### Possible molecular bound states: $\Lambda_cN$ and $\Lambda_c\Lambda_c$

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**Abstract.** One boson exchange approach is used to study bound state problems for the $\Lambda_cN$ and $\Lambda_c\Lambda_c$ systems. The coupling constants are determined by various methods while the phenomenological cutoffs at vertices are parameterized with a free cutoff parameter. The numerical results are sensitive to the cutoff parameter. Although we cannot determine it at present, it is possible to have molecular bound states $\Lambda_cN$ and $\Lambda_c\Lambda_c$ for a reasonable value.

1 Introduction

Study of possible molecule-like bound states containing heavy quark hadrons is now an interesting topic, especially after the observation of number of near-threshold heavy quark mesons like $X(3872)$. Such a study may help us to understand further the strong interaction. The deuteron, a shallow bound state of a proton and a neutron, is the well established baryon–baryon hadronic molecule. In the heavy quark sector, molecule-like bound states are more likely to be bound for two reasons. One is that the kinetic energy is suppressed because of the larger reduced mass of the systems, which is advantageous for the formation of hadronic molecules. The other is that the degeneracy of hadrons in the heavy quark limit, which renders the possible important coupled channel effects. In the present cases, the degeneracy means that $\Sigma_c$ and $\Sigma_c^-$ should have the same mass according to heavy quark spin symmetry. In general, coupled channel effects become important when two channels are close. As the lowest charmed baryon, $\Lambda_c$, does not decay through strong interaction. It is worthwhile to study possible bound states containing it by including the coupled channel effects. Here we study two systems, $\Lambda_cN$ and $\Lambda_c\Lambda_c$.

The interaction between $\Lambda_c$ and $N$ was first studied in Ref. [1] when the authors explored the $\Lambda_c$ hypernuclei. It was found that the results are model dependent. It is interesting to give the $\Lambda_cN$ a serious study with the modern effective theory and channel couplings to $\Sigma_cN$ and $\Sigma_c^0N$. We present the quantum numbers and coupled channels in Table 1.

<table>
<thead>
<tr>
<th>Channels</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J^P = 0^+$</td>
<td>$\Lambda_cN(S_0)$</td>
<td>$\Sigma_cN(S_0)$</td>
<td>$\Sigma_c^0N(D_0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J^P = 1^+$</td>
<td>$\Lambda_cN(S_1)$</td>
<td>$\Sigma_cN(S_1)$</td>
<td>$\Sigma_c^0N(S_1)$</td>
<td>$\Lambda_cN(D_1)$</td>
<td>$\Sigma_cN(D_1)$</td>
<td>$\Sigma_c^0N(D_1)$</td>
<td>$\Sigma_c^0N(D_1)$</td>
</tr>
</tbody>
</table>

**Table 1.** The $S$-wave $\Lambda_cN$ states and the channels which couple to them.

For the $\Lambda_c\Lambda_c$ state, its quantum numbers are $I(J^P) = 0(0^+)$. The coupled channels we are considering are given in Table 2. Although $\Lambda_c$ does not have strong decay channels, $\Lambda_c\Lambda_c$ may mix with $\Xi_cN$ by exchanging charmed mesons. Since we consider the possible loosely bound molecular state, such a mixing occurring at short distance may be unimportant and we neglect the $\Xi_cN$ channel in the present work. Possible bound state in the $\Xi_cN$ system was proposed in Ref. [2].

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The two systems we study here are different, but the frameworks are similar. One gets the non-relativistic meson-exchange potentials with effective Lagrangian and then solves the bound state problem numerically. Here we just present main ingredients and part of results. Details of the formulations and results are given in Refs. [3] and [4].

2 Framework

In the meson-exchange approach, the potential model generally incorporates the contributions from the pseudoscalar, scalar, and vector mesons. To construct effective Lagrangian and reduce the number of coupling constants, constraints from various symmetries are usually used. For the pseudoscalar pions, relevant interactions are protected by the well known chiral symmetry. For the charmed baryons, one may resort to heavy quark symmetry. The interactions for vector mesons can be constrained by hidden local symmetry. With these requirements, we obtain the following Lagrangian,

