Quark-Hadron Mixed Phase

V.D. Toneev

JINT, Dubna, 141980 Russia

Abstract. The concept of a mixed quark-hadron phase is considered. Dynamics of heavy-ion collisions in a large energy range is outlined. Various possible probes of the mixed phase manifestation as well as modern status of experimental signals are discussed.

I am grateful to my colleagues A.Khvorostukhin, V.Skokov and A.Shanenko for fruitful collaboration.
Quark-Hadron Mixed Phase

V. Toneev
Bogoliubov Laboratory of Theoretical Physics

- Introductory remarks
- Mixed-phase concept
- A glance at collision dynamics
- Status of signals for a phase transition
- Concluding remarks

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Helmholtz International Summer School, July 14-26, 2008

Introductory remarks
Phase diagrams

E. Fermi: Notes on thermodynamics and statistics, 1953


Condensed matter

Nuclear matter

Phase diagram – artist’s view

Phases of strongly interacting nuclear matter
Phase diagram for QCD matter (scales)

Mixed-phase concept
Dense Matter In Heavy Ion Collisions and Astrophysics (DM2008)

**Mixed-phase concept**

\[ f = \frac{F}{N} = -\frac{T \ln Z}{N} \]

\[ f_h = f_{pl} \]

Maxwell construction

The Gibbs mixed phase (spatially separated)

Generalized mixed phase (homogeneous)

**Deconfinement transition (an illustrative example)**

For the massless Stefan-Boltzmann gas

\[ P = g_l \left( \frac{\pi^2}{90} \right) T^4 \]

\[ \varepsilon = g_l \left( \frac{\pi^2}{30} \right) T^4 = 3P \]

\[ g_l = \text{degeneracy factor} \]

\[ \varepsilon^* = \frac{P}{\varepsilon} = 1/3 \]

**Two-phase model:**

- Pions \( g_\pi = 3 \)
  \[ P_\pi = 3 \left( \frac{\pi^2}{90} \right) T^4 \]
  \[ \varepsilon_\pi = 3 \left( \frac{\pi^2}{90} \right) T^4 = 3P_\pi \]

- Quarks+Gluons (pl)
  \[ g_{pl} = 2 \left( \frac{N_c N_f}{4} + N_c^2 - 1 \right) \]
  \[ P_{pl} = 37 \left( \frac{\pi^2}{90} \right) T^4 \]
  \[ \varepsilon_{pl} = 37 \left( \frac{\pi^2}{30} \right) T^4 + B = 3P_{pl} + 4B \]

Plasma

**Gibbs conditions for the first order phase transition**

\[ P_\pi = P_{pl} = P_{cr} \]

\[ T_\pi = T_{pl} = T_{cr} \]

\[ P_{cr} = \frac{B}{g_{pl} - g_\pi} \approx 35 \text{ MeV/fm}^3 \]

\[ T_{cr} = \frac{90B}{(g_{pl} - g_\pi)\pi^2} \approx (170 \text{ MeV})^4 \]

Latent heat \( \sim 1.5 \text{ GeV/fm}^3 \)

02005-p.5
Simplest example (no conserved charge)

Entropy density: $\sigma(\varepsilon)$

Inverse temperature: $\beta(\varepsilon) = \partial \sigma / \partial \varepsilon$

Equation of State

Pressure: $p(\varepsilon) = T \sigma - \varepsilon$

Pressure: $p(T)$


Familiar example (one conserved charge)

Nuclear equation of state $p_T(\rho)$

Critical end point

Spinodal
Nuclear phase diagram in different representation (one conserved charge)

\[ \rho \]

\[ \mu \]

\[ T \]

Two conserved charges

for an extensive thermodynamic quantity

\[ A = \lambda A_2 + (1-\lambda) A_1 \]

\[ \lambda = \frac{V_2}{V} \]

1 charge

2 charges

volume fraction

A.Sissakian, A.Sorin, V.Toneev, nucl-th/0608032
“Pasta” structures (finite size effects)

Compact stars, T=0

T. Maruyama et al., nucl-th/0605075

Dynamics of the first order phase transition

Evolution of a gas bubble in a mixed phase. Van der Waals equation of state.

