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The Strong Decay Properties of the 1^{-+} Hybrid Mesons

Peng-Zhi Huang[a], Hua-Xing Chen[b], and Shi-Lin Zhu[c]

Department of Physics and State Key Laboratory of Nuclear Physics and Technology
Peking University, Beijing 100871, China

Abstract. We calculate the coupling constants of the decay modes $1^{-+} \to \rho\pi, f_1\pi, b_1\pi, \eta\pi, \eta'\pi, a_1\pi, f_1\eta$ within the framework of the light-cone QCD sum rule. Then we calculate the partial widths of these decay channels, which differ greatly from the existing calculations using phenomenological models. For the isovector 1^{-+} state, the dominant decay modes are $\rho\pi, f_1\pi$. For its isoscalar partner, its dominant decay mode is $a_1\pi$. We also discuss the possible search of the 1^{-+} state at BESIII, for example through the decay chains $J/\psi(\psi') \to \pi_1 + \gamma$ or $J/\psi(\psi') \to \pi_1 + \rho$ where π_1 can be reconstructed through the decay modes $\pi_1 \to \rho\pi \to \pi^+\pi^-\pi^0$ or $\pi_1 \to f_1(1285)\pi^0$. Hopefully the present work will be helpful to the experimental establishment of the 1^{-+} hybrid meson.

1 Introduction

The quark model has been proved to be very successful in the classification of hadrons and calculation of hadron spectrum and their other properties. Yet quantum chromodynamics (QCD), which is widely accepted as the fundamental theory of the strong interaction, does not prohibit the existence of those hadron states which cannot be accommodated in the conventional quark model. These nonconventional hadrons include multiquark states ($qq\bar{q}\bar{q}$, $qqqq\bar{q}$, ...), glueballs (gg, ggg, ...), and hybrids ($q\bar{q}g$). Some of them are totally "exotic,", namely, their J^{PC} quantum numbers are excluded by the conventional quark model. A straightforward analysis of J^{PC} reveals that 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, $\cdots$ are the so called exotic ones. Hadrons with these exotic J^{PC} quantum numbers are widely studied since they do not mix with conventional hadrons

Recently, COMPASS collaboration observed a resonance with exotic quantum numbers $J^{PC} = 1^{-+}$ at $(1660 \pm 10^{+0}_{-64})$ MeV/c^2 with a width of $(269 \pm 21^{+42}_{-64})$ MeV/c^2 [1]. In literature, three isovector $J^{PC} = 1^{-+}$ exotic mesons, namely $\pi_1(1400)$[2], $\pi_1(1600)$[3][4], and $\pi_1(2015)$[4] have been reported.

The 1^{-+} hybrids have been studied in a few different theoretical schemes. The mass of the lowest-lying 1^{-+} hybrid meson was predicted to be around 1.9 GeV in the flux tube model [5]. Llanes-Estrada and Cotanch predicted the hybrid mass to be above 2.0 GeV utilizing a QCD inspired Coulomb gauge Hamiltonian [6]. The mass of the 1^{-+} hybrid were estimated in the framework of QCD sum rule[7]. The lattice QCD prediction for the mass of 1^{-+} falls into a wide range of $(1.5 \sim 2.2)$ GeV [8].

The 1^{-+} hybrids were also studied in the large N_c limit of QCD [9]. Cook and Fiebig presented a decay width calculation on lattice for the channel $1^{-+} \to a_1\pi$ in Ref. [10]. The partial widths of the ground 1^{-+} hybrid to $b_1\pi$ and $f_1\pi$ were predicted to be 400(120) MeV and 90(60) MeV, respectively, in Ref. [11]. The strong decay properties were explored in Ref. [12] (IKP) within the same framework. Page, Swanson, and Szczepaniak [13] (PSS) extended the original IKP flux tube model and studied the strong decays of hybrid mesons with different J^{PC} quantum numbers, including those of the 1^{-+}

[a] e-mail: `pzhuang@pku.edu.cn`
[b] e-mail: `chx@water.pku.edu.cn`
[c] e-mail: `zhusl@pku.edu.cn`

hybrids. Burns and Close [14] compared the flux tube model and lattice QCD for the S-wave decay of the 1^{-+} hybrid and found excellent agreement.

Some decay modes of the isoscalar and isovector 1^{-+} hybrids were studied using the three-point function sum rule [15]. In Ref. [16], the decay widths of the 1^{-+} hybrid were calculated using a three-point function sum rule evaluated at the symmetric Euclidean point. The mass of the strange hybrid was also studied in this paper within a light quark expansion formalism. The partial width of the channel $1^{-+} \to \rho\pi$ was predicted to be rather broad by using the three-point function at the symmetric point [16][17]. Zhu reexamined the decay channels $1^{-+} \to \rho\pi, f_1\pi$ by using the light-cone QCD sum rules and reduced the partial width of $1^{-+} \to \rho\pi$ significantly [18].

In this work, we employ the light-cone QCD sum rule (LCQSR) [19] to calculate the various coupling constants of the decay modes $1^{-+} \to \rho\pi, f_1\pi, b_1\pi, \eta\pi, \eta'\pi, a_1\pi, f_1\eta$. With the extracted coupling constants, we figure out the partial widths of these modes and compare them with the predictions obtained using other approaches. We also suggest the possible search for the 1^{-+} hybrid mesons at BESIII.

