Isolated Neutron Stars
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Abstract. Several aspects related to astrophysics of isolated neutron stars are discussed. We start with an introduction into the "new zoo" of young isolated neutron stars. In addition to classical radio pulsars, now we know several species (soft gamma-ray repeaters, anomalous X-ray pulsars, central compact objects in supernova remnants, close-by cooling neutron stars - aka "Magnificent seven", - RRATs, and some others). All these types are briefly discussed. In the second lecture a description of magneto-rotational evolution of neutron stars is given. Finally, in the third lecture we discuss population synthesis of isolated neutron stars. In some details we discuss population synthesis of young isolated radio pulsars and young close-by cooling neutron stars.
Lecture 1
Isolated Neutron Stars. Intro.

Sergei Popov (SAI MSU)

Dubna “Dense Matter In Heavy Ion Collisions and Astrophysics”, July 2008

Artistic view
Prediction ...

Neutron stars have been predicted in 30s:

L.D. Landau: Star-nuclei (1932) + anecdote

Baade and Zwicky: neutron stars and supernovae (1934)

Good old classics

For years two main types of NSs have been discussed: radio pulsars and accreting NSs in close binary systems

The pulsar in the Crab nebula  A binary system
The old zoo of neutron stars

In 60s the first X-ray sources have been discovered.

They were neutron stars in close binary systems, BUT ...
.... they were «not recognized»....

Now we know hundreds of X-ray binaries with neutron stars in the Milky Way and in other galaxies.

Rocket experiments
Sco X-1

Giacconi, Gursky, Hendel
1962

In 2002 R. Giacconi was awarded with the Nobel prize.
Discovery !!!!

Serendipitous discovery.

The pulsar in the Crab nebula
The new zoo of neutron stars

During last >10 years it became clear that neutron stars can be born very different. In particular, absolutely non-similar to the Crab pulsar.

- Compact central X-ray sources in supernova remnants.
- Anomalous X-ray pulsars
- Soft gamma repeaters
- The Magnificent Seven
- Unidentified EGRET sources
- Transient radio sources (RRATs)
- Calvera ....

Compact central X-ray sources in supernova remnants

Problem: small emitting area 6.7 hour period (de Luca et al. 2006, 0803.1373)
Puppis A

One of the most famous central compact X-ray sources in supernova remnants.
Age about 3700 years.
Probably the progenitor was a very massive star (mass about 30 solar).

\[ V_{\text{kick}} = 1500 \text{ km/s} \]

Winkler, Petre 2006
(astro-ph/0608205)

See a review on these objects in arxiv:0712.2209

Magnetars

- \[ \frac{dE}{dt} > \frac{dE_{\text{rot}}}{dt} \]
- By definition: The energy of the magnetic field is released
- P-Pdot
- Direct measurements of the field (Ibrahim et al.)

Magnetic fields \(10^{14} - 10^{15} \text{ G}\)
Known magnetars

**SGRs**
- 0526-66
- 1627-41
- 1806-20
- 1900+14
- +candidates

**AXPs**
- CXO 010043.1-72
- 4U 0142+61
- 1E 1048.1-5937
- CXOU J164710.3-
- 1 RXS J170849-40
- XTE J1810-197
- 1E 1841-045
- AX J1844-0258
- 1E 2259+586
- +candidates and transients

(CTB 109)

**SGRs: monitoring and extraG**

SRG detectors can contribute to observations of SGRs.

Now there are few other candidates (Mazets et al., Frederiks et al., Golenetskii et al., Ofek et al, Crider ....), including one in the direction of M31 (Mazets et al. arxiv:0712.1502).

[D. Frederiks et al. astro-ph/0609544]
QPOs after giant flares

A kind of quasi periodic oscillations have been found in tail of two events (aug. 1998, dec. 2004). They are supposed to be torsional oscillations of NSs, however, it is not clear, yet.


See a recent review in aXiv: 0710.2475

Transient radio emission from AXP

Radio emission was detected from XTE 1810-197 during its active state.

One another magnetar was reported to be detected at low frequencies in Pushchino, however, this result has to be checked.

(Camilo et al. astro-ph/0605429)
Another AXP detected in radio

1E 1547.0-5408
P= 2 sec
SNR G327.24-0.13

Transient radiopulsar

PSR J1846-0258
P=0.3 sec
B=5 \times 10^{13} \text{ G}

The pulsar increased its luminosity in X-rays. Magnetar-like X-ray bursts.
**ROSAT**

**ROentgen SATellite**

German satellite (with participation of US and UK).

Launched 01 June 1990. The program was successfully ended on 12 Feb 1999.

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**Close-by radioquiet NSs**

- Discovery: Walter et al. (1996)
- Proper motion and distance: Kaplan et al.
- No pulsations
- Thermal spectrum
- Later on: six brothers

RX J1856.5-3754
## Magnificent Seven

<table>
<thead>
<tr>
<th>Name</th>
<th>Period, s</th>
</tr>
</thead>
<tbody>
<tr>
<td>RX 1856</td>
<td>7.05</td>
</tr>
<tr>
<td>RX 0720</td>
<td>8.39</td>
</tr>
<tr>
<td>RBS 1223</td>
<td>10.31</td>
</tr>
<tr>
<td>RBS 1556</td>
<td>6.88?</td>
</tr>
<tr>
<td>RX 0806</td>
<td>11.37</td>
</tr>
<tr>
<td>RX 0420</td>
<td>3.45</td>
</tr>
<tr>
<td>RBS 1774</td>
<td>9.44</td>
</tr>
</tbody>
</table>

- Radioquiet (?)
- Close-by
- Thermal emission
- Absorption features
- Long periods

## Unidentified EGRET sources

Grenier (2000), Gehrels et al. (2000)

Unidentified sources are divided into several groups. One of them has sky distribution similar to the Gould Belt objects.

It is suggested that GLAST (and, probably, AGILE) can help to solve this problem.

Actively studied subject (see for example papers by Harding, Gonthier)

No radio pulsars in 56 EGRET error boxes (Crawford et al. 2006)
Pulsars invisible in radio?