V. Skokov, D. Voskresensky
Evolution of a mixed phase

Heat capacity (nuclear matter)

\[ T^{-1} = \frac{\partial S}{\partial E} \quad C_i^{-1} = -T^2 \frac{\partial^2 S}{\partial E^2} \]

\[ E_1 = E_1 + E_2 \quad T_1 = T_2 = T \]

\[ C_t \approx C_1 + C_2 = \frac{C_1^2}{C_1 - \sigma_1^2/T^2} \]

heat capacity is negative in the mixed phase


Statistical Tsallis model of multifragmentation

K.Gudima, A.Parvan, M.Ploszajczak
V.Toneev, PRL 85 (2000) 4691

http://www-linux.gsi.de/~skokov/playme.gif
Susceptibility

Nambu–Jona-Lasinio model

\[ \chi_q = -\frac{1}{V} \left( \frac{\partial^2 F}{\partial \mu_q^2} \right) = \frac{1}{V} (\langle N^2 \rangle - \langle N \rangle^2) \]

\[ \frac{\partial P}{\partial V} \bigg|_T = 0 : \text{isothermal} \]

\[ \frac{\partial P}{\partial V} \bigg|_S = 0 : \text{isentropic} \]

\[ T_c = 81 \text{ MeV} \]

\[ \mu = 330 \text{ MeV} \]

- Poles, negative branch
- Instability region shrinks toward the critical end point


Distillation in the mixed phase

- For unbound quarks in a fireball
- K*π+ excitation function, statistical equilibrium along the freeze-out line

\[ \chi_q / A^2 \]

\[ T = 30 \text{ MeV} \]

Strangeness separation


At CP the K*π+ ratio should jump down on the S=0 curve

Lattice QCD calculations (history)

The Evolving QCD Phase Transition

- Critical Temperature: 150 - 200 MeV ($\mu_B = 0$)
- Critical Density: 1/2-2 Baryons/Fm$^3$ ($T = 0$)

L. McLerran, Prama 60 (2003) 575

Lattice QCD predictions for the order of PT


S. Kim et al. hep-lat/510069

μ_B=0

μ_B≠0
Time evolution near the QCD critical point

Two-phase EoS (first order)

Two-phase EoS with CP

Critical point acts as an attracter of isentropic trajectories

C. Nonaka, M. Asakawa,

A glance at the collision dynamics
Evolution of nucleus-nucleus collisions

Nuclear interaction scales

\( d \) - repulsion \( NN \) force range
\( \Lambda = \frac{1}{\sigma} \) - (nucleon) mean free path
\( \rho_0 \simeq 0.16 \text{ fm}^{-3} \quad \sigma \simeq 40 \text{ mb} \rightarrow \Lambda \sim 1.5 \text{ fm} \)

Pauli principle compression ...

\( L \) - "macroscopic" length, 2-8 fm

<table>
<thead>
<tr>
<th>units</th>
<th>d</th>
<th>( \Lambda )</th>
<th>L</th>
<th>( d/\Lambda )</th>
<th>( Kn = \Lambda/L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>air ((10^{-8} \text{ cm}))</td>
<td>1</td>
<td>(10^5)</td>
<td>(10^8)</td>
<td>(10^{-3})</td>
<td>(10^{-3})</td>
</tr>
<tr>
<td>liquid ((10^{-8} \text{ cm}))</td>
<td>1</td>
<td>2-10</td>
<td>(10^8)</td>
<td>0.1-0.5</td>
<td>(10^{-7})</td>
</tr>
<tr>
<td>nuclei ((0.4/1.2 \text{ fm}))</td>
<td>1</td>
<td>1.5-2</td>
<td>2-8</td>
<td>0.2-0.6</td>
<td>1-0.2</td>
</tr>
</tbody>
</table>

\textit{kinetics} \leftarrow d \ll \Lambda \ll L \quad \Rightarrow \textit{hydrodynamics}

For nuclear case (intermediate energies) \( d < \Lambda < L \)
Relativistic Boltzmann equation

\[
(p_{\mu} \partial^{\mu}) f_i(x, p_i) = \sum_j C^{rel}(x, p_i) + \sum_r R_{r \rightarrow i}
\]

\[
= -f_i(x, p_i) \sum_j \int d\omega f_j(x, p_j) Q_{ij} \sigma^{ij} + \sum_{\delta \delta'} \int d\omega \int d\omega' f_i(x, p_i) \Gamma^{r \rightarrow i + \delta} \delta(p_r - p_i - k')
\]

hadron production rate

\[
\Phi(p_j p_k | x, p_i, \tau_f) = \int dx' f_k(x', p_i) f_j(x', p_j) Q_{ij} Q^{ij} \phi(x' | x, p_i, \tau_f)
\]

transition probability for a finite formation time

\[
\phi(x' | x, \tau_f) = \frac{1}{\sigma} \frac{d\sigma}{d\omega} \theta(t - t' - \tau_f) \delta^{(3)}(\vec{x} - \vec{x}') \frac{\eta}{E}(t - t') F(\tau_f)
\]

* multiple particle production ⇒ coupled set of equations for stable hadrons and resonances \( \{h_i\} \); new flavors

* finite formation time \( \theta(t - \tau_f) \), \( \tau_f = (E/m)_f^0 \) with \( \tau_f^0 \approx 1 \text{ fm} \);