2 Strong decays of π_1

We denote the isovector and the isoscalar $J^{PC} = 1^{-+}$ hybrid meson by π_1 and $\tilde{\pi}_1$, respectively. The adopted interpolating current for π_1, ρ, f_1, b_1, η, and η' read

$$
\begin{aligned}
J_\mu^{\pi_1}(x) &= \frac{1}{\sqrt{2}}\left[\bar{u}(x)\frac{\lambda^a}{2}g_s G^a_{\mu\nu}(x)\gamma^\nu u(x) - \bar{d}(x)\frac{\lambda^a}{2}g_s G^a_{\mu\nu}(x)\gamma^\nu d(x)\right], \\
J_\mu^{\rho}(x) &= \bar{d}(x)\gamma_\mu u(x), \\
J_\mu^{f_1}(x) &= \frac{1}{\sqrt{2}}\left[\bar{u}(x)\gamma_\mu\gamma_5 u(x) + \bar{d}(x)\gamma_\mu\gamma_5 d(x)\right], \\
J_\mu^{b_1}(x) &= \bar{d}(x)(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)\gamma_5 u(x), \\
J^{\eta} &= J^{\eta_8}\cos\theta - J^{\eta_1}\sin\theta, \\
J^{\eta'} &= J^{\eta_8}\sin\theta + J^{\eta_1}\cos\theta,
\end{aligned} \tag{1}
$$

where $\theta = -19^\circ$ is the mixing angle between η_8 and the SU(3) singlet η_1. J^{η_8} and J^{η_1} are defined as:

$$
\begin{aligned}
J^{\eta_8}(x) &= \frac{1}{\sqrt{6}}\left[\bar{u}(x)i\gamma_5 u(x) + \bar{d}(x)i\gamma_5 d(x) - 2\bar{s}(x)i\gamma_5 s(x)\right], \\
J^{\eta_1}(x) &= \frac{1}{\sqrt{3}}\left[\bar{u}(x)i\gamma_5 u(x) + \bar{d}(x)i\gamma_5 d(x) + \bar{s}(x)i\gamma_5 s(x)\right].
\end{aligned} \tag{2}
$$

There is another possible interpolating current for b_1: $J'_\mu(x) = \bar{d}(x)\sigma_{\mu\nu}u(x)$, which couples to b_1 through $\langle 0|J'_{\mu\nu}(0)|b_1(k,\lambda)\rangle = i f^T_{b_1}\varepsilon_{\mu\nu\rho\sigma}\epsilon^\rho_\lambda k^\sigma$. Notice that the same current couples to the ρ meson through $\langle 0|J'_{\mu\nu}(0)|\rho(k,\lambda)\rangle = i f^T_\rho(\epsilon^\lambda_\mu k_\nu - \epsilon^\lambda_\nu k_\mu)$. Unfortunately, the Lorentz structure of these two couplings will mix with each other in the correlation function

$$
\Pi'_{\mu\nu\beta}(k^2, p^2) = i\int d^4x e^{ik\cdot x}\langle \pi(q)|T\{J'_{\mu\nu}(x)J_\beta^{\pi_1\dagger}(0)\}|0\rangle, \tag{3}
$$

so that we are unable to separate the contribution of the b_1 part from that of the ρ part. This is the consideration behind our choice of the interpolating current $J_\mu^{b_1}$ for the b_1 meson instead of the tensor one. The axial-vector current for f_1 also couples to $I = 0$ pseudoscalar meson η/η'. However, we can differentiate the contribution of the P-wave channel $\pi_1 \to \eta/\eta'\pi$ to the correlation function for the channel $\pi_1 \to f_1\pi$ from that of $\pi_1 \to f_1\pi$ due to their different Lorentz structures.

The overlapping amplitudes between the above interpolating currents and the corresponding mesons are defined as

$$\begin{aligned}
\langle 0|J^{\pi_1}_\mu(0)|\pi_1(p,\lambda)\rangle &= \tilde{f}_{\pi_1}\eta^\lambda_\mu\,,\\
\langle 0|J^{\rho}_\mu(0)|\rho(k,\lambda)\rangle &= f_\rho m_\rho \epsilon^\lambda_\mu\,,\\
\langle 0|J^{f_1}_\mu(0)|f_1(k,\lambda)\rangle &= f_{f_1} m_{f_1} \epsilon^\lambda_\mu\,,\\
\langle 0|J^{b_1}_\mu(0)|b_1(k,\lambda)\rangle &= f_{b_1} \epsilon^\lambda_\mu\,,\\
\langle 0|J^{\eta}(0)|\eta(k)\rangle &= \lambda_\eta\,,\\
\langle 0|J^{\eta'}(0)|\eta'(k)\rangle &= \lambda_{\eta'}\,,
\end{aligned}\tag{4}$$

where η_μ and ϵ_μ are the polarization vectors of π_1 and the final mesons.

The decay amplitudes of these channels can be written as

$$\begin{aligned}
\mathcal{M}(\pi_1\to\rho\pi) &= \varepsilon_{\alpha\beta\gamma\delta}\epsilon^{*\alpha}\eta^\beta k^\gamma q^\delta g_\rho\,,\\
\mathcal{M}(\pi_1\to f_1/b_1+\pi) &= i g^1_{f_1/b_1}(\eta\cdot\epsilon^*) + i g^2_{f_1/b_1}(\eta\cdot k)(\epsilon^*\cdot p)\,,\\
\mathcal{M}(\pi_1\to\eta/\eta'+\pi) &= g_{\eta/\eta'}(\eta\cdot q)\,.
\end{aligned}\tag{5}$$

We consider the following correlation functions in our calculation:

$$\begin{aligned}
\Pi_\rho(k^2,p^2) &= i\int d^4x e^{ik\cdot x}\langle\pi(q)|T\{J^\rho_\alpha(x)J^{\pi_1\dagger}_\beta(0)\}|0\rangle = i\varepsilon_{\alpha\beta\gamma\delta}k^\gamma q^\delta G_\rho(k^2,p^2)+\cdots\,,\\
\Pi_{f_1/b_1}(k^2,p^2) &= i\int d^4x e^{ik\cdot x}\langle\pi(q)|T\{J^{f_1/b_1}_\alpha(x)J^{\pi_1\dagger}_\beta(0)\}|0\rangle\\
&= -G^1_{f_1/b_1}(k^2,p^2)g_{\alpha\beta} - G^2_{f_1/b_1}(k^2,p^2)p_\alpha k_\beta+\cdots\,,\\
\Pi_{\eta/\eta'}(k^2,p^2) &= i\int d^4x e^{ik\cdot x}\langle\pi(q)|T\{J^{\eta/\eta'}(x)J^{\pi_1\dagger}_\beta(0)\}|0\rangle = q^\beta G_{\eta/\eta'}(k^2,p^2)+\cdots\,,
\end{aligned}\tag{6}$$

where p and k are the momentum of the π_1 meson and the final mesons, respectively, $p = k + q$.