(Grenier astro-ph/0011298)

Discovery of radio transients

McLaughlin et al. (2006) discovered a new type of sources—RRATs (Rotating Radio Transients).
For most of the sources periods about few seconds were discovered.
The result was obtained during the Parkes survey of the Galactic plane.

These sources can be related to The Magnificent seven.

Thermal X-rays were observed from one of the RRATs (Reynolds et al. 2006). This one seems to me the youngest.
P-Pdot diagram for RRATs

McLaughlin et al. 2006 Nature

Estimates show that there should be about 400,000 Sources of this type in the Galaxy.
Young or old???
Relatives of the Magnificent seven? (astro-ph/0603258)

RRATs

- 11 sources detected in the Parkes Multibeam survey (McLaughlin et al 2006)
- Burst duration 2-30 ms, interval 4 min-3 hr
- Periods in the range 0.4-7 s
- Period derivative measured in 3 sources: \( B \sim 10^{12} - 10^{14} \) G, age \( \sim 0.1-3 \) Myr
- RRAT J1819-1458 detected in the X-rays, spectrum soft and thermal, \( kT \sim 120 \) eV (Reynolds et al 2006)
RRATs

- P, B, ages and X-ray properties of RRATs very similar to those of XDINSs
- Estimated number of RRATs \( \sim 3-5 \) times that of PSRs
- If \( \tau_{\text{RRAT}} \approx \tau_{\text{PSR}}, \beta_{\text{RRAT}} \approx 3-5 \beta_{\text{PSR}} \)
- \( \beta_{\text{XDINS}} > 3 \beta_{\text{PSR}} \) (Popov et al 2006)
- Are RRATs far away XDINSs?

RRATs. Recent data

X-ray pulses overlapped on radio data of RRAT J1819-1458.

(arXiv: 0710.2056, see there also a brief review on these sources)
Recently, Rutledge et al. reported the discovery of an enigmatic NS candidate dubbed Calvera.

It can be an evolved (aged) version of Cas A source, but also it can be a M7-like object, who’s progenitor was a runaway (or, less probably, hypervelocity) star.

No radio emission was found ([arxiv:0710.1788](http://arxiv.org/abs/0710.1788)).

**LOFAR**

Low Frequency Array (<250 MHz)
Perfect for wide-field observations, search for transients in radio etc.

See a brief review in [arxiv:0710.0675](http://arxiv.org/abs/0710.0675)
Gravitational waves from INSs

INS are expected to be sources of GWs:

- Radio pulsars
- Young magnetars
- .......?
- Know periods
- Rapid rotation and strong deformation
- Possible new types sources

Recent LIGO results for PSRs

1. **0805.4758** Beating the spin-down limit on gravitational wave emission from the Crab pulsar

   \( h_0^{95\%} < 3.5 \times 10^{-25} \) \( \varepsilon \leq 1.9 \times 10^{-4} \) (single template)

2. **0708.3818** All-sky search for periodic grav. waves in LIGO S4 data

   50-1000 HZ

   No evidence. Upper limits on isolated NSs GW emission.

   Very weak limits for “dark” sources (<50 pc for \( \varepsilon \sim 10^{-6} \))

3. **gr-qc/0702039** Upper limits on gravitational wave emission from 78 PSRs

   \( \varepsilon < 10^{-6} \) for PSR J2124-3358

   \( h < 2.6 \times 10^{-25} \) for PSR J1603-7202
GWs from young magnetars

A newborn magnetar with msec period and toroidal magnetic field B\sim 10^{16} \, G for few days can be a strong source detectable from the Virgo cluster distance by advanced LIGO.

Birthrate of magnetars in Virgo is about 0.3-1 per year.

Due to strong toroidal magnetic fields young magnetars can have prolate shape with ellipticity 10^{-3} \text{–} 10^{-4}.

Power of the signal goes as \sim P^{-6}.

\[ \tau_{sd} \equiv \frac{a}{2\omega} \approx 10^3 \frac{P_{2/3}^2}{B_{3,14}^2 (B_{4,16.3}^4 P_{1/2}^2)^{-1} d_{20}} \]

\[ h \sim 3 \times 10^{-26} \frac{P_{2/3}^{-1} P_{1/2}^{-2} B_{4,16.3}^2}{d_{20} a^{3/2} (B_{3,14}^4 + 1.15 B_{4,16.3}^4 P_{1/2}^2)^{3/2}} \]

Instantaneous signal

\[ \dot{\omega} = -K_{2/3} \omega^3 - K_{4/3} \omega^5 \]

astroph/0702075 Dall’Osso et al.
astroph/0511068 Stella et al.

Detectability by advanced LIGO

\[ h_c = h N^{1/2}, \text{ where } N \sim \tau_{sd}/P_4 \]

The signal is observable for several periods

So, the signal lasts for several days after a magnetar’s birth

It is possible to get very optimistic predictions for advanced LIGO. However, mining the data for a signal with unknown parameters from unknown place in the sky is very complicated.

Computational costs are very large.

astroph/0702075 Dall’Osso et al.
Magnetar parameters

Estimating GW emission from magnetars it is necessary to make assumptions about ellipticity. It is not that easy.

0712.2162 Pons et al.
Relativistic models of magnetars: structure and deformations

This authors estimate that for $B \sim 10^{15}$ G ellipticity is expected $\sim 10^{-6} – 10^{-5}$
In extreme cases (low-mass NSs) it can go up to $10^{-3}$
Crustal field are very important.

0806.2794 Regimbau, Mandic
Astrophysical sources of stochastic GW background

Magnetars produce strong (in comparison with expected cosmological signal) background around 1 kHz (depends on initial spin of magnetars).

Next generation of interferometers will give important results on NSs.

Population synthesis of close-by NS and GWs

Palomba [astro-ph/0503046]

The author assumes that some fraction of NSs are born with low magnetic fields, so that GW losses dominate in spin-down.

The author predicts that few NSs can be detected by Virgo (for not too small fraction of low-field NSs),
and tens – by advanced Virgo.