⇒ memory (retarded) effect (non-Markovian process)

\[\cdot\text{ new degrees of freedom (QCD) : quark/gluons, strings, formation of color rope}\]

\[\star \text{ Hadron as a string}\]

\[
V_{q\bar{q}} = -\frac{\alpha_{\text{eff}}}{r} + \kappa r
\]

\[H_{\text{yo-yo}} = \left| p_1 \right| + \left| p_2 \right| + \kappa \left| x_1 - x_2 \right|
\]

\[
\frac{dp_{1,2}}{dt} = \pm \kappa , \quad \frac{dx_{1,2}}{dt} = \pm 1
\]

\[
x^+ = \frac{p^+}{\kappa} , \quad x^- = \frac{p^-}{\kappa} , \quad S = \frac{p^+ p^-}{\kappa^2} = \frac{E^2 - p^2}{\kappa^2} = \frac{m^2}{\kappa^2}
\]
**Particularities of space-time evolution**

**CLASSICAL STRING THEORY**
- string fusion
- string rearrangement
- leading particle effect
- color rope formation

**DUAL TOPOLOGICAL MODEL**
- planar diagram
- cylindrical diagram

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**Kinetic results**


Central cell size 4x4x4/\(\gamma\) (Au+Au)

All dynamical models (w/o PT) demonstrate similar results
Rapidity distributions

Hydrodynamic approach

- Conservation laws (Gauss theorem) ⇒ Fluid dynamics

\[ \partial_\mu \, J_i^\mu = 0 \quad \text{net charge } i \text{ conservation} \]

\[ \partial_\mu \, T^{\mu\nu} = 0 \quad \text{energy momentum conservation} \]

- Tensor decomposition of the charge current \( J_i^\mu \) and energy-momentum tensor \( T^{\mu\nu} \) with respect to 4-velocity \( u^\mu \)

\[ J_i^\mu = n_i u^\mu + (g_i^{\mu\nu} - u^\mu u^\nu) \, J_i^\nu + \ldots \]

\[ T^{\mu\nu} = \varepsilon \, u^\mu u^\nu - P \, (g^{\mu\nu} - u^\mu u^\nu) + q^{\mu \nu} u^\mu + q^{\nu} u^\mu + \pi^{\mu\nu} + \ldots \]

\text{Navier-Stockes}

with

\[ J_i^\mu = \int \frac{d^3p}{p_0} \, p^\mu \, [f_i(x, p) - \bar{f}_i(x, p)] \]

\[ T^{\mu\nu} = \int \frac{d^3p}{p_0} \, p^\mu p^\nu \, [f(x, p) + \bar{f}(x, p)] \]

\[ f_i(x, p) = \frac{g_i}{(2\pi)^3} \exp \left[ \left( (u_\mu p^\mu(x) - \mu_i(x))/T(x) \right) \pm 1 \right]^{-1} \]
Gradient expansion

* Perfect hydro in local thermodynamical equilibrium

\[
\begin{align*}
J_i^\mu &= n_i u^\mu, \\
T^{\mu\nu} &= \varepsilon (u^\mu u^\nu - P (g^{\mu\nu} - u^\mu u^\nu)) \\
\text{perfect hydro} \\
+ \text{EoS} \quad \varepsilon(p)
\end{align*}
\]

* First order dissipative corrections (viscosity, heat capacity)

\[\Rightarrow \text{acausality} \quad \text{Navier-Stokes}\]

* Second order corrections \[\Rightarrow + 14 \text{ Grad equations} \quad \text{Israel-Stuart}\]

Spatial-temporal variation of the macro fields has to be SMALL

Relation between kinetics and hydrodynamics

- The non-relativistic case (for nucleons)

\[
\int d^3 p \begin{bmatrix} \rho \\
\varepsilon \\
\end{bmatrix} \frac{d}{dt} f(\vec{p}, \vec{x}, t) = \int d^3 p \begin{bmatrix} \rho \\
\frac{\varepsilon}{\rho} \\
\end{bmatrix} \left( \frac{1}{\varepsilon / m_N} + \frac{\vec{p} / m_N}{\rho^2 / 2 m_N} \right) \mid (\vec{p}, \vec{x}, t)
\]

Boltzmann equation + local equilibrium hypothesis

\[
\begin{align*}
\vec{v} &= \vec{u} + \vec{c} , \quad \text{hydro} \quad \vec{u} = < \vec{v} > , \quad \text{thermo} \quad < \vec{c} > = 0 \\
\rho < c_i c_k > &= P \delta_{ik} + \Pi_{ik} , \quad \rho < c^2 c_k > = Q_k
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \rho}{\partial t} - \frac{\partial}{\partial x_k} \rho u_k &= 0 \\
\frac{\partial \rho u_i}{\partial t} - \frac{\partial}{\partial x_k} \rho u_i u_k &= \left\{ \frac{\partial}{\partial x_k} \Pi_{ik} - \frac{\partial}{\partial x_i} P \right\} \\
\frac{\partial \varepsilon}{\partial t} - \frac{\partial}{\partial x_k} \varepsilon u_k &= \left\{ \frac{\partial}{\partial x_k} \Pi_{ik} u_i - \frac{\partial}{\partial x_i} P u_k - \frac{\partial}{\partial x_k} Q_k \right\}
\end{align*}
\]

hydro: \[ \Lambda \ll L ; \quad \text{Re} = \frac{\text{inertial-viscous}}{\text{viscous}} \simeq M / \Lambda \simeq 4 - 10 \text{ with } M = \nu / c_s \quad \text{and} \]

\[c_s = \sqrt{\frac{\partial P}{\partial \rho}} \mid \approx 0.2 \]

turbulent regime \[ \text{Re} \simeq 10^2 - 10^3 \]