\[ \mathcal{L}_B = \mathcal{L}_{B_3} + \mathcal{L}_S + \mathcal{L}_{int}, \]
\[ \mathcal{L}_{B_3} = \frac{1}{2} \text{tr} [\bar{B}_3 (i v \cdot D) B_3] + \mathcal{I}_{B_3} (\Gamma_{\mu} - \rho_{\mu}) B_3 + \ell_{\mu} \text{tr} [\bar{B}_3 \sigma B_3] \]
\[ \mathcal{L}_S = -\text{tr} [S^a (i v \cdot D - A_{\mu}) S_a] + \frac{3}{2} g_1 (i \nu_5) e^{-i m_{\pi} \xi} \text{tr} [S_\mu A_\nu S_\nu] + i \lambda_5 [S_\mu F^{\mu \nu} S_\nu] + \ell_{\nu} [S_\mu S_\nu] \]
\[ \mathcal{L}_{int} = g_4 [\bar{S}_\mu A_\nu B_3] + i G_{\mu \nu} e^{-i m_{\pi} \xi} \text{tr} [\bar{S}_\nu F_{\mu \alpha} B_3] + h.c. \]

In this Lagrangian,

\[ B_3 = \begin{pmatrix} 0 & A_+^\tau & \Xi_+^\tau \\ -A_+^{\tau \dag} & 0 & \Xi_0^\tau \\ -\Xi_+^{\tau \dag} & -\Xi_0^{\tau \dag} & 0 \end{pmatrix}, \]
\[ B_6 = \begin{pmatrix} \Sigma^{\tau \dag} & \frac{1}{2} \frac{1}{\sqrt{2}} \xi^{\tau \dag} & \frac{1}{3} \xi^{\tau \dag} \\ \frac{1}{2} \frac{1}{\sqrt{2}} \xi^{\tau \dag} & 0 & \frac{1}{3} \xi^{\tau \dag} \\ \frac{1}{3} \xi^{\tau \dag} & \frac{1}{3} \xi^{\tau \dag} & 0 \end{pmatrix}, \]
\[ B_7 = \begin{pmatrix} \Sigma^{\tau \dag} & \frac{1}{2} \xi^{\tau \dag} & \frac{1}{3} \xi^{\tau \dag} \\ \frac{1}{2} \xi^{\tau \dag} & 0 & \frac{1}{3} \xi^{\tau \dag} \\ \frac{1}{3} \xi^{\tau \dag} & \frac{1}{3} \xi^{\tau \dag} & 0 \end{pmatrix}, \]
\[ \Pi = \sqrt{2} \begin{pmatrix} \rho_0 & \frac{n}{\sqrt{2}} & \frac{\pi^+}{\sqrt{2}} \\ \frac{n}{\sqrt{2}} & \rho_0 & \frac{\pi^-}{\sqrt{2}} \\ \frac{\pi^+}{\sqrt{2}} & \frac{\pi^-}{\sqrt{2}} & K^0 \end{pmatrix}, \]
\[ \mathcal{V} = \begin{pmatrix} \rho_0 & \frac{n}{\sqrt{2}} & \frac{\pi^+}{\sqrt{2}} \\ \frac{n}{\sqrt{2}} & \rho_0 & \frac{\pi^-}{\sqrt{2}} \\ \frac{\pi^+}{\sqrt{2}} & \frac{\pi^-}{\sqrt{2}} & K^0 \end{pmatrix}, \]
\[ \xi = \exp [\frac{\mu}{\sqrt{2}}], \quad A_\mu = \frac{1}{4} (\partial_\mu \xi + (\partial_\mu \xi) \xi^\tau), \quad \Gamma_\mu = \frac{1}{4} (\xi^\tau (\partial_\mu \xi) - (\partial_\mu \xi) \xi^\tau), \quad \xi^\tau = -\frac{1}{\sqrt{2}} (\gamma_\mu + \nu_\mu) \gamma^5 \xi, \]
\[ F_{\mu \nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu], \quad S_\mu = B_6^{\mu \nu} - \frac{1}{\sqrt{2}} (\gamma_\mu + \nu_\mu) \gamma^5 \xi, \]
\[ D_\mu B_3 = \partial_\mu B_3 + \Gamma_\mu B_3 + B_3 \Gamma_\mu, \quad D_\mu S_\nu = \partial_\mu S_\nu + \Gamma_\mu S_\nu + S_\nu \Gamma_\mu, \quad \]
where \( v_\mu \) is the velocity of the heavy baryon, \( \Delta \rho \) is the mass difference between the sextet and the antitriplet baryons in the heavy quark limit, and \( f = 92.3 \text{ MeV} \) is the pion decay constant. The constant \( g_5 = m_\pi / (\sqrt{2} f_\pi) = 5.8 \) is derived with the vector meson dominance (VMD). There are eight coupling constants in the Lagrangian. To constrain their values, we have used several methods: strong decay of the heavy baryons, the quark model, the chiral multiplet assumption, VMD, and the QCD sum rule results. For the numerical evaluation, we take \( g_4 = 0.999, \quad q_1 = 0.94, \quad \ell = -3.1, \quad \beta = -6.2, \quad (\beta g_4) = -5.04, \quad (\beta_5 g_4) = -10.08, \quad (\lambda_5 g_4) = 19.2 \text{ GeV}^{-1}, \) and \( (\lambda g_4) = -6.8 \text{ GeV}^{-1}. \)

