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Sigma terms in Pion Weak-production near Threshold by Charged Current

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Abstract. We investigated pion production by charged weak current, which comprises vector and axial vector current. In particular, the contribution by the axial vector current is detailed at the pion threshold in terms of multipole amplitudes, which are useful tools for understanding the pion weak-production by combining the multipole amplitudes deduced from other probes. By exploiting the chiral Ward identities based on the QCD Lagrangian, we derived relevant multipole amplitudes in closed forms and showed their numerical results. In the amplitudes, scalar and pseudo scalar form factor generating sigma terms manifest by themselves. Physical implications and roles of the scalar and the pseudo scalar form factor in pion weak-production are also discussed.

1 Introduction

Pion weak-production is pion production through the weak current mediated by vector bosons, Z^0 and $W^{\pm}$, in neutrino (ν) or antineutrino ($\bar{\nu}$) scattering. Since the pioneering seminal work by Adler [1], these projectiles on the nucleon target have been considered as efficient tools for studying the nucleon structure at the intermediate energy nuclear physics albeit experimental difficulties. Since the weak current comprises vector and axial vector current, it may complement the information about the nucleon structure obtained by electro-magnetic probes, such as pion photo- and electro-production. Moreover, two different types of the scattering, *i.e.* neutral-current (NC) and charged current (CC) scattering of $\nu(\bar{\nu})$ on the nucleon provides some valuable and unique clues on the responses of the nucleon target to the weak current type [2,3].

Beyond the pion threshold energy of incident neutrinos, inelastic channel processes like pion production and Δ resonance are opened in the reaction. For instance, $N(\nu(\bar{\nu}), \nu^{'}(\bar{\nu}^{'})\pi^0)N$ and $N(\nu(\bar{\nu}), \nu^{'}(\bar{\nu}^{'})\pi^{\pm})N^{'}$ for NC reaction, and $N(\nu_l(\bar{\nu}_l), \ell^{\mp}\pi^{\pm})N$ and $N(\nu_l(\bar{\nu}_l), \ell^{\mp}\pi^0)N^{'}$ for CC reaction are possible, respectively.

Various low energy theorem (LET) on pion electro-magnetic production, such as classical LETs based on the current algebra [4] and their modern versions by the chiral perturbation theory (χPT) [5], which is known as the most systematic approach for the non-perturbative QCD, had shed meaningful lights on the internal structure of the nucleon. Following the approaches in other LETs, our discussion in this paper is restricted to the pion threshold. In the experimental side, detecting final leptons, $e^{\pm}$ or $\mu^{\pm}$, with outgoing pions is more feasible than detecting final $\nu(\bar{\nu})$ itself, so that we focus on the CC reaction at the pion threshold.

Our analysis is carried out in terms of multipole amplitudes, whose approach has yielded many fruitful results for pion production by electro-magnetic probes in the field of intermediate energy nuclear physics governed by the chiral symmetry over the last two decades [4–6].

In Sect. 2, our transition amplitude for pion weak-production is derived from the chiral Ward identity under the real pion limit. This is one of the motivation for this paper compared to Born approximation and soft pion limit usually adopted in other approaches [1,7]. In Sect. 3, sigma terms and their primitive forms, which represent explicit chiral symmetry breaking due to finite quark masses

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on the nucleon structure, are shown to naturally appear by this approach. In Sect. 4, multipole amplitudes and their numerical results near pion threshold are presented, in a compact closed form, from the transition amplitude in terms of simple dimensionless kinematical variables. The χPT calculations by Bernard *et al.* [8] also show efficient analytical results for the amplitudes. But they contain undetermined low energy constants in higher orders, which could not be pinned down easily. Our summary and conclusions are given with the comparison to the amplitude derived by the χpT in Sect. 5.

2 Transition matrix elements for pion weak production

To describe pion weak-production near threshold, $\nu(\bar{\nu}) + N \rightarrow l(\bar{l}) + \pi^a + N^{'}$, where $l = \nu^{'}$ for the NC reaction and $l = e^{\pm}$ (or $\mu^{\pm}$) for the CC reaction, we start from an invariant amplitude, $\mathcal{M}_{ab} = \epsilon_\nu \mathcal{M}^\nu_{ab} = \epsilon_\nu \langle N(p_2), \pi_a(q)|W^\nu_b(0)|N(p_1)\rangle$, with a weak current W^ν_b, a polarization vector of the lepton part ϵ_ν, and isospin indices a and b. Since we consider pion production near threshold, the Δ current is excluded in this work. The weak current W^ν_b, which takes a $V^\nu_b - A^\nu_b$ current form according to the electro-weak theory, has isoscalar and isovector parts for the NC interaction.