(near a phase transition ?)
Main ingredients of hydro approach

Non-equilibrium and initial state

- to postulate an initial state (Landau, Bjorken, ...)
- to calculate it in kinetic models (QGSM, UrQMD)
- to introduce many-fluid dynamics

\[ f(x, p) = \sum_j f_j(x, p) \]

A single fluid may consist of several particle spaces. Different fluids may be of the same particle spaces.

Equation of state (hadronic EoS)

\[ \varepsilon(n_B, T) = \varepsilon_{\text{gas}}(n_B, T) + W(n_B), \]

\[ P(n_B, T) = P_{\text{gas}}(n_B, T) + n_B \frac{dW(n_B)}{dn_B} - W(n_B) \]

nuclear potential \( W(n_B) = n_B m_N \left[ -b \left( \frac{n_B}{n_0} \right) + c \left( \frac{n_B}{n_0} \right)^{\gamma+1} \right] \)

Freeze-out procedure (locally \( \varepsilon > \varepsilon_f \))

3-fluid hydro:
Yu.B. Ivanov, V.N. Russikh, V.D. Toneev
3-fluid hydrodynamic model, cont’d


Phase diagram (hadronic EoS)

Mean particle multiplicity

- Freeze out (local $\varepsilon < \varepsilon_f$) $\Rightarrow$ observable

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Hybrid model:

- Initial state of nucleus-nucleus interaction

Entropy evolution and a fireball formation are calculated within the transport code QGSM which defines an initial state for subsequent hydrodynamic stage

V. Skokov, V. Toneev,
Hybrid model, cont’d

Hydrodynamic expansion stage
Isentropic expansion of the formed fireball is described by the 3D relativistic hydrodynamics with the above calculated energy and baryon densities as well as the velocity profile. The statistical quark-hadron equation of state with crossover phase transition is used.

Status of signals of a phase transition in HIC
Onset of the deconfinement

\[ S \sim n_\pi \]

\[ F = (\sqrt{s_{NN}^2 - 2m_N})^{3/4} / (s_{NN})^{1/4} \approx 2.25 \text{ GeV}^{1/2} \]
\[ (E_{\text{beam}} \approx 30 \text{ AGeV}) \]

M. Gazdzicki, arXiv:0712.3001

Excitation functions of \( \pi^\pm, K^\pm, (\Lambda + \Sigma^0) \) yields

Pion yield is overestimated by HSD below AGS energy and by UrQMD at the top AGS and above (deviations are < 20%).

Reasonable description of strangeness by both models.

E. Bratkovskaya
Strangeness enhancement

- **Strangeness production in a hadronic world (at low energy):**
  \[ N+N \rightarrow N+\Lambda+K \] requires \( \Delta E = 2M_N-(M_K+M_\Lambda+M_N) = 670 \text{ MeV} \)
  \[ \pi+N \rightarrow \Lambda+K \] \[ \Delta E = (M_\pi+M_N)-(M_K+M_\Lambda) = 535 \text{ MeV} \]

- **Strangeness production in a QGP:**
  bare mass of strange quark \( m_S \approx 150 \text{ MeV} \)
  \[ \rightarrow s\bar{s} \text{ pair production} \]
  by q-qbar annihilation \( q\bar{q} \rightarrow s\bar{s} \)
  needs only \( \Delta E = 300 \text{ MeV} \)
  \[ \rightarrow s\bar{s} \text{ pair can be also produced by} \]
  gluon fusion \( g+g \rightarrow s\bar{s} \)

\[ \Rightarrow \text{Strong enhancement of strangeness production in a QGP!} \]

\[ \Rightarrow \text{strangeness enhancement increases with strangeness content –} \]
stronger effect for multi-strange hadrons \( \Xi(uss), \Omega(sss) \)

Strange-to-nonstrange particle ratio

\[ \frac{K^+}{\bar{K}^+} \] vs \( \sqrt{s_{NN}} \) (GeV)

\[ \text{M. Gazdzicki, arXiv:0712.3001} \]
Excitation functions of $K^+/\pi^+$, $K^-/\pi^-$ and $(\Lambda+\Sigma^0)/\pi$ ratios

Experimental $K^+/\pi^+$ ratios show a peak at $\sim 30$ A GeV ("horn") which is not reproduced by hadronic transport models HSD and UrQMD