Expected distances (for Virgo) are 100-300 pc, and frequencies about 200-600 Hz.
Conclusion

- There are several types of sources: CCOs, M7, SGRs, AXPs, RRATs ...
- Magnetars (?)
- Significant fraction of all newborn NSs
- Unsolved problems:
  1. Are there links?
  2. Reasons for diversity

Main reviews

- NS basics: physics/0503245
  astro-ph/0405262
- SGRs & AXPs: astro-ph/0406133
  astro-ph/0311526
  0712.2209
- Quark stars: astro-ph/0608360
- The Magnificent Seven:
  astro-ph/0502457
  astro-ph/0609066
  0801.1143
- RRATs: astro-ph/0511587
- Cooling of NSs: astro-ph/0508056
  astro-ph/0402143
- NS structure
- NS interiors
- Magnetic field of NSs
  arXiv: 0705.2708
  arXiv: 0802.2227
  0711.3650
- EoS
- NS atmospheres
  astro-ph/0612440
  astro-ph/0206025
Lecture 2
Spin evolution of NSs

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Dubna “Dense Matter In Heavy Ion Collisions and Astrophysics”, July 2008

Hard life of neutron stars

There are about $6 \times 10^8$ persons on Earth. How many do you know?
There are about $1 \times 10^9$ NSs in the Galaxy. How many do we know? Why?

We know PSRs, SGRs, AXPs, CCOs, M7, RRATs, .... They are young.

Dialogue of two magnetars:

-We are not getting younger, man....
-Yeh, at first you lose spin, then – magnetic field, and then you just cool down...
-....and nobody cares about you any more ....

Evolution is important!!!
Evolution of neutron stars

Thermal  Magneto-rotational

Observational appearance of a NS can depend on:
- Temperature
- Period
- Magnetic field
- Velocity

Evolution of NSs: temperature

First papers on the thermal evolution appeared already in early 60s, i.e. before the discovery of radio pulsars.

[Yakovlev et al. (1999) Physics Uspekhi]
**Magnetic rotator**

Observational appearances of NSs (if we are not speaking about cooling) are mainly determined by $P$, $P_{\text{dot}}$, $V$, $B$, (probably the inclination angle $\chi$), and properties of the surrounding medium. B is not evolving significantly in most cases, so it is important to discuss spin evolution.

Together with changes in $B$ (and $\chi$) one can speak about **magneto-rotational evolution**

We are going to discuss the main stages of this evolution, namely: Ejector, Propeller, Accretor, and Georotator following the classification by Lipunov.
Magneto-rotational evolution of radio pulsars

For radio pulsar magneto-rotational evolution is usually illustrated in the P-Pdot diagram. However, we are interested also in the evolution after this stage.

\[
L_m = \frac{2 \mu^2 \omega^4}{3} \varepsilon \sin^2 \beta = \kappa_i \mu^2 R_i^3 \omega,
\]

\[
B \sim 3 \times 10^{19} (PdP/dt)^{1/2} \text{ G}.
\]

Spin-down. Rotational energy is released. The exact mechanism is still unknown.

Magneto-rotational evolution of NSs

Ejector → Propeller → Accretor → Georotator

1 – spin down
2 – passage through a molecular cloud
3 – magnetic field decay

See the book by Lipunov (1987, 1992)
Critical radii -I

Transitions between different evolutionary stages can be treated in terms of critical radii

- Shvartsman radius.  $R_{sh}$.
- Propeller stage. Corotation radius. $R_{co}$
- Accretor stage. Magnetospheric (Alfven) radius. $R_A$
- Georotator stage. Magnetospheric (Alfven) radius. $R_{A}$

As observational appearance is related to interaction with the surrounding medium the radius of gravitational capture is always important. $R_{sc}=2GM/V^2$.

Schwarzshild radii is typcall unimportant.

\[
R_s = \frac{2GM}{c^2} \approx 2.95 \times \frac{M}{M_{sc27}} \text{ km}
\]

Critical radii-II

1. Shvartsman radius
   It is determined by relativistic particles wind

\[
R_{sh} = \left( \frac{8\kappa \mu^2 G^2 M^2 \omega^2}{M_c v_{∞}^2 c^4} \right)^{1/2}, \quad R_{sh} > R_G
\]

2. Corotation radius

\[
\omega R_{st} < \sqrt{GM_x/R_{st}}.
\]

\[
R_c = \left( \frac{GM_x}{\omega^2} \right)^{1/3} \sim 2.8 \times 10^8 m_x^{1/3} (P/1 \text{ s})^{2/3} \text{ cm}
\]

3. Alfven radius

\[
P_a(R_{st}) = P_m(R_{st})
\]

\[
R_{A} = \begin{cases} 
\left( \frac{2 \mu^2 G^2 M^2}{M_c v_{∞}^2} \right)^{1/6}, & R_A > R_G \\
\left( \frac{\mu^2}{2M_c \sqrt{2GM}} \right)^{2/7}, & R_A \leq R_G 
\end{cases}
\]
### Classification

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Type</th>
<th>Characteristic radial relation</th>
<th>Accretion rate</th>
<th>Observational appearances</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Ejector</td>
<td>$R_a &gt; R_G$ $R_a &gt; R_t$</td>
<td>$\dot{M}<em>e \leq \dot{M}</em>\alpha$</td>
<td>Radiopulsars, Soft $\gamma$-ray repeaters, Cyg X-3? LSI+61 303?</td>
</tr>
<tr>
<td>P</td>
<td>Propeller</td>
<td>$R_e &lt; R_a$ $R_a \leq R_G$ $R_a \leq R_t$</td>
<td>$\dot{M}<em>e \leq \dot{M}</em>\alpha$</td>
<td>X-ray transients? Rapid burster? $\gamma$-bursters??? Magnetic Ap-stars</td>
</tr>
<tr>
<td>A</td>
<td>Accretor</td>
<td>$R_a &lt; R_G$ $R_e \leq R_t$</td>
<td>$\dot{M}<em>e = \dot{M}</em>\alpha$</td>
<td>X-ray pulsars, bursters, cataclysmic variables, intermediate polars</td>
</tr>
<tr>
<td>G</td>
<td>Georotator</td>
<td>$R_G &lt; R_a$ $R_a \leq R_e$</td>
<td>$\dot{M}<em>e \leq \dot{M}</em>\alpha$</td>
<td>Earth, Jupiter</td>
</tr>
<tr>
<td>M</td>
<td>Magnator</td>
<td>$R_a &gt; a$ $R_e &gt; a$</td>
<td>$\dot{M}<em>e \leq \dot{M}</em>\alpha$</td>
<td>AM Her, polars</td>
</tr>
</tbody>
</table>

We can define a stopping radius $R_{st}$, at which external and internal pressures are equal.