For the nucleon-nucleon interaction part, we use the following SU(2) Lagrangian,

\[ \mathcal{L}_N = \frac{G_A}{2f} N^\dag \gamma^\mu \gamma^5 \partial_\mu (\pi^{+ \dag}) N - h_3 N \phi N - h_4 \bar{N} \gamma^\mu (\pi^{+ \dag} + \omega_\mu) N - h_5 \bar{N} \gamma^\mu (\pi^{+ \dag} + \omega_\mu) N, \]
where $\tau^i$ is the Pauli matrix, representing the isospin. For the values of the coupling constants, we use $h_\sigma = 10.95$, $h_V = 3.0$, and $h_T = 6.4 \text{ GeV}^{-1}$.

One derives the non-relativistic potentials from the t-channel meson-exchange diagrams with the above Lagrangian. We have neglected $O(1/M_\Lambda)$ corrections and thus the final potentials are independent of the heavy baryon masses. We also neglect the $\delta$-functional terms since we are considering loosely bound molecules and the very short range interactions should in principle have small contributions. To include the size effects of the hadrons, we have introduced a phenomenological cutoff $\Lambda$ at each interacting vertex through the monopole-type form factor

$$F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2} \quad (11)$$

where $m$ is the mass of the exchanged meson and $q$ is its 4-momentum. The cutoffs for various vertices in principle should be different and their values may be around 1 GeV. To reduce the number of unknown parameters, in the following parts, we just assume four independent cutoffs denoted by the exchanged mesons: $\Lambda_\pi$, $\Lambda_\sigma$, $\Lambda_\rho$, and $\Lambda_\omega$ and parameterize them with one free parameter.

### 3 Possible $\Lambda_cN$ molecular bound states

The variational method [5] is used when we solve the Schrödinger equation. For the formation of a loosely bound molecule, the long-range pion interaction plays a crucial role. For comparison, we use both the one-pion-exchange potential (OPEP) model and the one-boson-exchange potential (OBEP) model in our study.

#### 3.1 OPEP model

We first consider the results in the OPEP model. In this model, the information of the intermediate- and short-range interactions is encoded in the cutoff $\Lambda_\pi$. The direct one pion exchange is forbidden in the dominant channel $\Lambda_cN$. But one may still get binding solutions by including the coupled channels $\Sigma_cN$ and $\Sigma^*_cN$. But one may still get binding solutions by including the coupled channels $\Sigma_cN$ and $\Sigma^*_cN$.

<table>
<thead>
<tr>
<th>$\Lambda_c$ (GeV)</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.E. ($J = 0$) (MeV)</td>
<td>0.64</td>
<td>6.16</td>
<td>18.51</td>
<td>38.88</td>
<td>68.29</td>
</tr>
<tr>
<td>$\sqrt{\langle r^2 \rangle}$ (fm)</td>
<td>5.2</td>
<td>1.9</td>
<td>1.2</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>Prob. (%)</td>
<td>(98.2/0.6/1.2)</td>
<td>(94.0/2.3/3.7)</td>
<td>(89.3/4.6/6.1)</td>
<td>(84.5/7.2/8.3)</td>
<td>(80.1/9.8/10.1)</td>
</tr>
</tbody>
</table>

Table 3. Binding solutions for the $J^P = 0^+$ case with channel coupling in the OPEP model. The binding energies (B.E.) are given with relative to the $\Lambda_cN$ threshold. The probabilities correspond to $\Lambda_cN(^1S_0)$, $\Sigma_cN(^1S_0)$, and $\Sigma^*_cN(^2D_0)$, respectively.

