Faraday rotation and magneto-optical figure of merit for the magnetite magnetic fluids

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Abstract. In the present paper, using magnetite magnetic fluids as examples, we consider the optical and magneto-optical properties of magnetic fluids based on particles of magnetic oxides, for the optical constants of the material of which, $n$ and $k$, the relation $k^2 \ll n^2$ holds. In this work the Faraday rotation is represented within the theoretical Maxwell-Garnett model. A theoretical analysis has shown that Faraday rotation for magnetic fluids is related to the Faraday rotation on the material of particles by the simple relation. According to this result in specific experimental conditions the values of the Faraday rotation proportion to $q$, which is the occupancy of the volume of the magnetic fluid with magnetic particles and spectral dependences of effect in magnetic fluid and in the proper bulk magnetic are similar. We also show that the values of the magneto-optical figure of merit for ultrafine medium and for the bulk material are equal.

1 Introduction

Magneto-optical research methods are successfully used to solve different types of task in the physics of magnetic phenomena. They are the powerful means to be used for the observation of domains and the boundaries of domains, as well as to examine the electronic energy of the structure of the solids [1].

Nowadays, magneto-optical research methods are widely used for examination of ultrafine magnetic structures with insertions that have the sizes of structural $n$ and magnetic heterogeneities less than 1000 $A$. Magneto-optical investigation of these types of structures are subject of overall interest, which is conditioned by both theoretical and practical importance.

In general, the magneto-optical properties of magnetic fluids are very different from the properties of the bulk ferromagnetic. This fact causes the changes in spectral and angular dependences of the magneto-optical effects. The reason for this is that magnetic fluids represent an example of a magnetic ultrafine medium composed with magnetic particles, the sizes of which are much less than the light wavelength. Therefore, it is important to define clearly the conditions of the magneto-optical experiment to make easier the interpretation of the magneto-optical spectra, which in its turn, provides with the information of surface layer of magnetized magnetic fluids, electronic energy structure of fine magnetic particles, as well as of the properties of carrier fluids. Besides, it is significant to examine the electronic energy structures of fine magnetic particles not only to investigate the temporary stability of magnetic fluids, but also to analyze the chemical processes on the surface of the magnetic particles. On the other hand, in some cases the fine magnetic particles could be used as a probe.

2 Theory

In general, magneto-optical effects can be divided into two groups: 1)Faraday and Cotton-Mutton Effects produced by the light which propagates through magnetic crystal, and 2)Magneto-Optical Kerr Effects which are created by the light reflected from the magnetic crystal. In reference [2] using magnetite magnetic fluids as examples, was considered the magneto-optical reflection effect for magnetic fluids based on particles of magnetic oxides, for the optical constants of the material of which, $n$ and $k$, the relation $k^2 \ll n^2$ holds. In this research the equatorial Kerr effect is represented within the theoretical Maxwell-Garnett model. A theoretical analysis has shown that in the case of magnetic oxides, for which the condition $k^2 \ll n^2$ is fulfilled, the equatorial Kerr effect $\delta_n(q)$ for magnetic fluids with the ratio of the volume occupied by magnetic particles is related to the equatorial Kerr effect $\delta_n$ on the material of particles by the following simple relation:

$$\delta_n(q) = q \delta_n$$

(1)

It follows from the relation obtained that in the case of magnetic oxides the character of the frequency dependences $\delta_n(q)$ is independent of $q$ and therefore on
magnetization itself. Besides, the spectral dependences of magneto-optical effects are similar on the magnetic fluids and on the substance of particles. The experimental and theoretical results of the magnetite magnetic fluids are in good agreement.

We did analogous researches for Faraday rotation. When considering the magneto-optical effects, which are odd function of magnetization, in magnetic fluids, it should be kept in mind that magnetic fluid is a variety of the ultrafine media [3].