The stage is determined by relation of this radius to other critical radii.
Ejector Propeller

Accretor Georotator

R=R_{co} \cos^{-2/3} \theta
R_{co}=(GM/\omega^2)^{1/3}

Light cylinder R_l=\omega/c
Unified approach to spin-down

One can find it comfortable to represent the spin-down moment by such a formula:

\[ -\kappa_1 \frac{\mu^2}{R_s^3} \]

\( \kappa_1 \) and \( R_s \) are different for different stages. \( \kappa_1 \) can be also frequency dependent.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \dot{M} )</th>
<th>( \kappa_1 )</th>
<th>( R_\kappa )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E, SE</td>
<td>0</td>
<td>( \gtrsim 2/3 )</td>
<td>( R_t )</td>
</tr>
<tr>
<td>P, SP</td>
<td>0</td>
<td>( \gtrsim 1/3 )</td>
<td>( R_m )</td>
</tr>
<tr>
<td>A</td>
<td>( \dot{M}_c )</td>
<td>( \gtrsim 1/3 )</td>
<td>( R_c )</td>
</tr>
<tr>
<td>SA</td>
<td>( \dot{M}_c(R_c/R_s) )</td>
<td>( \sim 1/3 )</td>
<td>( R_c )</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
<td>( \sim 1/3 )</td>
<td>( R_A )</td>
</tr>
<tr>
<td>M</td>
<td>( \dot{M}_c )</td>
<td>( \sim 1/3 )</td>
<td>a</td>
</tr>
</tbody>
</table>

Spin-up/down at the stage of accretion

\[ \frac{dI}{dt} = \dot{M} \kappa_{\text{w}} - \kappa_1 \frac{\mu^2}{R_s^3} \]

\[ \kappa_{\text{w}} = \begin{cases} (GM_d R_d)^{1/2}, & \text{Keplerian disk accretion,} \\ \eta_b \Omega R_d^2, & \text{wind accretion in a binary,} \\ \sim 0, & \text{a single magnetic rotator.} \end{cases} \]

For a single rotator (i.e. an isolated NS) spin-up can be possible due to turbulence in the interstellar medium.

In the case of isolated accreting NS one can estimate the accretion rate as:

\[ \dot{M}_c = 4\pi R_c^2 \rho_\infty v_\infty \]
Equilibrium period

\[
\mathcal{M} k_{su} = -\kappa^2 \frac{\mu^2}{R_i^3}
\]

The hypothesis of equilibrium can be used to determine properties of a NS.

The corotation radius is decreasing as a NS is spinning up. So, before equilibrium is reached the transition to the propeller stage can happen.

Looking at this formula (and remembering that for Accretors \( R_i = R_{co} \)) it is easy to understand why millisecond PSRs have small magnetic field. Spin-up can not be very large (Eddington rate). So, to have small spin periods (and so small corotation radii), it is necessary to have small magnetic fields. High magnetic field NS can not be spun-up to millisecond periods.

Critical periods for isolated NSs

\[
P_E(E \rightarrow P) \simeq 10 \mu_{30}^{1/2} n^{-1/4} v_{10}^{1/2} \text{ s}
\]

Transition from Ejector to Propeller (supersonic)

\[
P_A(P \rightarrow A) \simeq 420 \mu_{30}^{6/7} n^{-3/7} v_{10}^{9/7} \text{ s}
\]

Transition from supersonic Propeller to subsonic Propeller or Accretor

\[
P_{eq} = 2.6 \times 10^3 v_{(t)10}^{-2/3} \mu_{30}^{2/3} n^{-2/3} v_{10}^{13/3} \text{ s}
\]

A kind of equilibrium period for the case of accretion from turbulent medium

\[
v < 410 n^{1/10} \mu_{30}^{-1/5} \text{ km s}^{-1}
\]

Condition for the Georotator formation (instead of Propeller or Accretor)

(see, for example, astro-ph/9910114)
Accreting isolated neutron stars

Why are they so important?

- Can show us how old NSs look like
  1. Magnetic field decay
  2. Spin evolution
- Physics of accretion at low rates
- NS velocity distribution
- New probe of NS surface and interiors
- ISM probe

Expected properties

1. **Accretion rate**

An upper limit can be given by the Bondi formula:
\[
\text{Mdot} = \pi R_0^2 \rho v, \quad R_0 \sim v^{-2}
\]
\[
\text{Mdot} = 10^{11} \text{ g/s} \quad (v/10 \text{ km/s})^{-3} n
\]
\[
L = 0.1 \text{ Mdot c}^2 \sim 10^{31} \text{ erg/s}
\]

However, accretion can be smaller due to the influence of a magnetosphere of a NS.

2. **Periods**

Periods of old accreting NSs are uncertain, because we do not know evolution well enough.

\[
\rho_A = \frac{2^{5/14}}{\pi} (GM)^{-5/7} (\mu^2 / \dot{M})^{3/7} \sim
\]
\[
R_A = R_{\infty} \quad 300 \mu_{30}^{6/7} (v/10 \text{ km s}^{-1})^{9/7} n^{-3/7} \text{ s}.
\]
Subsonic propeller

Even after $R_{\text{co}} > R_A$ accretion can be inhibited. This has been noted already in the pioneer papers by Davies et al.