For the $J^P = 0^+$ case, we present the binding energies, the root-mean-square (RMS) radius, and the probabilities of each channel in Table 3. The binding energy is given with relative to the $\Lambda_cN$ threshold. If a deuteron-like molecule composed of two baryons exists, the baryons should not be very close. Here we only list the results with the RMS radius larger than 0.8 fm. Other tighter solutions may be not reasonable. As an example, we show the wave functions of different channels for $\Lambda_c = 1.3 \text{ GeV}$ in Fig. 1 (a). That the probability of the channel $\Sigma^*_cN(^2D_0)$ is larger than that of the channel $\Sigma_cN(^1S_0)$ indicates the importance of the tensor force in the model. As a check, we have calculated the two channel case: $\Lambda_cN(^1S_0)$ and $\Sigma_cN(^1S_0)$ and we do not find any binding solutions.

For the $J^P = 1^+$ case, we present the results in Table 4. One finds that the binding energies are slightly different between the singlet and triplet cases. The calculation indicates again that the coupled
Table 4. Binding solutions for the $J^P = 1^+$ case with channel coupling in the OPEP model. The binding energies (B.E.) are given with relative to the $\Lambda_c N$ threshold. The probabilities correspond to $\Lambda_c N(^3S_1)$, $\Sigma_c N(^3S_1)$, $\Sigma_c^* N(^3S_1)$, $\Lambda_c N(^3D_1)$, $\Sigma_c N(^3D_1)$, $\Sigma_c^* N(^3D_1)$, and $\Sigma_c^* N(^5D_1)$, respectively. We also present the total $D$-wave probability.

channel effects are important. The 4-th channel, $\Lambda_c N(^3D_1)$, has small contribution. Actually, if one ignores this channel, the resultant binding energy changes little. We show the wave functions of different channels with the cutoff $\Lambda_\pi = 1.3$ GeV in Fig. 1 (b).

For comparison, we illustrate the sensitivity of the binding energy to the cutoff parameter in Fig. 2 in a diagrammatic form. Because there is no $\Lambda_c A_\sigma$ coupling, the results indicate that the channel coupling is very important for the formation of the $\Lambda_c N$ molecule. Especially, the molecular bound state with $J^P = 0^+$ results completely from the coupled channel effects. By comparing the two diagrams in Fig. 2, one finds that the binding energies for the two cases are similar with the same cutoff.

### 3.2 OBEP model

We may perform similar calculation in the OBEP model. In this model, to reduce the number of parameters, we consider two cases for the parametrization of the cutoffs: (1) common cutoff $A_\sigma = A_\rho = A_\omega = A_{\text{com}}$ and (2) scaled cutoff $A_\sigma = \alpha_\sigma$, $A_\rho = \alpha_\rho$, $A_\omega = \alpha_\omega$ with $A_{\text{meson}} = m_{\text{meson}} + \alpha A_{\text{QCD}}$. For the calculation, we have considered both the case without channel coupling (w/o) and the one with channel coupling (w). Therefore, we have totally six cases. One may find results in various cases in Ref. [3]. Here we just show the cutoff dependence of the results on the cutoff parameters (see Figs. 3 and 4).

In fact, the results for the spin-singlet and triplet are also not significantly different in the OBEP model with the same cutoff parameter. To make a comparison among different cases, we tune the cutoff parameters to get similar binding energies. Table 5 shows the comparison.
Fig. 2. The sensitivity of the binding energy (B.E.) to the cutoff $\Lambda_{c}$ in the OPEP model for the $J^P = 0^+$ and $J^P = 1^+$ cases. The cases without (w/o) and with (w/) channel coupling are both shown. ($^3D_1$) means there is no $S - D$ mixing when one considers only the $\Lambda cN$ channel.

Fig. 3. The sensitivity of the binding energy (B.E.) to the cutoff $\Lambda_{c} = \Lambda_{c'}, A_{0c} = A_{0c'}$, in the OBEP model for the $J^P = 0^+$ and $J^P = 1^+$ cases. The cases without (w/o) and with (w/) channel coupling are both shown. ($^3D_1$) means there is no $S - D$ mixing when one considers only the $\Lambda cN$ channel.

4 Possible $\Lambda_{c}/\Lambda_{c'}$ molecular bound state

When studying this system, we use only OPEP model, that is, consider only the long-range interaction mediated by pion meson exchange. The procedure for the calculation is similar to the $\Lambda_{c}N$ system. Because the dominant channel has two identical baryons, we also need the antisymmetrization of the wave function for the 5-th channel

$$|\Sigma_{c}\Sigma_{c^*}^>\rangle = \frac{1}{\sqrt{2}}\left(\Sigma_{c}\Sigma_{c}^I_{S=0} - \Sigma_{c}^*\Sigma_{c}^I_{S=0}\right), \quad (12)$$

where the minus sign comes from the exchange of two Fermions.
Fig. 4. The sensitivity of the binding energy (B.E.) to the parameter \( \alpha \) in the OBEP model for the \( J^P = 0^+ \) and \( J^P = 1^+ \) cases. The cases without (w/o) and with (w/) channel coupling are both shown. (\( ^3D_1 \)) means there is no \( S-D \) mixing when one considers only the \( \Lambda_c N \) channel.