When $k^2, p^2 \ll 0$, $G(k^2, p^2)$s can be calculated by operator product expansion (OPE) near the light-cone $x^2 = 0$, with the π light-cone wave functions as its input. Furthermore, $G(k^2, p^2)$s can be related to the corresponding coupling constants by the dispersion relations. Here we consider the channel $\pi_1 \to \rho\pi$ to illustrate our calculation.

$$\begin{aligned}
G_\rho(k^2,p^2) &= \int_0^\infty ds_1\int_0^\infty ds_2\frac{\rho(s_1,s_2)}{(s_1-k^2-i\epsilon)(s_2-p^2-i\epsilon)}\\
&\quad+\int_0^\infty ds_1\frac{\rho_1(s_1)}{s_1-k^2-i\epsilon}+\int_0^\infty ds_2\frac{\rho_2(s_2)}{s_2-p^2-i\epsilon}+\cdots\,,
\end{aligned}\tag{7}$$

where

$$\rho(s_1,s_2) = f_\rho\tilde{f}_{\pi_1}m_\rho g_\rho\delta(s_1-m^2_\rho)\delta(s_2-m^2_{\pi_1})+\cdots\,.\tag{8}$$

After invoking the double Borel transformation $\mathcal{B}^{M_1^2}_{k^2}\mathcal{B}^{M_2^2}_{p^2}$, we extract the double dispersion relation part of Eq. (7):

$$\begin{aligned}
&f_\rho\tilde{f}_{\pi_1}m_\rho g_\rho e^{\bar{u}_0 m^2_\rho/M^2+u_0 m^2_{\pi_1}/M^2}+\cdots\\
&= e^{u_0\bar{u}_0 m^2_\pi/M^2}\Big\{\frac{f_\pi}{\sqrt{2}}(\mathcal{A}^{[\alpha_2]}_\perp-\mathcal{A}^{[\alpha_1]}_\perp-\mathcal{V}^{[\alpha_1]}_\parallel-\mathcal{V}^{[\alpha_2]}_\parallel+\mathcal{V}^{[\alpha_1]}_\perp+\mathcal{V}^{[\alpha_2]}_\perp)m^2_\pi M^2\\
&\quad-\frac{f_\pi}{36\sqrt{2}}[\phi_\pi(u_0)+\phi_\pi(\bar{u}_0)]\langle g_s^2G^2\rangle\Big\}\\
&\approx\frac{f_\pi}{\sqrt{2}}(\mathcal{A}^{[\alpha_2]}_\perp-\mathcal{A}^{[\alpha_1]}_\perp-\mathcal{V}^{[\alpha_1]}_\parallel-\mathcal{V}^{[\alpha_2]}_\parallel+\mathcal{V}^{[\alpha_1]}_\perp+\mathcal{V}^{[\alpha_2]}_\perp)m^2_\pi M^2-\frac{f_\pi}{36\sqrt{2}}[\phi_\pi(u_0)+\phi_\pi(\bar{u}_0)]\langle g_s^2G^2\rangle\,,
\end{aligned}\tag{9}$$

where $u_0 = M_1^2/(M_1^2+M_2^2)$, $M^2 = M_1^2 M_2^2/(M_1^2+M_2^2)$ and $\bar{x} \equiv 1-x$. Hereafter we ignore the factor $e^{u_0\bar{u}_0 m_\pi^2/M^2}$ because $m_\pi^2/M^2 < 0.01$ in our calculations. The definitions of $\mathcal{F}^{[\alpha_i]}$s are

$$\mathcal{F}^{[\alpha_1]} \equiv \int_0^{\bar{u}_0} \mathcal{F}(\alpha_1, u_0, \bar{u}_0-\alpha_1)\, d\alpha_1\,, \qquad \mathcal{F}^{[\alpha_1,u]} \equiv \int_0^{\bar{u}_0}\int_0^{u} \mathcal{F}(\alpha_1, \alpha_2', \bar{\alpha}_1-\alpha_2')\, d\alpha_2' d\alpha_1\,,$$
$$\mathcal{F}^{[\alpha_2]} \equiv \int_0^{\bar{u}_0} \mathcal{F}(u_0, \alpha_2, \bar{u}_0-\alpha_2)\, d\alpha_2\,, \qquad \mathcal{F}^{[\alpha_2,u]} \equiv \int_0^{\bar{u}_0}\int_0^{u} \mathcal{F}(\alpha_1', \alpha_2, \bar{\alpha}_2-\alpha_1')\, d\alpha_1' d\alpha_2\,. \tag{10}$$

We present the Feynman diagrams corresponding to the quark-level calculation of $\Pi_\rho(k^2,p^2)$ in Fig. 1.