Following the approach adopted in previous papers [6,9], we exploited the Green functions [9], C^ν_{ab}, P^ν_{ab}, and $T^{\mu\nu}_{ab}$, from nucleon matrix elements of time ordered products of relevant currents, *i.e.* the weak current W^ν_b, the vector current V^ν_a and the axial current A^ν_a, and the pseudo scalar (PS) quark density P_a with $a = 1, ..., 8$

$$\begin{aligned} C^\nu_{ab} &= \int d^4x e^{iqx} \delta(x_0) \langle p_2|[A^0_a(x), W^\nu_b(0)]|p_1\rangle \, , \\ P^\nu_{ab} &= i\int d^4x e^{iqx} \langle p_2|T(P_a(x) W^\nu_b(0))|p_1\rangle \, , \\ T^{\mu\nu}_{ab} &= i\int d^4x e^{iqx} \langle p_2|T(A^\mu_a(x) W^\nu_b(0))|p_1\rangle \, . \end{aligned} \tag{1}$$

The Greens functions satisfy the chiral Ward identity, $q_\mu T^{\mu\nu}_{ab} = \hat{m} P^\nu_{ab} + C^\nu_{ab}$ with $\hat{m} = (m_u + m_d)/2$. This identity is derived from the QCD Lagrangian in the presence of external electro-magnetic (EM) fields, which method was successfully applied for pion electro-production at the previous papers [6,9]. Consequently, the modern gauged electro-weak theory can be included in a reasonable way contrary to a simple minimal coupling scheme [10,11], and recent developments by the χPT [5] are safely incorporated because it enables to calculate poles contributions by using the effective Lagrangian approach (ELA). One more point is that we exploit a real pion limit [4,7], which leads to more elaborate results than the soft pion limit used in conventional calculations [1,4].

Invariant transition matrix element $\mathcal{M}^\nu_{ab}$ is obtained by the LSZ reduction formula with the interpolating pion field $\Phi_a(x)$ within the Bjorken-Drell (BD) convention as follows

$$f_\pi \mathcal{M}^\nu_{ab} = [(C^\nu_{ab} + q_\mu T^{\mu\nu}_{ab}) - \frac{q^2}{m^2_\pi}(C^\nu_{ab} + q_\mu T^{\mu\nu}_{ab})] \,|_{\, q^2 \rightarrow m^2_\pi} \, . \tag{2}$$

The above result can be also derived from the pre-QCD PCAC hypothesis $\partial_\mu A^\mu_a = m^2_\pi f_\pi \Phi_a$. Or one can start from a given model lagrangian that satisfies the PCAC hypothesis [10]. Those approaches, however, corresponds just to the inclusion of the EM interactions by the minimal coupling scheme [11], so that they may not have any direct relation to the QCD and may lead to an incompatible failure with the chiral symmetry as discussed in Ref.[9]. But the approach adopted here is based on the chiral Ward identity from the QCD lagrangian.

By the equal time commutator (ETC), C^ν_{ab} in Eq.(2) is reduced to a sum of axial and vectorial parts as follows

$$C^\nu_{ab} = \bar{u}(p_2)\frac{I_{ab}}{2}[G_A(t)\gamma^\nu\gamma_5 + \frac{G_P(t)}{2M_N}(k-q)^\nu\gamma_5 - F^V_1(t)\gamma^\nu - i\frac{\sigma_{\mu\nu}(k-q)^\mu}{2}F^V_2(t)]u(p_1) \, , \tag{3}$$

where $t = (k-q)^2 = (p_2 - p_1)^2$ with outgoing pion momentum q is a 4 momentum transfer by the lepton. Form factors embedded here are detailed at Ref. [12]. The 2nd term in Eq.(3), which contributes to charged pion electro-production and $\nu(\bar{\nu})$ reaction via CC, is the induced PS part. But in pion weak-production by the CC it is usually discarded by negligible lepton masses compared to participating hadron masses.

