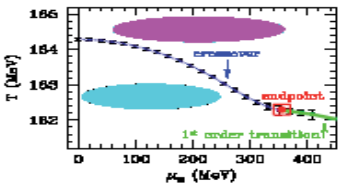
Models, lattice

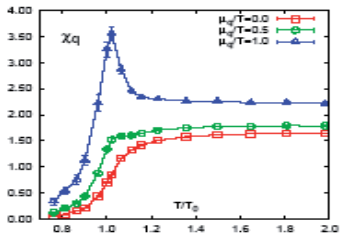


Freezeout points from thermal model fits to SIS/AGS/SPS/RHIC data
(Braun-Munzinger, Redlich, Stachel)

M.Stephanov, CEOD, Darmstadt, July 9-13, 2007

Critical point on the lattice





There is a peak in χ_q but not in χ_I

Several approaches:

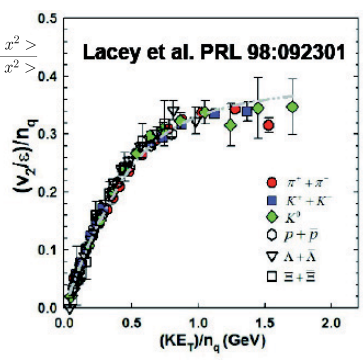
- Reweighting: Fodor-Katz
  - 2001: $\mu_B \sim 725$ MeV
  - 2004: $\mu_B \sim 360$ MeV (smaller m_q and larger V)
- Taylor expansion: Bielefeld-Swansea (to μ^6)
  - 2003: $\mu_B \sim 420$ MeV
  - 2005: 300 MeV $\lesssim \mu_B \lesssim$ 500 MeV
- Taylor expansion: Gavai-Gupta (to μ^8)
  - From convergence radius: $\mu_B \sim 180$ MeV (more precisely > 180 MeV)
- Imaginary μ: deForcrand-Philipsen, Lombardo, *et al*
  - Sensitive to m_s, perhaps $\mu_B \gg 300$ MeV
- Fixed density: deForcrand, Kratochvila; Density of states: Fodor, Katz, Schmidt.
  - ? ($N_f = 4$, small volumes)

M.Stephanov, CEOD, Darmstadt, July 9-13, 2007

Viscosity-to-entropy ratio

minimum bias Au+Au, √s=200 GeV

$$\varepsilon = \frac{<y^2> - <x^2>}{<y^2> + <x^2>}$$



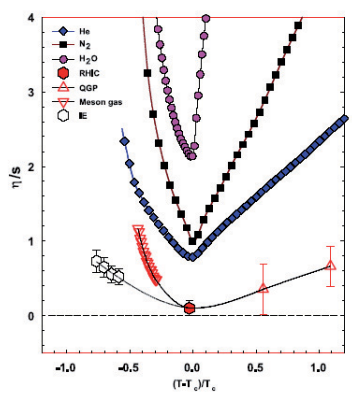
Hydrodynamic scaling

$$T^{ij} = \delta^{ij} P - \eta \left(\partial^i u^j + \partial^j u^i - \frac{2}{3}\delta^{ij}\partial_l u^l \right) - \zeta \delta^{ij} \partial_l u^l$$

Partonic fluid $\frac{\eta}{s} \sim T \lambda_f c_s, \quad \sim 1.3 \times \left(\frac{1}{4\pi} \right)$

Lower bound of η/s=1/4π in the strong coupling limit (P.Kovtun et al. PRL **94** (2005) 111601)

η/s for several substances

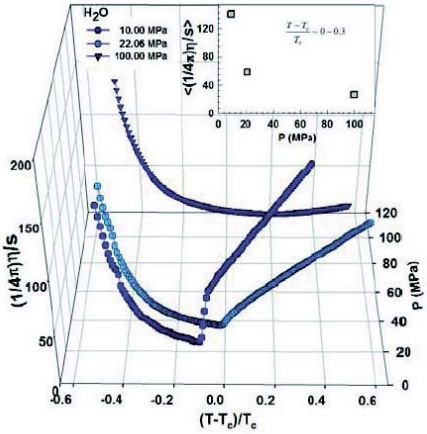


Strong indication for a minimum in the vicinity of T_c

L.P.Csernai et al. PRL **97** (2006) 152303; R.Lacey at al. PRL **98** (2007) 092301

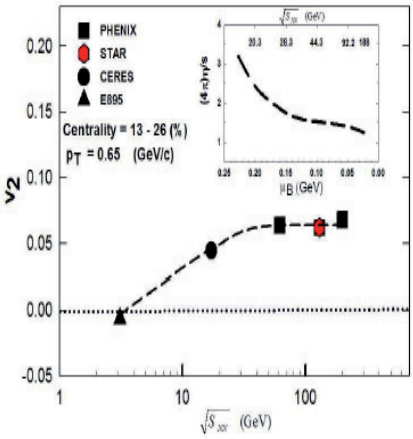
Location of the CEP (?)

Results for an isobar at the critical pressure P_c and one above/below it



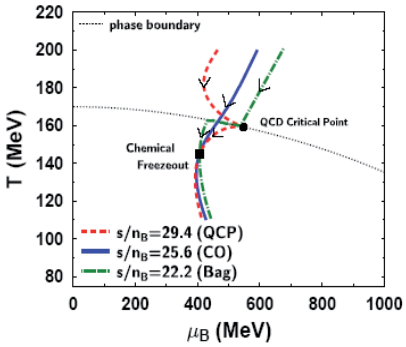
R.Lacey et al. arXiv:0708.3512

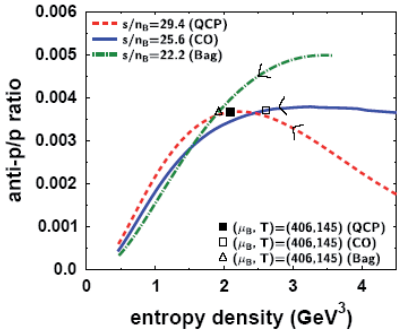
Flow excitation function



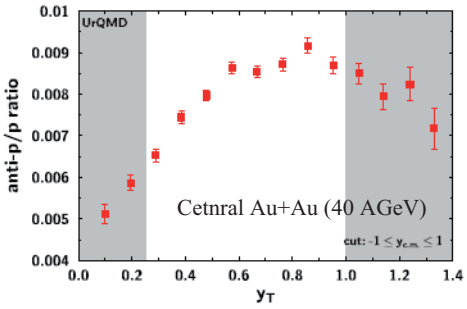
η/s is a potential signal of the CEP

T-μ_B correlates at the freeze-out. First esatimate T~(165-170) MeV, μ_B~(150 -180) MeV





Transverse rapidity dependence of the proton-antiproton ratio



An unusual y_T dependence of the p/anti-p ratio in a narrow beam energy window would signal the presence of the CEP

Asakawa et al., arXiv:0803.2449

Concluding remarks

- PT in HIC is not stable state but transient one
- PT evolves in finite space/time
- after PT the system continues to evolve

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- There is not a single decisive (crucial) signal
- Every signal is essentially washed out due to subsequent interactions

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Scanning of all signals in energy and impact parameter