Due to rapid (however, subsonic) rotation a hot envelope is formed around the magnetosphere. So, a new critical period appears.

$$ P_{\text{br}} \approx 450 \mu_{30}^{16/21} \dot{M}_{15}^{5/7} m^{-4/21} \text{ s.} \quad \text{(Ikhansov astro-ph/0310076)} $$

If this stage is realized (inefficient cooling) then

• accretion starts later
• accretors have longer periods

Expected properties-2

3. Temperatures

Depend on the magnetic field. The size of polar caps depends on the field and accretion rate: $\sim R (R/R_A)^{1/2}$

4. Magnetic fields

Very uncertain, as models of the field decay cannot give any solid predictions

5. Flux variability.

Due to fluctuations of matter density and turbulent velocity in the ISM it is expected that isolated accretors are variable on a time scale

$\sim R_{\nu}/v \sim$ days - months

Still, isolated accretors are expected to be numerous at low fluxes (their total number in the Galaxy is large than the number of coolers of comparable luminosity). They should be hotter than coolers, and have much longer spin periods.
Properties of accretors

In the framework of a simplified model (no subsonic propeller, no field decay, no accretion inhibition, etc.) one can estimate properties of isolated accretors.

Slow, hot, dim, numerous at low fluxes (<10⁻¹³ erg/cm²/s)

Reality is more uncertain.

(astro-ph/0009225)

Where and how to look for

As sources are dim even in X-rays, and probably are extremely dim in other bands it is very difficult to find them.

In an optimistic scenario they outnumber cooling NSs at low fluxes. Probably, for ROSAT they are to dim. We hope that eROSITA will be able to identify accreting INs.

Their spatial density at fluxes ∼10⁻¹⁵ erg/cm²/s is expected to be ∼few per sq.degree in directions close to the galactic plane.

It is necessary to have an X-ray survey at ∼100-500 eV with good resolution.
Magnetic fields of NSs are expected to decay due to decay of currents which support them.

Crustal field of core field?

It is easy to decay in the crust.

In the core the field is in the form of superconducting vortices. They can decay only when they are moved into the crust (during spin-down).

Still, in most of model strong fields decay.

Period evolution with field decay

An evolutionary track of a NS is very different in the case of decaying magnetic field.

The most important feature is slow-down of spin-down. Finally, a NS can nearly freeze at some value of spin period.

Several episodes of relatively rapid field decay can happen.

Number of isolated accretors can be both decreased or increased in different models of field decay. But in any case their average periods become shorter and temperatures lower.
Magnetic field decay vs. thermal evolution

Magnetic field decay can be an important source of NS heating. Heat is carried by electrons. It is easier to transport heat along field lines. So, poles are hotter. (for light elements envelope the situation can be different).

Ohm and Hall decay

\[ B = B_0 \frac{\exp(-t/\tau_{\text{Ohm}})}{1 + \frac{\tau_{\text{Ohm}}}{\tau_{\text{Hall}}} (1 - \exp(-t/\tau_{\text{Ohm}}))} \]

arxiv:0710.0854 (Aguilera et al.)

Thermal heating for everybody?

It is important to understand the role of heating by the field decay for different types of INS. In the model by Pons et al. the effect is more important for NSs with larger initial B.

Note, that the characteristic age estimate (p/2 pdot) are different in the case of decaying field!

arXiv: 0710.4914 (Aguilera et al.)
We are really in trouble with spin-down models for pulsars!
Radio pulsar braking: braking index

\[ n_{br} = \frac{\ddot{\Omega}}{\dot{\Omega}^2} \]  \hspace{1cm} \text{Braking index (definition)}

\[ n_{br} = 3 + 2 \cot^2 \chi, \]  \hspace{1cm} \text{Magneto-dipole formula}

\[ n_{br} = 1.93 + 1.5 \tan^2 \chi. \]  \hspace{1cm} \text{Longitudinal current losses}

For well-measured braking indices \( n < 3 \).
However, for many pulsars they are very large.
This can be simply an observational effect (microglitches, noise, etc.),
but it can also be something real.
For example, related to the magnetic field evolution.

Conclusions

• We have some framework for spin evolution of NSs.
  They are expected to passe several well-defined stages:
  Ejector (including radion pulsar),
  Propeller (probably, with subsonic substage),
  Accretor.
  NSs with large velocities (or fields) after the Ejector stage
  can appear as Georotators.

• In binaries we observe Ejectors, Propellers and Accretor.
  For isolated NSs – only Ejectors (even, mostly radiopulsars).

• There are still many uncertainties related to the spin evolution:
  1. Spin-down rate and angle evolution for radio pulsars
  2. Subsonic propeller stage for isolated NSs
  3. Inhibition of accretion at low rates
  4. The role of the field decay
Conclusions-2

- Observations of isolated accreting NSs can help a lot to understand all unknown questions of NS spin evolution and low-rate accretion.
- Magnetic field decay can be important also for young NSs, especially for highly magnetized ones, as a source of energy.

So, we have some coherent picture ..... But ..... 

A lot of funny thing a still waitng for us!

Papers and books to read

- Lupinov V.M. “Astrophysics of neutron stars”
  Astrophysics and Space Science Reviews (1996)
  http://xray.sai.msu.ru/~mystery/articles/review/ 
- Pons, Geppert “Magnetic field dissipation in neutron star crusts: from magnetars to isolated neutron stars ” astro-ph/0703267 
Databases on NSs

1. ATNF. Pulsar catalogue
2. Magnetar database in McGill
3. Be/X-ray binaries
   http://xray.sai.msu.ru/~raguzova/BeXcat/

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Lecture 3
Population synthesis

Sergei Popov (SAI MSU)

Dubna “Dense Matter In Heavy Ion Collisions and Astrophysics”, July 2008
Population synthesis in astrophysics

A population synthesis is a method of a direct modeling of relatively large populations of weakly interacting objects with non-trivial evolution. As a rule, the evolution of the objects is followed from their birth up to the present moment.

see astro-ph/0411792 and Physics-Uspekhi 50, 1123

(УФН 2007 р., N11; http://www.ufn.ru)

Why PS is necessary?

1. No direct experiments $\Rightarrow$ computer experiments
2. Long evolutionary time scales
3. Selection effects. We see just a top of an iceberg.
4. Expensive projects for which it is necessary to make predictions
Tasks

1. To test and/or to determine initial and evolutionary parameters. To do it one has to compare calculated and observed populations. This task is related to the main peculiarity of astronomy: we cannot make direct experiments under controlled conditions.