Hyperon enhancements at 160 A GeV/c

Significant centrality dependence of strangeness enhancements for all hyperons except for $\Lambda$

systematic errors:
10% for $\Lambda$, $\Xi$
15% for $\Omega$

Most peripheral class:
$<N_{\text{wound}}> = 62 \pm 4$
**In-medium effect in RMF**

Baryon density and temperature dependence of hadron effective masses

Relativistic mean-field model with scaled hadron masses and couplings simulates a partial restoration of the chiral symmetry


**In-medium (anti)particle number densities vs. \(T, n_B\)**

**SHMC:**
- Solid => particles
- Dashed - dotted => antiparticles

**Ideal Gas:**
- Dashed => particles
- Dashed – double - dotted => antiparticles

Strong increase of the hadron yield at \(T\) above \(\sim 160\ MeV\)
Metastable Exotic Multistrange Object (W. Greiner)

Each baryon sits in the $1s_{1/2}$ state.

PAULI BLOCKED!

→ METASTABLE!

Properties:
- Strangeness content S/A = 1
- Charge $z = \sigma$
- Density $\rho = 4\rho$

Transverse temperature

$$\frac{dN}{m_T dm_T} \sim \exp(-m_T/T),$$

M. Gazdzicki, arXiv:0712.3001
m_T spectra for central Au+Au collisions from AGS to RHIC energies

Pions slopes are only slightly underestimated by hadronic transport models HSD and UrQMD

Kaon slopes are too small above 5 A GeV!

Inverse T slopes of K^+ and K^- spectra

PRL 92 (2004) 013302

In HSD and UrQMD hadronic rescattering has only a small impact on the slope of kaon spectra.

Cronin effect – initial semi-hard gluon radiation – leads to the substantial broadening of the m_T spectra at RHIC, however, has a very small effect at low energies.

The hadron-string picture fails? → New degrees of freedom
3-fluid hydrodynamic model, central Au+Au

\[ \frac{d^2N}{dmt_p dy} \propto (m_T)^\lambda \exp \left(-\frac{m_T}{T}\right) \]

Hadronic EoS
Dashed lines:
\[ \pi \quad \lambda = -1 \]
\[ p \quad \lambda = +1 \]

Collectivity is important!

Yu.Ivanov, V.Russkikh, nucl-th/0607070

Transport model with 3-body collisions

Central Au+Au

Inclusion of 3-body collisions is important

Remark: Blast-wave description of spectra

Model: thermalization plus hydro-dynamical transverse flow description

\[
\frac{d^2 N_j}{m_T dy dm_T} = \int^R G A_j m_T \cdot K_1 \left( \frac{m_T \cosh \rho}{T} \right) \cdot I_0 \left( \frac{p_i \sinh \rho}{T} \right) r dr
\]

\[
\rho(r) = \tanh^{-1} \beta_\perp(r)
\]

\[
\beta_\perp(r) = \beta_S \left( \frac{r}{R_G} \right)^n r \leq R_G
\]

Uniform particle density

\[
\langle \beta_\perp \rangle = \frac{2}{2 + n} \beta_S
\]


Parameters: \( \langle \beta_T \rangle, T \)

Blast wave description of the inverse slope T values at 40 and 160 A GeV

- indication of an early freeze-out of the \( \Xi, \Omega \)
**Fluctuations**

- in the vicinity of the critical point: all length scales are equally important.

- fluctuation scales become arbitrarily large $\rightarrow$ measurable.

- at the critical point: light is strongly scattered.

**Lattice QCD predictions:** Fluctuations of the quark number density (susceptibility) at $\mu_B > 0$ (C. Allton et al., PR D68 (2003) 014507)

\[
\frac{\chi_q}{T^2} = \left[ \frac{\partial^2}{\partial (\mu_q/T)^2} \frac{P}{T^4} \right]_{T_{\text{fixed}}}
\]

$\chi_q$ (quark number density fluctuations) will diverge at the critical end point

**Experimental observation:**
- Baryon number fluctuations
- Charge number fluctuations
**Multiplicity fluctuations: experimental status**

\[ \omega_i = \frac{\langle N_i^2 \rangle - \langle N_i \rangle^2}{\langle N_i \rangle} \]

\[ J\ell s [\text{GeV}] \]

**Strong dependence on acceptance**

V. Konchakovski et al., nucl-th/0703052

**Event-by-event fluctuations, cont’d**

**p_T fluctuation**

\[ \sigma(p_T)_{dyn} = \sqrt{\langle \Delta p_T \Delta p_T \rangle / \bar{p}_T} \]

**The relative deviation of the K/\pi fluctuation width from the mixed event background width**

\[ \sigma_{dyn} = \sqrt{\sigma_{data}^2 + \sigma_{mix}^2} \]