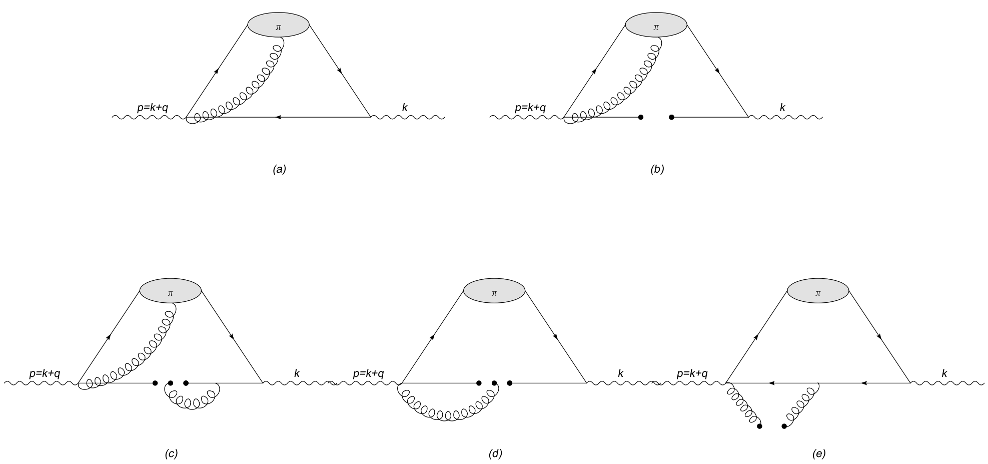

Fig. 1. The Feynman diagrams for $\Pi_\rho(k^2,p^2)$.

The spectral density $\rho(s_1,s_2)$ can be derived after two continuous double Borel transformation:

$$\rho(s_1,s_2) = \mathcal{B}_{\sigma_1}^{\frac{1}{s_1}} \mathcal{B}_{\sigma_2}^{\frac{1}{s_2}} \mathcal{B}_{k^2}^{\frac{1}{\sigma_1}} \mathcal{B}_{p^2}^{\frac{1}{\sigma_2}} G_\rho(k^2,p^2)\,. \tag{11}$$

According to quark-hadron duality, we can subtract the contribution of the excited states and the continuum from Eq. (9) and arrive at

$$f_\rho \tilde{f}_{\pi_1} m_\rho g_\rho e^{\bar{u}_0 m_\rho^2/M^2 + u_0 m_{\pi_1}^2/M^2} = \int_0^{s_{01}} ds_1 \int_0^{s_{02}} ds_2\; e^{-s_1\sigma_1} e^{-s_2\sigma_2}\; \mathcal{B}_{\sigma_1}^{\frac{1}{s_1}} \mathcal{B}_{\sigma_2}^{\frac{1}{s_2}} \mathcal{B}_{k^2}^{\frac{1}{\sigma_1}} \mathcal{B}_{p^2}^{\frac{1}{\sigma_2}} G_\rho(k^2,p^2)\,, \tag{12}$$

where s_{01} and s_{02} are the continuum thresholds of the mass rules of the ρ meson and the hybrid state π_1, respectively.

The large mass difference between the π_1 hybrid and the ρ meson inclines us to work at an asymmetric point of the Borel parameter M_1^2 and M_2^2, leading to a sophisticated subtraction of the continuum contribution [20]. The terms of $\mathcal{B}_{k^2}^{\frac{1}{\sigma_1}} \mathcal{B}_{p^2}^{\frac{1}{\sigma_2}} G_\rho(k^2,p^2)$ have general form $c u_0^m (M^2)^n = c\sigma_2^m/(\sigma_1+\sigma_2)^{m+n}$. Here we assume $m, n > 0$ to illustrate the procedure of the continuum subtraction:

$$\begin{aligned}
&\int_0^{s_{01}} ds_1 \int_0^{s_{02}} ds_2\; e^{-s_1\sigma_1} e^{-s_2\sigma_2}\; \mathcal{B}_{\sigma_1}^{\frac{1}{s_1}} \mathcal{B}_{\sigma_2}^{\frac{1}{s_2}} \frac{\sigma_2^m}{(\sigma_1+\sigma_2)^{m+n}} \\
&= \int_0^{s_{01}} ds_1 \int_0^{s_{02}} ds_2\; e^{-s_1\sigma_1} e^{-s_2\sigma_2}\; \frac{1}{\Gamma(m+n)} \left[-\frac{\partial \delta(s_1-s_2)}{\partial s_1}\right]^m s_1^{m+n-1} \\
&= 2\int_0^{s_{01}} ds_+ \int_{-s_+}^{s_+} ds_-\; e^{-s_+ M^2} e^{s_- M_-^2}\; \frac{(s_+-s_-)^{m+n-1}}{2^m \Gamma(m+n)} \left(\frac{\partial}{\partial s_-}\right)^m \delta(2s_-) \\
&= \frac{M^{2n}}{2^m} \sum_{i=0}^{m} \frac{m!}{i!(m-i)!} (2u_0-1)^i f_{n-1+i}\left(\frac{s_{01}}{M^2}\right)\,,
\end{aligned} \tag{13}$$

where $s_+ = (s_1 + s_2)/2$, $s_- = (s_2 - s_1)/2$, $1/M_-^2 = 1/M_1^2 - 1/M_2^2$ and we assume $s_{01} < s_{02}$. $f_n(x)$ is the subtraction function defined as $f_n(x) = 1 - e^{-x} \sum_{i=0}^{n} x^i/i!$.

The 1^{-+} hybrid has not been firmly established experimentally. Both theoretical predictions and experimental measurements suggest that the mass of π_1 falls within the range $1.6 \sim 2.0$ GeV. In this work the mass of π_1 is taken to be $m_{\pi_1} = 1.6$, 1.8, and 2.0 GeV. We adopt $\tilde{f}_{\pi_1} = 0.15$ GeV^4 in our numerical analysis [7]. The π decay constant $f_\pi = 131$ MeV. The mass and the decay constant of the ρ meson are $m_\rho = 0.77$ GeV and $f_\rho = 0.216$ GeV. $\mu_\pi \equiv m_\pi^2/(m_u + m_d) = (1.573 \pm 0.174)$ GeV is given in Ref. [21].