The magneto-optical properties of ultrafine media were analyzed in refs. [3, 4]. It was shown that when considering the magneto-optical properties of medium consisting of magnetic colloidal particles the sizes of which are much less than the light wavelength, one should introduce the tensor of the effective dielectric permittivity:

\[ \mathbf{\varepsilon}_{\text{eff}} = \begin{bmatrix} \varepsilon_{\text{eff}} & -i\varepsilon_{\text{eff}} & 0 \\ i\varepsilon_{\text{eff}} & \varepsilon_{\text{eff}} & 0 \\ 0 & 0 & \varepsilon_{\text{eff}} \end{bmatrix} \]  

(2)

where, \( \varepsilon_{\text{eff}} = \varepsilon_{\text{m}} - i\varepsilon_{\text{m}} \), \( \varepsilon_{\text{m}} = \varepsilon_{\text{m}} - i\varepsilon_{\text{m}} \) and \( \varepsilon_{\text{m}} = \varepsilon_{\text{m}} - i\varepsilon_{\text{m}} \) is the diagonal and non-diagonal tensor components of the effective dielectric permittivity of the material of magnetic colloidal particles, \( \varepsilon_{\text{m}} \) is the dielectric permittivity of the fluid phase, and \( q \) is the ratio of the volume, occupied by magnetic particles, to the total volume of the magnetic fluid.

In the research, the Faraday rotation is represented within the framework of the theoretical Maxwell-Garnett model.

The Faraday effect is related to the tensor components of the effective dielectric permittivity as follows [3]:

\[ \alpha_F = \frac{\pi}{\lambda} \text{Re} \frac{\varepsilon_{\text{f}}}{\varepsilon_{\text{f}}^2} \]  

(5)

where \( \varepsilon_{\text{f}} = \varepsilon_{\text{ef}} - i\varepsilon_{\text{ef}} \) and \( \varepsilon_{\text{f}} = \varepsilon_{\text{ef}} - i\varepsilon_{\text{ef}} \).

In this case, considering the following relation \( k^2 \ll n^2 \) and formula (3), Faraday rotation can be expressed in this way:

\[ \alpha_F = \frac{\pi}{\lambda n^2_{\text{f}}} (\varepsilon_{\text{ef}} + \varepsilon_{\text{ff}}) \]  

(6)

Taking into account formulas (4) for real and imaginary parts of the tensor of the effective dielectric permittivity \( \varepsilon_{\text{f}} \) together with \( k^2 \ll n^2 \) and formula (6) takes the form:

\[ \alpha_F = \frac{\pi q n^2_{\text{f}}}{\lambda} \left[ \frac{\varepsilon_{\text{m}} n + 4 y k\varepsilon_{\text{f}}}{y(n^2 - n_{\text{f}}^2 + n_{\text{m}}^2)} \right] \]  

(7)

where, \( y = \frac{1 - q}{3} \) and \( n_{\text{f}} \) is the reflective index of liquid carrier.

If \( n_{\text{f}} \approx n \), then

\[ \alpha_F = \frac{\pi q}{\lambda n} [\varepsilon_{\text{m}} n + 4 y k\varepsilon_{\text{f}}] \]  

(8)

Note that we have the following correlation: when \( q < 0.4 \), then \( 4y \approx 1 \). Thus the expression for the Faraday rotation could be written down in the simpler way:

\[ \alpha_F (q) = q \alpha_{FM} \]  

(9)

where, \( \alpha_F (q) \) is Faraday rotation for magnetic fluids with the ratio of the volume occupied by magnetic particles \( q \); \( \alpha_{FM} \) - Faraday rotation on the material of particles.

It is necessary to say that received results (9) represent truth if only these two conditions are followed: 1) \( k^2 \ll n^2 \); 2) \( n_{\text{f}} \approx n \).

It follows from the relation obtained that in the specific experimental conditions the amount of Faraday effect is proportional to \( q \) and therefore on magnetization itself. Also, the character of spectral dependences of magneto-optical effects, are similar for the magnetic fluids and the substance of particles.