**No hint at CEP phenomena**

C. Pruneau et al., NP A774 (2006) 661

**Exhibits a steep rise towards lower energies \((\sqrt{s}=6.2 \text{ GeV})\)**

S. Das et al., SQM06
Hanbury-Brown and Twiss correlations of identical particles

\[ q = p_1 - p_2, \quad \Delta x = x_1 - x_2 \]

\[ A_{12} = \frac{1}{\sqrt{2}} \left[ e^{i p_1 x_1} + i p_2 x_2 + (-1)^s e^{i p_1 x_2} + i p_2 x_1 \right], \quad s - \text{spin of a pair} \]

\[ |A_{12}|^2 = 1 + (-1)^s \cos[q(x_1 - x_2)] \]

\[ C_2(q) = \int d^4 x_1 \int d^4 x_2 |A_{12}|^2 \rho(x, y) \]

Correlation function

\[ C_2 = 1 + (-1)^s \langle \cos q\Delta x \rangle \]

\[ \rightarrow 1 + \lambda \exp(-R_{\text{long}}^2 q_{\text{long}}^2 - R_{\text{side}}^2 q_{\text{side}}^2 - R_{\text{out}}^2 q_{\text{out}}^2 - 2 R_{\text{out}} q_{\text{out}} q_{\text{long}}) \]

**HBT correlations: experimental status of (π⁻ π⁻)**

NA49 and CERES data are contradictory

Increase of the lifetime \( \Rightarrow \)

Softening of the equation of state due to phase transition?
**Directed flow \( v_1 \) & elliptic flow \( v_2 \)**

Non central Au+Au collisions:
interaction between constituents leads to a
pressure gradient => spatial asymmetry is converted
to an asymmetry in momentum space =>
collective flow

\[
\frac{dN}{dy d\eta d\phi} = \frac{dN}{2\pi} \left( 1 + 2v_1 \cos(\phi) + 2v_2 \cos(2\phi) + \ldots \right)
\]

\( v_1 = \frac{\langle p_x \rangle}{p_T} \) - directed flow

\( v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{p_x^2 + p_y^2} \) - elliptic flow

\( V_2 > 0 \) indicates in-plane emission of particles

\( V_2 < 0 \) corresponds to a squeeze-out perpendicular
to the reaction plane (out-of-plane emission)

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**Collective flow: \( v_2 \) excitation function**

- Proton \( v_2 \) at low energy shows sensitivity to the nucleon potential.
- Cascade codes fail to describe the exp. data.
- AGS energies: transition from squeeze-out to in-plane elliptic flow
v₁ and v₂ flows for Pb+Pb at 40 AGeV

Small wiggle in v₁ at midrapidity is not described by HSD and UrQMD

Too large elliptic flow v₂ at midrapidity from HSD and UrQMD for all centralities!

Experiment (NA49): breakdown of elliptic v₂ flow at midrapidity!

Signature for a first order phase transition

H. Stoecker et al., JPG 31 (2005) S929

3 fluid hydro with hadronic EoS

Softening of EoS is needed

Y. Ivanov, V. Russkikh, arXiv:0710.3708
Flow scaling at partonic level

\[ v_2^P (p_T) = \frac{v_2^B (3p_T)}{3} \]
\[ v_2^P (p_T) = \frac{v_2^M (2p_T)}{2} \]
\[ v_2^P (p_T) = \frac{v_2^h (np_T)}{n} \]

... at intermediate \( p_T \)!

Idea of flow per constituent - Coalescence/Recombination
At RHIC energies elliptic flow develops at partonic level.
What about 40 AGeV?

Chiral condensate and dileptons

\[ < \bar{q} q >_0 = (-230 \text{ MeV})^3 \]
\[ m_V^2 \sim < \bar{q} q > \quad \text{(G.Brown, M.Rho)} \]
Characteristics of dilepton experiments

HADES: C+C $\rightarrow$ e$^+$ e$^- + X$ at 1 & 2 A GeV

Dilepton production

In-medium effect


R. Rapp and J. Wambach,

$$\frac{d^4N_{ee}}{dq^4} = - \int dx^4 \mathcal{L}(M) \frac{\alpha^2}{\pi^3 q^2} \frac{Im \Pi_{em}(q, T(x), \mu_B(x))}{e^{\beta_0/T(x)} - 1}$$

$$q^2 = M^2 = q_0^2 - \vec{q}^2$$
**ρ-meson spectral function**

**NA60 μ⁺μ⁻ data at 158 AGeV**

High precision NA60 data allow to distinguish among in-medium models

**Clear evidence for a broadening of the ρ-meson spectral function**
(chiral symmetry restoration?)

NA60 Collaboration, PRL 96 (2006) 162302

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**Dropping ρ-mass scenarios**

In the one-loop approximation

\[ Im\Pi_{em}(M) = \frac{m_\rho^4}{g^2} \frac{Im\Pi}{(M^2 - m_\rho^2)^2 + (Im\Pi)^2} \]