2. To predict properties of unobserved populations. Population synthesis is actively used to define programs for future observational projects: satellites, telescopes, etc.

Two variants

Evolutionary and Empirical

1. Evolutionary PS. The evolution is followed from some early stage. Typically, an artificial population is formed (especially, in Monte Carlo simulations)

2. Empirical PS. It is used, for example, to study integral properties (spectra) of unresolved populations. A library of spectra is used to predict integral properties.
Examples

1. PS of radiopulsars
2. PS of gamma-ray pulsars
3. PS of close-by cooling NSs
4. PS of isolated NSs
5. PS of close binary systems

Population synthesis of radio pulsars

The idea was to make an advance population synthesis study of normal radio pulsar to reproduce the data observed in PMBPS and Swinburne. Comparison between actual data and calculations should help to understand better the underlying parameteres and evolution laws.

Only normal (non-millisecond, non-binary, etc.) pulsars are considered. Note, however, that the role of pulsars originated in close binaries can be important.

Ingredients
- Velocity distribution
- Spatial distribution
- Galactic model
- Initial period distribution
- Initial magnetic field distribution
- Field evolution (and angle)
- Radio luminosity
- Dispersion measure model
- Modeling of surveys

The observed PSR sample is heavily biased. It is necessary to model the process of detection, i.e. to model the same surveys in the synthetic Galaxy.

A synthetic PSR is detected if it appears in the area covered by one of the survey, and if its radio flux exceeds some limit.

2/3 of known PSRs were detected in PMBPS or and SM (914 and 151).

(following Faucher-Giguere and Kaspi astro-ph/0512585)
Velocity distribution

The authors checked different velocity distributions: single maxwellian, double maxwellian, lorentzian, paczynski mode, and double-side exponential. The last one was taken for the reference model. Single maxwellian was shown to be inadequate.

\[ p(v) = \frac{1}{2\langle v^2 \rangle} \exp \left( -\frac{|v|^2}{2\langle v^2 \rangle} \right). \]

\[ \theta = B_{\lambda} \cos \Delta \phi \]

\[ \phi = \Delta \phi + \Delta \phi \]

Spatial distribution

Initial spatial distribution of PSRs was calculated in a complicated realistic way.

- exponential dependences (R and Z) were taken into account
- Spiral arms were taken into account
- Decrease of PSR density close to the Galactic center was used

<table>
<thead>
<tr>
<th>Arm Number</th>
<th>Name</th>
<th>k</th>
<th>r₀</th>
<th>θ₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Norma</td>
<td>4.25</td>
<td>3.48</td>
<td>1.57</td>
</tr>
<tr>
<td>2</td>
<td>Carina-Sagittarius</td>
<td>4.25</td>
<td>3.48</td>
<td>4.71</td>
</tr>
<tr>
<td>3</td>
<td>Perseus</td>
<td>4.89</td>
<td>4.90</td>
<td>4.09</td>
</tr>
<tr>
<td>4</td>
<td>Crux-Scytus</td>
<td>4.89</td>
<td>4.90</td>
<td>0.95</td>
</tr>
</tbody>
</table>

\[ \theta(r) = k \ln(r/r₀) + \theta₀. \]

However, some details are still missing. For example, the pattern is assumed to be stable during all time of calculations (i.e. corotating with the Sun).
Galactic potential

The potential was taken from Kuijken and Gilmore (1989):
• disc-halo
• bulge
• nuclei

\[ \phi_G(r, z) = \phi_{dh}(r, z) + \phi_b(r) + \phi_n(r), \]

\[ \phi_{dh}(r, z) = \frac{-GM_{dh}}{\sqrt{\left(\sqrt{z^2 + b_{dh}^2}\right)^2 + b_{dh}^2 + r^2}}. \]

\[ \phi_{b,n}(r) = \frac{-GM_{b,n}}{\sqrt{b_{b,n}^2 + r^2}}. \]

\[ \vec{x} = -\nabla \phi_G, \]

<table>
<thead>
<tr>
<th>Constant</th>
<th>Disc-Halo (dh)</th>
<th>Bulge (b)</th>
<th>Nucleus (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>( 1.45 \times 10^{11} ) M(_{\odot})</td>
<td>( 9.3 \times 10^9 ) M(_{\odot})</td>
<td>( 1.0 \times 10^{10} ) M(_{\odot})</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_1 )</td>
<td>0.325 kpc</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0.090 kpc</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_3 )</td>
<td>0.125 kpc</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_0 )</td>
<td>2.4 kpc</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b )</td>
<td>5.5 kpc</td>
<td>0.25 kpc</td>
<td>1.5 kpc</td>
</tr>
</tbody>
</table>

Initial spin periods and fields

Spin periods were randomly taken from a normal distribution. Magnetic fields – also from a normal distribution for \( \log B \).

The authors do not treat separately the magnetic field and inclination angle evolution.

Purely magneto-dipole model with \( n=3 \) and \( \sin \chi = 1 \) is used. \( R_{NS}=10^6 \) cm, \( l=10^{45} \).

\[ P \dot{P} = \left( \frac{8\pi^2 \mu^6}{3Ic^3} \right) B^2 \sin^2 \chi, \]

\[ P \sim (P_0^2 + K t)^{1/2} \]

The death-line is taken in the usual form:

\[ \frac{B}{P^2} = 0.17 \times 10^{12} \text{ G s}^{-2} \]
Radio luminosity and beaming

Model I

\[
p(L) \propto \begin{cases} 
L_\alpha & \text{for } L \in [L_{\text{low}}, L_{\text{to}}] \\
L_\beta & \text{for } L \in [L_{\text{to}}, \infty) \\
0 & \text{otherwise}
\end{cases}
\]

\[
L_{\alpha} = 2 \text{ mJy kpc}^2 \\
\alpha_1 = -19/15 \\
\alpha_2 = 2 \\
L_{\text{low}} = 0.1 \text{ mJy kpc}^2
\]

Model II

\[
\log L = \log \left( L_0 P^{\text{sp}} P_{15}^{\text{tr}} \right) + L_{\text{corr}}.
\]

\[
f(P) = 0.09 [\log (P/s) - 1]^2 + 0.03.
\]

Average beaming fraction is about 10%.