The parameters which appear in the π distribution amplitudes are listed below [21]. We use the values at the scale $\mu = 1$ GeV in our calculation.

a_2	η_3	ω_3	η_4	ω_4	h_{00}	v_{00}	a_{10}	v_{10}	h_{01}	h_{10}
0.25	0.015	−1.5	10	0.2	−3.33	−3.33	5.14	5.25	3.46	7.03

It is reasonable to let $M_1^2 = \beta m_\rho$ and $M_2^2 = \beta m_{\pi_1}$, where β is a dimensionless scale parameter. Then we have $u_0 = m_\rho^2/(m_\rho^2 + m_{\pi_1}^2)$ and $M^2 = \beta m_\rho^2 m_{\pi_1}^2/(m_\rho^2 + m_{\pi_1}^2)$.

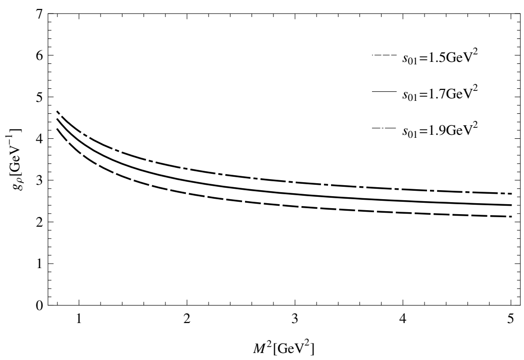

Fig. 2. The sum rule for g_ρ with $m_{\pi_1} = 1.6$ GeV, $2.3 < M^2 < 2.7$ GeV^2, and $s_{01} = 1.5, 1.7, 1.9$ GeV^2.

From the requirement of the stability of the coupling constant to the variation of the Borel parameter M^2 and the requirement that the pole contribution is larger than 40%, we get the working interval of M^2. The resulting sum rule is plotted with $s_{01} = 1.5, 1.7, 1.9$ GeV^2 in Fig. 2 in the case of $m_{\pi_1} = 1.6$ GeV. The sum rules for $m_{\pi_1} = 1.8, 2.0$ GeV are similar. The numerical values of g_ρ are presented here with their variations determined by the working interval of the Borel parameter $2.3 < M^2 < 2.7$ GeV^2 and the range of the threshold $1.5 < s_{01} < 1.9$ GeV^2.

m_{π_1}[GeV]	1.6	1.8	2.0
g_ρ[GeV^{-1}]	$2.5 \sim 3.2$	$2.6 \sim 3.3$	$2.6 \sim 3.4$

In a similar way, we obtain the sum rules for the coupling constants of other channels, ignoring the terms $\sim O(m_\pi^4)$ in our calculation. We point out here that some of these sum rules are not stable, namely, there is no working interval of M^2 for them. As an example, the pole contribution to $g_{b_1}^2$ plotted in Fig. 4 displays the instability of the corresponding sum rule, contrary to the case of g_ρ which is plotted in Fig. 3. Here we can't find an interval of M^2 in which the pole contribution is large than 40%.

The adopted values of the parameters $f_{f_1} = 0.17$ GeV and $f_{b_1} = 0.18$ GeV^3 are obtained from Eq. (4.52) and Eq. (A.20) in Ref. [22], respectively. We take $m_\eta = 0.547$ GeV, $\lambda_\eta = 0.23$ GeV^2, $m_{\eta'} = 0.958$ GeV, and $\lambda_{\eta'} = 0.33$ GeV^2 in our numerical analysis [23]. The extracted values of $g_{f_1}^1$, $g_{f_1}^2$, $g_{b_1}^1$, $g_{b_1}^2$, g_η, and $g_{\eta'}$ are collected in Table 1.

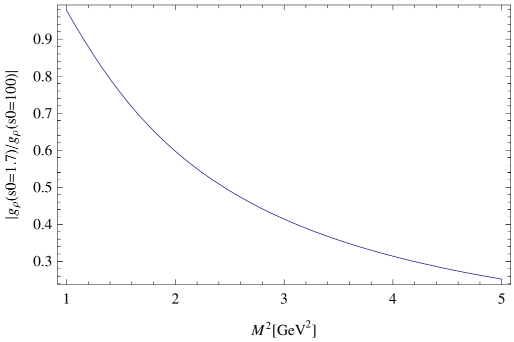

Fig. 3. The pole contribution in the sum rules for g_ρ with $m_{\pi_1} = 2.0$ GeV.

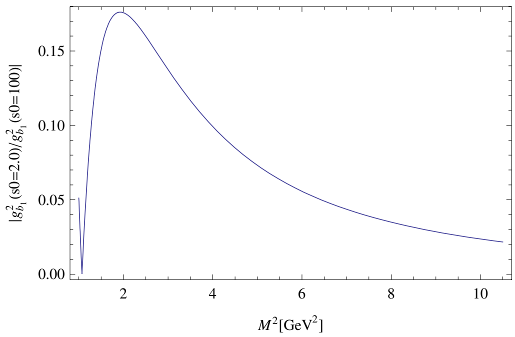

Fig. 4. The pole contributions in the sum rules for $g^2_{b_1}$ with $m_{\pi_1} = 2.0$ GeV.

m_{π_1} [GeV]	1.6	1.8	2.0	M^2[GeV^2]
$g^1_{f_1}$ [GeV]	$-4.2 \sim -5.6$	$-4.4 \sim -5.9$	$-4.6 \sim -6.1$	$1.7 \sim 2.1$
$g^2_{f_1}$ [GeV^{-1}]	$2.0 \sim 2.4$	$2.1 \sim 2.5$	$2.3 \sim 2.7$	$2.3 \sim 2.7$
$g^1_{b_1}$ [GeV]	$\ll 0.1$	$\ll 0.1$	$\ll 0.1$	-
$g^2_{b_1}$ [GeV^{-1}]	1.9	1.8	1.6	-
g_η	0.45	0.45	0.42	-
$g_{\eta'}$	−0.16	−0.15	−0.15	-

Table 1. The numerical values of $g^1_{f_1}$, $g^2_{f_1}$, $g^1_{b_1}$, $g^2_{b_1}$, g_η, and $g_{\eta'}$. The working intervals of the Borel parameter M^2 are listed in the right column. Here "-" indicates the nonexistence of a stable working interval for the corresponding sum rule. The values presented in these cases are determined by $M^2 = 2.0$ GeV^2. The range of the threshold is $2.0 < s_{01} < 2.4$ GeV^2.