ρ-meson self-energy for free pions

\[ Im\Pi = \frac{g_{\rho\pi\pi}^2}{48\pi} \frac{(M^2 - 4m_\rho^2)^{3/2}}{M} \]
\[ g_{\rho\pi\pi} = 6.05 \quad g = 5.03 \quad m_\rho = 770 \text{ MeV} \]

\[ m_\rho \rightarrow m_\rho^*, \quad \frac{m_\rho^*}{g} = \frac{m_\rho^*}{g^*} \]

\[ m_\rho^* = m_\rho(1 - 0.15 \frac{n_B(x)}{n_0})\left(1 - \left[\frac{T(x)}{T_c}\right]^2\right)^{0.5} \]


---

Remarks

- Brown-Rho scaling 
\[ \frac{m_p^*}{m_p} = \left( \frac{\langle \bar{q}q^* \rangle}{\langle \bar{q}q \rangle} \right)^{1/2} \]

To be consistent with QCD sum rules, both the collision broadening and dropping ρ-mass should be taken into account (relation to the quark condensate?)

- Dileptons carry a direct information on the ρ-meson spectral function only if the assumption on vector dominance is valid (G.E.Brown and M.Rho, nucl-th/0509001; nucl-th/0509002). This is not so in the Harada-Yamawaki vector manifestation of Hidden Local Symmetry – HLS (M.Harada and K.Yamawaki, Phys. Rept. 381 (2003) 1). In this case the electromagnetic coupling is governed by the parameter \( a \) of HLS theory which runs with temperature and density (see G.E. Brown et al., arXiv:0804.3196).

- Experimental subtraction of the hadronic cocktail: what about in-medium ω-mesons? \( ρ-ω \) interference?

Quark(anti-quark)-hadron bremsstrahlung

In the soft photon approximation (similarly to the pn bremsstrahlung)

\[ \frac{d\sigma_{qN}^t}{dM^2} = \frac{\alpha^2}{3\pi^2} \frac{\sigma(s)}{M^2} \ln \left[ \frac{s^{1/2} - m_N - m_q}{M} \right] \]

\[ \sigma(s) \approx 2\sigma_{el}^0 \left( \frac{s}{(m_N + m_q)^2} \right)^2 \]

\[ \sigma_{el}^0 = \frac{18 (mb \cdot GeV) m_N}{s - (m_N + m_q)^2} + 10 mb \times \frac{1}{3} \]

Dilepton emission rate

\[ \frac{dR_{qN}^{t\bar{t}}}{dM^2} = \int d^3k_q f(k_q) \int d^3k_N f(k_N) \frac{d\sigma_{qN}^{t\bar{t}}(s, M)}{dM^2} v_{rel} \]

Intermediate mass dileptons. Quark-antiquark annihilation from the plasma phase (K.Dusling and I.Zahed, arXiv:0712.1982; hep-ph/0701253) were considered but not from the q-h mixed phase!
First estimate of the q-h bremsstrahlung (preliminary)

K=10
Further study is needed

Charmonium in heavy-ion collisions

Charmonium creation versus absorption

‘hidden’ and ‘open’ charm will be discussed in other lectures
Production of two correlated photons

A sign of chiral symmetry restoration

$\sigma \to \pi\pi, \sigma \to \gamma\gamma$

$m_\sigma(T) - m_\pi(T) = 2m_\pi(T)$

Invariant mass distribution for $\sigma \to \gamma\gamma$

S. Chiki, T. Hatsuda, PR D58 (1998) 076001

M. Volkov et al., PL B424 (1998) 235
\[ \pi^+ \pi^- \rightarrow \gamma \gamma \text{ in NJL model} \]

\[ \sigma_{\pi^+ \pi^- \rightarrow \gamma \gamma}(s) = \sigma_{\text{Born}}(s) + \sigma_{\text{interf}}(s) + \sigma_{\text{res}}(s) \]

\[ \sigma_{\text{Born}}(s) = 16\sigma_0 \left( 2 - \kappa^2 - \frac{1}{2\kappa} \ln \left[ \frac{1 + \kappa}{1 - \kappa} \right] \right) \]

\[ \sigma_{\text{interf}}(s) = 4\sigma_0 \Re \left[ A_{\pi^+ \pi^- \rightarrow \gamma \gamma}(s) \right] \left( \frac{1 - \kappa^2}{\kappa} \ln \left[ \frac{1 + \kappa}{1 - \kappa} \right] \right) \]

\[ \sigma_{\text{res}}(s) = \sigma_0 s^2 \left| A_{\pi^+ \pi^- \rightarrow \gamma \gamma}(s) \right|^2 \]

where \( \sigma_0 = \frac{\pi \alpha^2}{4s\kappa} \), \( \kappa^2 = 1 - 4M_{\pi^2}(T, \mu)/s \), and

\[ A_{\pi^+ \pi^- \rightarrow \gamma \gamma} = \frac{1}{(6\pi f_\pi(T, \mu))^2} \left[ \frac{40m(T, \mu)}{M^2_{\sigma}(T, \mu) - s - i\sqrt{s} \Gamma_\sigma(s|T, \mu)} \times f_1(\mu, T) - f_2(\mu, T) \right] \]