Table 5. Comparison among different cases. The meaning of the numbers are [cutoff \( \Lambda \) in GeV or dimensionless \( \alpha \): binding energy in MeV, RMS radius in fm].

<table>
<thead>
<tr>
<th>( J^P )</th>
<th>( \Lambda_c N ) (S-wave)</th>
<th>( \Lambda_c N - \Sigma_c N - \Sigma_c^* N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0^+ )</td>
<td>OPEP (( \Lambda ))</td>
<td>( \times )</td>
</tr>
<tr>
<td></td>
<td>OMEP (( \Lambda ))</td>
<td>[0.900; -1.24, 3.86]</td>
</tr>
<tr>
<td></td>
<td>OMEP (( \alpha ))</td>
<td>[1.533; -0.25, 8.13]</td>
</tr>
<tr>
<td>( 1^+ )</td>
<td>OPEP (( \Lambda ))</td>
<td>( \times )</td>
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<td></td>
<td>OMEP (( \Lambda ))</td>
<td>[0.900; -1.24, 3.86]</td>
</tr>
<tr>
<td></td>
<td>OMEP (( \alpha ))</td>
<td>[1.618; -0.80, 4.72]</td>
</tr>
</tbody>
</table>

Fig. 5. Diagrams (a), (b), and (c) show diagonal and transition potentials of \( S \)-wave case, \( S-D \) transition case, and \( D \)-wave case with the cutoff parameter \( \Lambda_c = 1.0 \) GeV, respectively. (\( ij \)) denotes the potential \( V_{ij}(\Lambda_c, r) \).

From the shapes of the potentials (see Fig.5), one observes that all the diagonal potentials are repulsive and the tensor forces are strong. Thus, if there are binding solutions, they result from a pure coupled channel effect. By solving the coupled channel equations with all the five channel contributions, we get the results given in Table 6. If we drop the \( D \)-wave channels, we do not find any binding solution. If only one \( D \)-wave channel is included, binding solution is obtained with appropriate cutoff. The 5-th channel \( \Sigma_c^* \Sigma_c^* (^5D_0) \) plays a more important role than \( \Sigma_c^* \Sigma_c^* (^3D_0) \). So it is clear that the tensor force from \( S-D \) mixing is essential in getting binding solutions.
To see the sensitivity of the binding energy on the cutoff parameter $A_x$, we present a diagrammatic form for the results in Fig. 6, where we also show the binding energies for the uncoupled channel $\Sigma^*_c\Sigma^*_c$ (with $S$-$D$ mixing but without mixing $\Sigma^*_c\Sigma_c$ or $\Lambda^*_c\Lambda^*_c$ channels). There is no binding solution in other uncoupled channels since the potentials are all repulsive. From the figure, if $A_x \geq 1.0$ GeV is reasonable, a bound state is possible although the diagonal potentials are all repulsive and there are no binding solutions in individual channels.

### 5 Conclusions

We present our studies on the $\Lambda_c N$ and $\Lambda_c\Lambda_c$ bound state problem in the meson exchange approach. For the former system, we use one-boson-exchange potentials which are derived from an effective Lagrangian reflecting the heavy quark symmetry, chiral symmetry and hidden local symmetry. For the latter system, we use only one-pion-exchange potential. To include the extended structure of the hadrons, a phenomenological cutoff has been introduced at each interacting vertex. To investigate the coupled channel effects, we consider the contributions from the intermediate states $\Sigma_c$ and $\Sigma^*_c$. After solving the Schrödinger equation, one finds that the channel coupling has important effects on the formation of the possible molecular bound states.

The resulting binding energies are sensitive to the cutoff parameters. Although we do not have enough information to determine them for heavy quark baryon systems, we have interesting results in a reasonable range of the parameters. We get implications for the existence of the $\Lambda_c N$ hadronic molecules and it is possible to have a bound state of two $\Lambda_c$'s. It will be a challenging subject at GSI, J-PARC, RHIC, or Belle to find such bound states.
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References