3 Partial decay widths

It is straightforward to calculate the partial widths of π_1 with the extracted coupling constants. The resulting partial widths are collected in Table 2, together with the results obtained using other phenomenological models, e.g. Ref. [13] (PSS) and Ref. [12] (IKP). Here we reproduce Table XIII of Ref. [24] which contains the existing experimental results on the total decay width of π_1 in Table 3 in order to compare with our predictions.

Our results on the partial width of π_1 differ greatly from those obtained using lattice QCD [25] and other phenomenological approaches such as the IKP and PSS flux tube model. In the flux tube model, the π_1 coupling to the two S-wave mesons is suppressed, leading to a small branch ratio of the mode $\pi_1 \to \rho\pi$. One flux tube model prediction [30] for widths for π_1 with $m_{\pi_1} = 2.0$ GeV is (in MeV)

$$\pi f_1 : \pi b_1 : \pi\rho : \eta\pi : \eta'\pi \;=\; 60 : 170 : 5 \sim 20 : 0 \sim 10 : 0 \sim 10\,. \tag{14}$$

The quenched lattice QCD simulation also predicted a large width of the channel $b_1\pi$ [25], although the linear extrapolation approximation adopted there may lead to an overestimated width for this channel, as pointed out in Ref. [14]. However, the $b_1\pi$ channel is severely suppressed in our calculation. Moreover, Chung, Klempt, and Korner argued that the channel $\pi_1 \to \eta\pi$ is forbidden due to the requirement of Bose symmetry and J^{PC} conservation in the limit that the η is a pure SU(3) octet [31]. The tiny mixing between η_8 and η_1 should not reverse the widths of these two channels. In contrast, our prediction of the width of $\eta\pi$ is at least 1 order of magnitude larger than that of the channel $\eta'\pi$. Experimentally, the relative branching ratios for $\pi_1(1600)$'s three channels $b_1\pi$, $\eta'\pi$, and $\rho\pi$ are [3]

$$b_1\pi : \eta'\pi : \rho\pi \;=\; 1 : 1 \pm 0.3 : 1.5 \pm 0.5. \tag{15}$$

m_{π_1}	1.6 GeV			1.8 GeV			2.0 GeV			
	IKP [12]	PSS [13]	This work	IKP [12]	PSS [13]	This work	IKP [12]	PSS [13]	Lattice [25]	This work
$\rho\pi$	8	9	73 ~ 120	12	13	138 ~ 222		16		216 ~ 370
$f_1\pi$	14	5	69 ~ 122	21	9	96 ~ 175		10.2	90 ± 60	109 ~ 195
$b_1\pi$	59	24	0.14	62	38	1.2		43	400 ± 120	3.7
$\eta\pi$	0	0	0.36	0.02	0.02	0.44		0.02		0.45
$\eta'\pi$	0	0	0.02	0	0.01	0.02		0.01		0.03

Table 2. The partial widths of the single-pion channels of π_1 in units of MeV, where IKP and PSS refer to the methods used in Ref. [12] and Ref. [13], respectively. The partial widths for the channels $b_1\pi$ and $f_1\pi$ cited from IKP and PSS are simply the sum of their S-wave and D-wave widths. The numerical values of the coupling constants of are stable with the variation of m_{π_1}. The partial widths of the two modes $\pi_1 \to \rho\pi, f_1\pi$ increase rapidly with m_{π_1} due to their enlarged two-body phase spaces.

Mode	Mass (GeV)	Width (GeV)	Experiment	Reference
$\rho\pi$	1.593 ± 0.08	0.168 ± 0.020	E852	[3]
$\eta'\pi$	1.597 ± 0.010	0.340 ± 0.040	E852	[3]
$f_1\pi$	1.709 ± 0.024	0.403 ± 0.080	E852	[4]
$b_1\pi$	1.664 ± 0.008	0.185 ± 0.025	E852	[4]
$b_1\pi$	1.58 ± 0.03	0.30 ± 0.03	VES	[26]
$b_1\pi$	1.61 ± 0.02	0.290 ± 0.03	VES	[3]
$b_1\pi$	~ 1.6	~ 0.33	VES	[27]
$b_1\pi$	1.56 ± 0.06	0.34 ± 0.06	VES	[28]
$f_1\pi$	1.64 ± 0.03	0.24 ± 0.06	VES	[28]
$\eta'\pi$	1.58 ± 0.03	0.30 ± 0.03	VES	[26]
$\eta'\pi$	1.61 ± 0.02	0.290 ± 0.03	VES	[27]
$\eta'\pi$	1.56 ± 0.06	0.34 ± 0.06	VES	[28]
$b_1\pi$	~ 1.6	~ 0.23	CBAR	[3]
$\rho\pi$	1.660 ± 0.010	0.269 ± 0.021	COMPASS	[1]
all	$1.662^{+0.015}_{-0.011}$	0.234 ± 0.050	PDG	[29]

Table 3. Reported masses and widths of the $\pi_1(1600)$ from the E852 experiment, the VES experiment and the COMPASS experiment. The PDG average from 2008 is also reported. (This table was reproduced from Ref. [24].)