Optimal model and simulations

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial Distribution Model</td>
<td>Yusifov &amp; König</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.55 kpc</td>
</tr>
<tr>
<td>$a$</td>
<td>1.64</td>
</tr>
<tr>
<td>$b$</td>
<td>4.01</td>
</tr>
<tr>
<td>Birth Height Distribution ($z_0$)</td>
<td>Exponential</td>
</tr>
<tr>
<td>50 pc</td>
<td></td>
</tr>
<tr>
<td>Birth Velocity Distribution ($v_{3D}$)</td>
<td>Exponential</td>
</tr>
<tr>
<td>380 km s$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Birth Spin Period Distribution ($P_0$)</td>
<td>Normal</td>
</tr>
<tr>
<td>300 ns</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{P_0}$</td>
<td>150 ns</td>
</tr>
<tr>
<td>Magnetic Field Distribution ($\log (B/G)$)</td>
<td>Log-Normal</td>
</tr>
<tr>
<td>12.65</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\log B}$</td>
<td>0.55</td>
</tr>
<tr>
<td>Luminosity Model</td>
<td>$P - P$ Power Law</td>
</tr>
<tr>
<td>$L_0$</td>
<td>0.18 mJy kpc$^3$</td>
</tr>
<tr>
<td>$\epsilon_P$</td>
<td>-1.5</td>
</tr>
<tr>
<td>$\epsilon_{P_0}$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_{\text{corr}}$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

The code is run till the number of “detected” synthetic PSR becomes equal to the actual number of detected PSRs in PBMP and SM.

For each simulation the “observed” distributions of $b,l$, $DM$, $S_{1400}$, $P$, and $B$, are compared with the real sample.

It came out to be impossible to to apply only statistical tests. Some human judgement is necessary for interpretation.
Results

Solid lines – calculation, hatched diagrams - real observations

Discussion of the results

1. No significant field decay (or change in the inclination angle) is necessary to explain the data.
2. Results are not very sensitive to braking index distribution
3. Birthrate is 2.8+/-0.1 per century.
   If between 13% and 25% of core collapse SN produce BHs, then there is no necessity to assume a large population of radio quiet NSs. 120 000 PSRs in the Galaxy
Population synthesis of gamma-ray PSRs

Ingredients
1. Geometry of radio and gamma beam
2. Period evolution
3. Magnetic field evolution
4. Initial spatial distribution
5. Initial velocity distribution
6. Radio and gamma spectra
7. Radio and gamma luminosity
8. Properties of gamma detectors
9. Radio surveys to compare with.

Tasks
1. To test models
2. To make predictions for GLAST and AGILE

(following Gonthier et al astro-ph/0312565)

Population of close-by young NSs

■ Magnificent seven
■ Geminga and 3EG J1853+5918
■ Four radio pulsars with thermal emission (B0833-45; B0656+14; B1055-52; B1929+10)
■ Seven older radio pulsars, without detected thermal emission.

To understand the origin of these populations and predict future detections it is necessary to use population synthesis.
To build an artificial model of a population of some astrophysical sources and to compare the results of calculations with observations.

Population synthesis: ingredients

- Birth rate of NSs
- Initial spatial distribution
- Spatial velocity (kick)
- Mass spectrum
- Thermal evolution
- Interstellar absorption
- Detector properties

Task:

Population synthesis – I.

Gould Belt : 20 NS Myr⁻¹
Gal. Disk (3kpc) : 250 NS Myr⁻¹

Cooling curves by Blaschke et al.
Mass spectrum

Arzoumanian et al. 2002
The Gould Belt

- Poppel (1997)
- R=300 – 500 pc
- Age 30-50 Myrs
- Center at 150 pc from the Sun
- Inclined respect to the galactic plane at 20 degrees
- 2/3 massive stars in 600 pc belong to the Belt
- 20-30 SN per Myr

Mass spectrum of NSs

- Mass spectrum of local young NSs can be different from the general one (in the Galaxy)
- Hipparcos data on near-by massive stars
- Progenitor vs NS mass:
  Timmes et al. (1996);
  Woosley et al. (2002)

astro-ph/0305599
In our study we use a set of cooling curves calculated by Blaschke, Grigorian and Voskresenski (2004) in the frame of the Nuclear medium cooling model.
Some results of PS-I:
Log N – Log S and spatial distribution

Log N – Log S for close-by ROSAT NSs can be explained by standard cooling curves taking into account the Gould Belt.

More than ½ are in +/- 12 degrees from the galactic plane.
19% outside +/- 30°
12% outside +/- 40°

(Popov et al. 2005 Ap&SS 299, 117)
1. Spatial distribution of progenitor stars

- a) Hipparcos stars up to 500 pc
  [Age: spectral type & cluster age (OB ass)]
- b) 49 OB associations: birth rate $\sim N_{\text{star}}$
- c) Field stars in the disc up to 3 kpc

We use the same normalization for NS formation rate inside 3 kpc: 270 per Myr.

Most of NSs are born in OB associations.

For stars <500 pc we even try to take into account if they belong to OB assoc. with known age.