V. Yudichev, Round Table I, Dubna, 2005

The gap equation

\[ m(T, \mu) - m_0 = 8m(T, \mu)G l_1^\Lambda(m|T, \mu) \]

\[ l_1^\Lambda(m|T, \mu) = \frac{N_c}{4\pi^2} \int^\Lambda \frac{k^2}{E(k)} \left( 1 - n(k; T, \mu) - \bar{n}(k; T, \mu) \right) dk \]

\[
\begin{array}{|c|c|c|c|}
\hline
T = \mu = 0 & G \ [\text{GeV}^{-2}] & \Lambda \ [\text{MeV}] & m_0 \ [\text{MeV}] \\
\hline
\text{Type I, } M_\pi f_\pi, \langle \psi \psi \rangle & 11.7205 & 618.7 & 5.76 \\
\text{Type II, } M_\pi f_\pi, \rho \rightarrow \pi \pi & 3.4105 & 1037.4 & 2.08 \\
\hline
\end{array}
\]
Pion annihilation into 2 photons (preliminary)

V. Toneev and V. Skokov, Round Table I, Dubna (2005)

Locating the QCD critical point: theoretical predictions

Freezeout points from thermal model fits to SIS/AGS/SPS/RHIC data
(Braun-Munzinger, Redlich, Stachel)

M. Stephanov, CEOD, Darmstadt, July 9-13, 2007
Critical point on the lattice

Several approaches:

- **Reweighting: Fodor-Katz**
  - 2001: $\mu_B \sim 725$ MeV
  - 2004: $\mu_B \sim 300$ MeV
  (smaller $m_q$ and larger $V$

- **Taylor expansion: Bielefeld-Swansea (to $\mu^6$)**
  - 2003: $\mu_B \sim 420$ MeV
  - 2005: $300 \, \text{MeV} \lesssim \mu_B \lesssim 500 \, \text{MeV}$

- **Taylor expansion: Gavai-Gupta (to $\mu^6$)**
  - From convergence radius: $\mu_B \sim 180$ MeV (more precisely $> 180$ MeV)

- **Imaginary $\mu$: deForcrand-Phillips-Lombardo, et al**
  - Sensitive to $m_q$, perhaps $\mu_B \gg 300$ MeV

- **Fixed density: deForcrand, Kratochvila**
  - Density of states: Fodor, Katz, Schmidt.
  - $? (N_f = 4, \text{small volumes})$

M. Stephanov, CEOD, Darmstadt, July 9-13, 2007

Viscosity-to-entropy ratio

minimum bias Au+Au, $\sqrt{s}=200$ GeV

<table>
<thead>
<tr>
<th>$T_{ij}$</th>
<th>$\eta/s$ for several substances</th>
</tr>
</thead>
<tbody>
<tr>
<td>$= \frac{\eta}{s}$</td>
<td>$\sim T\lambda_f c_\sigma$,</td>
</tr>
<tr>
<td>Lower bound of $\eta/s=1/4\pi$ in the strong coupling limit (P.Kovtun et al. PRL 94 (2005) 111601)</td>
<td></td>
</tr>
</tbody>
</table>

Hydrodynamic scaling

$T_{ij} = \delta_{ij} P - \frac{q_i}{q_j} \left( \partial^i u^j + \partial^j u^i - \frac{2}{3} \delta^{ij} \partial^k u_k \right) - \zeta \delta^{ij} \partial^k u^k$

Partonic fluid

$L_P = \zeta (1/4\pi)$

Strong indication for a minimum in the vicinity of $T_c$

L.P. Csernai et al. PRL 97 (2006) 152303; R. Lacey at al. PRL 98 (2007) 092301
Location of the CEP (?)

Results for an isobar at the critical pressure $P_c$ and one above/below it

Flow excitation function

$\eta/s$ is a potential signal of the CEP

$T-\mu_B$ correlates at the freeze-out. First esatimate $T\sim(165-170)$ MeV, $\mu_B\sim(150 -180)$ MeV

Transverse rapidity dependence of the proton-antiproton ratio

An unusual $y_T$ dependence of the p/anti-p ratio in a narrow beam energy window would signal the presence of the CEP

Asakawa et al., arXiv:0803.2449
Concluding remarks

• PT in HIC is not stable state but transient one
• PT evolves in finite space/time
• after PT the system continues to evolve

↓

• There is not a single decisive (crucial) signal
• Every signal is essentially washed out due to subsequent interactions

↓

Scanning of all signals in energy and impact parameter