In a summary on the VES results, Amelin [28] obtained the relative branch ratios for the $\pi_1(1600)$ as follows:

$$b_1\pi : f_1\pi : \rho\pi : \eta'\pi \ = \ 1.0 \pm .3 : 1.1 \pm .3 :< .3 : 1. \tag{16}$$

4 Strong decays of the isoscalar 1^{-+} hybrid state

Now we consider the strong decays of $\tilde{\pi}_1$, the isoscalar partner of π_1. We notice that $I^G J^{PC}$ conservation restricts the possible decay channels to the S-wave $\tilde{\pi}_1 \to a_1(1260)\pi$, $f_1(1285)\eta$, and the P-wave

$\tilde{\pi}_1 \to \eta\eta'$, $\pi(1300)\pi$, $\eta(1295)\eta$. The partial widths of the three P-wave channels are supposed to be relatively small due to their small phase spaces. In addition, the channel $\tilde{\pi}_1 \to f_1\eta$ is kinematically forbidden if the mass of $\tilde{\pi}_1$ is smaller than 1.83 GeV. Hence, the dominant decay mode of $\tilde{\pi}_1$ is $\tilde{\pi}_1 \to a_1\pi$.

We use the following interpolating currents for the $\tilde{\pi}_1$ meson and the a_1 meson:

$$J_\mu^{\tilde{\pi}_1}(x) = \frac{1}{\sqrt{2}}\left[\bar{u}(x)\frac{\lambda^a}{2}g_s G^a_{\mu\nu}(x)\gamma^\nu u(x) + \bar{d}(x)\frac{\lambda^a}{2}g_s G^a_{\mu\nu}(x)\gamma^\nu d(x)\right],$$
$$J_\mu^{a_1}(x) = \frac{1}{\sqrt{2}}\left[\bar{u}(x)\gamma_\mu\gamma_5 u(x) - \bar{d}(x)\gamma_\mu\gamma_5 d(x)\right]. \quad (17)$$

We define the coupling constants $g^1_{a_1}$ and $g^2_{a_1}$, similar to the coupling constants $g^1_{f_1}$ and $g^2_{f_1}$ of the channel $\pi_1 \to f_1\pi$. $g^1_{a_1} \approx g^1_{f_1}$ and $g^2_{a_1} \approx g^2_{f_1}$ if we simply adopt $\tilde{f}_{\tilde{\pi}_1} = \tilde{f}_{\pi_1} = 0.15$ GeV^4 and $f_{a_1} = f_{f_1} = 0.17$ GeV in our numerical analysis. This leads to the estimate of the partial width of the mode $\tilde{\pi}_1 \to a_1\pi$: $\Gamma_{\tilde{\pi}_1\to a_1\pi} \approx 3\Gamma_{\pi_1\to f_1\pi}$. Here the factor 3 comes from the difference between the two channels' final states, namely $\tilde{\pi}_1 \to a_1^+\pi^-, a_1^-\pi^+, a_1^0\pi^0$ versus $\pi_1^0 \to f_1\pi^0$.

We adopt $f_\eta = 130$ MeV and $\mu_\eta = 1.47$ GeV [21]. The mass of strange quark is taken to be $m_s = 0.15$ GeV. The input parameters for the η light-cone distribution amplitudes involved in our calculation are as follows ($\mu = 1$ GeV) [21]:

a_2	η_3	ω_3	η_4	ω_4	h_{00}	v_{00}	a_{10}	v_{10}	h_{01}	h_{10}
0.2	0.013	−3	0.5	0.2	−0.17	−0.17	0.17	0.26	0.15	0.38

The numerical values of $g^1_{f_1\eta}$ and $g^2_{f_1\eta}$ with $2.0 < s_{01} < 2.4$ GeV^2 are given here:

$m_{\tilde{\pi}_1}$[GeV]	1.6	1.8	2.0	M^2[GeV^2]
$g^1_{f_1\eta}$[GeV]	$-1.6 \sim -2.2$	$-1.7 \sim -2.3$	$-1.7 \sim -2.3$	$2.6 \sim 3.0$
$g^2_{f_1\eta}$[GeV^{-1}]	$0.8 \sim 1.0$	$0.9 \sim 1.1$	$1.0 \sim 1.2$	$2.6 \sim 3.0$

Our calculation of $\tilde{\pi}_1$'s widths are straightforward if we take advantage of the width formulas for $\pi_1 \to f_1\pi$:

$$\Gamma_{\tilde{\pi}_1^0\to a_1^+\pi^-} = \Gamma_{\pi_1^0\to f_1\pi^0}(g^1_{f_1} \to g^1_{a_1}, g^2_{f_1} \to g^2_{a_1}, m_{f_1} \to m_{a_1}, m_{\pi_1} \to m_{\tilde{\pi}_1}),$$
$$\Gamma_{\tilde{\pi}_1^0\to f_1\eta} = \Gamma_{\pi_1^0\to f_1\pi^0}(g^1_{f_1} \to g^1_{f_1\eta}, g^2_{f_1} \to g^2_{f_1\eta}, m_{\pi_1} \to m_{\tilde{\pi}_1}, m_\pi \to m_\eta). \quad (18)$$

The partial widths of $\tilde{\pi}_1$ are listed in Table 4, along with the results obtained using other approaches. As mentioned above, the only dominant channel for $\tilde{\pi}_1$ is $\tilde{\pi}_1 \to a_1\pi$ whose width varies from 200 to 600 MeV, depending heavily on the mass of $\tilde{\pi}_1$ we adopt.