It is well-known that magneto-optical figure of merit \( F \) is introduced to draw a comparison between magneto-optical materials. The former is the ratio of twice specific Faraday rotation to the absorption coefficient of the material:

\[ F = 2\alpha_F / K \]  

(10)
Within the framework of the theoretical Maxwell-Garnett model in order to find the link between \( k \) and \( k_{ef} \), we discuss the formula for imaginary parts of the diagonal components of the tensor of effective dielectric permittivity written in the following way:

\[
\varepsilon_{ef} = \frac{q}{\text{y}(\varepsilon_1 - \varepsilon_0) + \varepsilon_0} \]

(11)

when \( k^2 << n^2 \) and \( \varepsilon_0 \approx \varepsilon_1 \) from (11) we can derive a simple relation:

\[
k_{ef} = qk
\]

(12)

The absorption indicators of the discussed medium \( k_{ef} \) and \( k \) are linked to natural (decimal) absorption coefficient \( K_{ef} \) and \( K \) with the formula

\[
k_{ef} = \lambda K_{ef} / 4\pi , \quad k = \lambda K / 4\pi
\]

(13)

Thus formula (12) could be put as following:

\[
K_{ef} = qK
\]

(14)

From (11) and (13) formula come:

\[
\frac{\alpha_{ef}(q)}{K_{ef}} = \frac{\alpha_{efM}}{K}
\]

(15)

As \( 1/K_{ef} = l_{ef} \) and \( 1/K = l \) characterise the depth of permeability of light in the medium, we can conclude from the formula (5) that the amounts of Faraday rotation in ultrafine medium and in subsequent bulk magnetic measured in thickness are equal. In the case of \( k^2 << n^2 \) the amounts of magneto-optical figure of merit in the ultrafine medium and bulk magnetic are equal.

### 3 Results and discussion

Fig. 1 represents the results of calculations of \( \alpha_{ef}(q) \) made by means of the formula (5) for magnetite particles in the air (a) and in the liquid silicon-organic compound (b), and by the formula (9) (c). Calculations were carried out for different \( q \). From the fig.1 it is obvious that the magneto-optical maxima for magnetite particles shift to the side of large energies. If we increase \( \varepsilon_0 \), this shift diminishes, and for the liquid silicon-organic compound \( (\varepsilon_0 = 2.56) \) the shift approximates to the zero. The results derived from our research indicate that calculations conducted according to (5) for large \( \varepsilon_0 \) \( (\varepsilon_0 = 2.56) \), coincide with the calculations carried out by the simplified formula (9).

We did analogous calculations using formula (5) and formula (9) for magnetite particles in order to verify the linear dependence of Faraday rotation on \( q \). Fig. 2 shows the correlations of Faraday rotation to \( q \) for magnetite particles, which are calculated by formula (5) and (9) at the quantum energy of incident light \( h\omega = 1.7 \text{ eV} \).

![Fig. 1. Dependences of the Faraday rotation on the quantum energy of incident light \( h\omega \), calculated by formula (5) for magnetite particles in the air (a) and in the liquid silicon-organic compound (b), and by the formula (9) (c) with \( q=0,1(1); 0,3(2); 0,5(3); 1(4) \).](image1)

![Fig. 2. Dependences of the Faraday rotation on \( q \), calculated by formula (5) for magnetite particles in the air (1) and in the liquid silicon-organic compound (2), and by the formula (9) (3) at \( h\omega = 1.7 \text{ eV} \).](image2)

From the fig. 2 it is obvious that the linear dependences of Faraday rotation on \( q \) have been observed only for large \( \varepsilon_0 \) \( (\varepsilon_0 = 2.56) \).

Our results are in a good relation with the experiments [5], where Faraday rotation has been studied in magnetite magnetic fluids with different concentrations.

### 4 Conclusions

In the present paper, on the example of magnetite magnetic fluids we considered Faraday rotation in magnetic fluids based on particles of magnetic oxides.
Within the framework of the theoretical Maxwell-Garnett model the simple relation between the Faraday rotation for ultra-fine medium and bulk material is revealed. The conditions of the magneto-optical experiment were defined for the essential analytical simplification of the subsequently received results.

In conclusion, we would like to underline that this result are expected to be suitable to all magnetic ultrafine medium with the same relation between optical constants: $k^2 \ll n^2$.

References

1. Krinchik G.S. Physical Principles of Magnetic Phenomena, Moscow State University (1985)