3. Spatial distribution of ISM ($N_H$)

Instead of: $N_H$ inside 1 kpc

Now: (see astro-ph/0609275 for details)

Modification of the old one

Hakkila
50 000 tracks, new ISM model

(Posselt et al. arXiv: 0801.4567)

Log N – Log S as an additional test

- Standard test: Age – Temperature
  - Sensitive to ages <10^5 years
  - Uncertain age and temperature
  - Non-uniform sample
- Log N – Log S
  - Sensitive to ages >10^5 years (when applied to close-by NSs)
  - Definite N (number) and S (flux)
  - Uniform sample
- Two test are perfect together!!!
Model I

- Pions.
- Gaps from Takatsuka & Tamagaki (2004)
- $T_s - T_{in}$ from Blaschke, Grigorian, Voskresenky (2004)

Can reproduce observed Log N – Log S
(astro-ph/0411618)

Model II

- No Pions
- Gaps from Yakovlev et al. (2004), $^3P_2$ neutron gap suppressed by 0.1
- $T_s - T_{in}$ from Tsuruta (1979)

Cannot reproduce observed Log N – Log S
Sensitivity of Log N – Log S

- Log N – Log S is very sensitive to gaps
- Log N – Log S is not sensitive to the crust if it is applied to relatively old objects (>10^{4.5} yrs)
- Log N – Log S is not very sensitive to presence or absence of pions

We conclude that the two test complement each other

Results for HySs application

<table>
<thead>
<tr>
<th>Model</th>
<th>Δα [MeV]</th>
<th>α</th>
<th>BC</th>
<th>T – t</th>
<th>Log N – Log S</th>
<th>M_{\rm sym} ≤ 1.5 M_{\odot}</th>
<th>All tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>10</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td>0.1</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>III</td>
<td>0.1</td>
<td>2</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>IV</td>
<td>5</td>
<td>25</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

One model among four was able to pass all tests.
Isolated neutron star census

**Task.**

- To calculate distribution of isolated NSs in the Galaxy over evolutionary stages: Ejector, Propeller, Accretor, Georotator
- Predict the number of accretors

**Ingredients.**

- Galactic potential
- Initial NS spatial distribution
- Kick velocity
- ISM distribution
- Spin initial distribution, evolution and critical periods
- Magnetic field initial distribution and evolution

---

**Stages**

Rather conservative evolutionary scheme was used.

For example, supersonic propellers have not been considered (Ikhsanov 2006).

*astro-ph/9910114*
Accreting isolated NSs

At small fluxes $< 10^{-13}$ erg/s/cm$^2$ accretors can become more abundant than coolers. Accretors are expected to be slightly harder: 300-500 eV vs. 50-100 eV. Good targets for eROSITA!

From several hundreds up to several thousands objects at fluxes about few $10^{-14}$, but difficult to identify.

Monitoring is important.

Also isolated accretors can be found in the Galactic center (Zane et al. 1996, Deegan, Nayakshin 2006).

astro-ph/0009225

Population synthesis of binary systems

Interacting binaries are ideal subject for population synthesis studies:

- The are many of them observed
- Observed sources are very different
- However, they come from the same population of progenitors...
- ... who’s evolution is non-trivial, but not too complicated.
- There are many uncertainties in evolution ...
- ... and in initial parameters
- We expect to discover more systems
- ... and more types of systems
- With new satellites it really happens!
There are several groups in the world which study evolution of close binaries using population synthesis approach.

**Examples of topics**

- Estimates of the rate of coalescence of NSs and BHs
- X-ray luminosities of galaxies
- Calculation of mass spectra of NSs in binaries
- Calculations of SN rates
- Calculations of the rate of short GRBs

(Lipunov et al.)
Looking for new magnetars

There are many archival XMM-Newton (and Chandra) observations.
Why not use them to search for magnetars?
Just using the fact that all known magnetars.
have a period of the order of a few milliseconds.

L = 3 \times 10^{33} \text{ erg/s}

Filled symbols: \( A_{1000} = 0.7 \)
Open symbols: \( A_{1000} = 0.15 \)

By the way, they also can put contraints on M7-like sources....
Looking for new M7-like sources

M7-like objects are very interesting by themselves and are important for studies of NS physics.

Several campaigns have been made to look for more sources.

- Agueros et al. (astro-ph/0511659)
- Chieregato et al. (astro-ph/0502292)

Looking for blank field soft X-rays sources (extreme $f_x/f_{opt}$ ratio).

Chieregato et al. searched for blank field sources with the ROSAT HRI data (only ~1.8% of the sky, mostly at high galactic latitudes). Several candidates have been figured out.

Agueros et al. used ROSAT All-sky Survey and SDSS. Also several candidates have been found.

Predictions for future searches and candidates

(Posselt et al. arXiv: 0801.4567)
Looking for isolated accretors

Many programs aimed to find accreting isolated NSs have been made in 90s (see a review in Treves et al. (2000) PASP 112, 297). Since then researches became a little bit pessimistic about the subject. However, with present day abilities and prospects for near future it is important to remember about the possibility to detect such interesting sources.

For example, looking for new M7-like NSs which are expected to be more abundant in the optimistic scenario) at fluxes \(<10^{-13} \text{ erg/cm}^2\text{s}\).

Recently, Pires and Motch (0710.5192) in the 2XMMp catalogue. One interesting object. Most probably, it is a cooling INS (work in progress).

Looking for radio pulsar counterparts for EGRET unidentified sources

Recently Crawford et al. (astro-ph/0608225) tried to find dim radio pulsars in 56 relatively small error boxes of EGRET unidentified sources. Nothing came out.

Then, Keith et al. (0807.2088) made a search at high frequencies for three cases and discovered a new pulsar! Probably, it is important to use high frequencies (~few GHz) GLAST is in orbit now and everything is working. Hopefully, soon we’ll have more gamma-ray selected isolated neutron stars (radio pulsars, coolers, ....). More population studies will be necessary which take into account all possible types of NSs.
Conclusions

- Population synthesis is a useful tool in astrophysics
- Many theoretical parameters can be tested only via such modeling
- Many parameters can be determined only via PS models
- Actively used to study NSs
- Actively used for predicting future observations and setting on observational programs

Dorothea Rockburne
Papers to read

- Popov, Prokhorov "Population synthesis in Astrophysics"
  Physics-Uspekhi 50 (11), 1123 (2007)
- Faucher-Giguere, Kaspi "Birth and evolution of isolated radio pulsars"
  astro-ph/0512585
- Postnov, Yungelson “The Evolution of Compact Binary Star Systems ”
- Lipunov, Postnov, Prokhorov “The Scenario Machine:
  Binary Star Population Synthesis”
  Astrophysics and Space Science Reviews (1996)
  http://xray.sai.msu.ru/~mystery/articles/review/