5 Search for the 1^{-+} state at BESIII

Since the BESIII detector has an excellent photon resolution, it is very interesting to search for the 1^{-+} state in the J/ψ radiative process $J/\psi(\psi') \to h_{1,0} + \gamma$. The photon spectrum peaks around $E_\gamma = (m^2_{J/\psi} - m^2_h)/(2m_{J/\psi})$ with a width $(m_h\Gamma_h)/m_{J/\psi} \sim (100 - 200)$ MeV. Such a process may be described by the following effective Lagrangian:

$$\mathcal{L} = c_0 F_{\mu\nu}\psi^\mu h^\nu_{1,0} + c_0' F_{\mu\nu}\psi^{\nu\alpha} h_{1,0}{}^\mu_\alpha \quad (19)$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor, ψ_μ and $h^\mu_{1,0}$ are the J/ψ and 1^{-+} field. Naively one expects the above branching ratio to be around $10^{-5} \sim 10^{-4}$.

$m_{\tilde{\pi}_1}$[GeV]	1.6			1.8			2.0		
	IKP [12]	PSS [13]	This work	IKP [12]	PSS [13]	This work	IKP [12]	PSS [13]	This work
$a_1\pi$			$207 \sim 366$	72	28.2	$288 \sim 525$		30.6	$327 \sim 585$
$f_1\eta$	-	-	-	-	-	-		8	$10 \sim 19$

Table 4. The partial widths of $\tilde{\pi}_1$ in units of MeV, where "-" indicates that the corresponding channel is kinematically forbidden.

The isovector 1^{-+} state π_1 can also be produced associated with other hadrons X at BESIII through the process $J/\psi(\psi') \to \pi_1 + X$. For the production of the neutral component of π_1, the quantum numbers of X are $I^G = 1^+, C = -$. Moreover, $m_X \le m_{J/\psi} - m_{\pi_1} \sim 1.5$ GeV. From the above constraint, we get $X = \rho^0, b_1^0, \rho(1450)$ if it is a single resonance or $X = \pi^+\pi^-$, etc. Let us focus on the case $X = \rho$. Such a production may be described by the following effective Lagrangian:

$$\mathcal{L} = c_1\psi_{\mu\nu}\mathbf{h}_1^{\mu} \cdot \rho^{\nu} + c_2\psi_{\mu}\mathbf{h}_1^{\mu\nu} \cdot \rho^{\nu} + c_3\psi_{\mu}\mathbf{h}_1^{\nu} \cdot \rho^{\mu\nu} + c_4\psi_{\mu\nu}\mathbf{h}_1^{\nu\alpha} \cdot \rho_{\alpha}^{\mu} \tag{20}$$

Naively one expects the above branching ratio to be around $10^{-4} \sim 10^{-3}$.

From Table 2, the dominant decay modes of the isovector 1^{-+} meson are $\rho\pi, f_1\pi$. We urge our BESIII colleagues to search for π_1 through the decay chain: $J/\psi(\psi') \to \pi_1 + \gamma, \pi_1 \to \rho\pi \to \pi^+\pi^-\pi^0$, or $\pi_1 \to f_1(1285)\pi^0$. $f_1(1285)$ is a narrow state with a width of 24.3 MeV. The $f_1\pi^0$ mode is also useful in the search of π_1, although $f_1(1285)$ mainly decays into multiple particle final states $4\pi, \eta\pi\pi$. The other important decay chain is $J/\psi(\psi') \to \pi_1 + \rho \to \rho + \rho + \pi \to 2(\pi^+\pi^-)\pi^0$. Once enough data are accumulated, one may also try to look for π_1 in the $b_1\pi, \eta\pi, \eta'\pi$ modes.

The isoscalar 1^{-+} state $\tilde{\pi}_1$ can also be produced associated with other hadrons X' at BESIII through the process $J/\psi(\psi') \to \tilde{\pi}_1 + X'$. Now the quantum numbers of X' are $I^G = 0^-, C = -$. The possible candidates are $X' = \omega, \phi, h_1(1170), \omega(1470), \pi^+\pi^-\pi^0$, etc. The $\tilde{\pi}_1$ state mainly decays into $a_1\pi$. Search of $\tilde{\pi}_1$ through the hadronic decay chain $J/\psi(\psi') \to \tilde{\pi}_1 + \omega/\phi \to a_1 + \pi + \omega/\phi$ is challenging since it involves too many pions in the final states. BESIII collaboration may also search for $\tilde{\pi}_1$ through the radiative decay chain: $J/\psi(\psi') \to \tilde{\pi}_1 + \gamma \to a_1 + \pi + \gamma$.

6 Conclusion

We have studied the major strong decay modes of the $J^{PC} = 1^{-+}$ hybrid mesons, including the isovector and the isoscalar cases. The coupling constants for these modes are calculated with the light-cone QCD sum rule approach. Some of the sum rules obtained are stable with the variations of the Borel parameter M^2 and the continuum threshold s_{01}. For the other sum rules, we can not find a stable working interval of M^2.

Some possible sources of the errors in our calculation include the inherent inaccuracy of LCQSR: the omission of the higher twist terms in the OPE near the light-cone, the variation of the coupling constant with the continuum threshold s_{01} and the Borel parameter M^2 in the working interval, the omission of the higher conformal partial waves in the light-cone distribution amplitudes of the pion or the η, and the uncertainty in the parameters that appear in these light-cone distribution amplitudes. The uncertainty in the overlapping amplitudes between the interpolating currents and the corresponding final mesons is another source of errors. We merely give an estimated range for each involved coupling constant. The uncertainty of $\tilde{f}_h$, which was not taken into account in this work, may broaden these ranges significantly. It's also understood that the α_s correction may turn out to be quite large. We also omit the $O(m_\pi^4)$ terms in the derivation of the sum rules for most of the coupling constants involved in our calculation.

Our predictions of the partial widths of the 1^{-+} hybrid are quite different from those obtained using other methods like the flux tube model and lattice QCD, etc. So far, the experimental data on the strong decays of the 1^{-+} hybrid is still not so accurate. We suggest the possible search of the isovector and the isoscalar 1^{-+} hybrids in $J/\psi(\psi')$ decay processes at BESIII.

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