3 Sigma terms in pion weak-production

Under soft pion limit $q^2 \to 0$, the 2nd parenthesis in Eq.(2), $-\frac{q^2}{m_\pi^2}(C^\nu_{ab} + q_\mu T^{\mu\nu}_{ab})$, has no contribution, so that one easily obtains a classical low energy theorem from the 1st parenthesis, $C^\nu_{ab} + q_\mu T^{\mu\nu}_{ab}$ [7, 13]. For instance, under k_μ and $q_\mu \to 0$ limit, one directly obtains from C_{ab}

$$M^\nu_{ab} = \frac{g^{\pi NN}}{4M_N}\bar{u}(p_2)[\lambda_a, \lambda_b](\gamma^\nu\gamma_5 - \frac{\gamma^\nu}{g_A})u(p_1)\ , \tag{4}$$

where the 1st term corresponds to the Kroll-Rudermann term and the 2nd term to the πN scattering length. Under real pion limit, $q_\mu \to m_\pi$, however, the 2nd parenthesis in Eq.(2) can be divided as [7, 14]

$$\frac{iq^0}{m_\pi^2}\int d^4x e^{iqx}\langle p_2|\ \delta(x_0)[\partial_\mu A^\mu_a(x), W^\nu_b(0)]|p_1\rangle + \ iq_\mu\int d^4x e^{iqx}\langle p_2|T(f_\pi\partial^\mu\Phi_a(x)W^\nu_b(0))|p_1\rangle\ , \tag{5}$$

where we used the chiral Ward identity. Here the 2nd term, with the $q_\mu T^{ab}_{\mu\nu}$ in the 1st parenthesis of Eq.(2), results in

$$iq_\mu\tilde{T}^{\mu\nu}_{ab} = iq^\mu\int d^4x e^{iqx}\langle p_2|T(\tilde{A}^\mu_a(x)W^\nu_b(0))|p_1\rangle\ , \tag{6}$$

where $\tilde{A}^a_\mu = A^a_\mu(x) - f_\pi\partial_\mu\Phi^a(x)$ is the axial current with the pion axial current subtracted. Consequently, one does not retain the pion pole structure in the $iq^\mu\tilde{T}^{ab}_{\mu\nu}$ any longer. Therefore, our transition amplitude is finally summarized as

$$f_\pi\mathcal{M}^\nu_{ab} = C^\nu_{ab} + iq_\mu\tilde{T}^{\mu\nu}_{ab} + \frac{iq_0}{m_\pi^2}(\Sigma^\nu_{ab} - i\sigma^\nu_{ab})\ , \tag{7}$$

where σ^ν_{ab} and Σ^ν_{ab} terms generating the sigma (σ) and sigma-like (Σ) term, respectively, are given as

$$\begin{aligned} i\sigma^\nu_{ab} &= \int d^4x\ e^{iqx}\delta(x_0)\langle p_2|\ [\partial_\mu A^\mu_a(x), A^\nu_b]|p_1\rangle\ , \\ \Sigma^\nu_{ab} &= \int d^4x\ e^{iqx}\delta(x_0)\langle p_2|\ [\partial_\mu A^\mu_a(x), V^\nu_b]|p_1\rangle\ . \end{aligned} \tag{8}$$

Since the divergence of the axial current goes to be zero in the chiral limit, both terms are explicit chiral symmetry breaking (ESB) terms. Eq.(7) just corresponds to the extension of our previous results for pion electro-production [6]. With the 1st and the last term in Eq.(7), one can construct contact terms and pion pole terms in the figures 1-3 at Ref. [15], and the terms by nucleon poles are generated from the 2nd term which does not retain pion pole terms any more.

In the following, we detail physical meanings of the σ^ν_{ab} and the Σ^ν_{ab} term in Eq.(8) and their roles in pion production. Time components of both terms, $\Sigma^{\nu=0}_{ab}$ and $\sigma^{\nu=0}_{ab}$, are reduced to nucleon expectation values (NEV) of the scalar and the PS quark nucleon density because the equal time commutator (ETC) at Eq.(5) can be easily calculated. As a result, they are expressed in terms of the scalar and the PS form factor, $\sigma(t)$ and $\Sigma(t)$, as follows

$$\begin{aligned} i\sigma^0_{ab} &= \hat{m}\ ih_{ab}\langle p_2|S_0(0)|p_1\rangle = i\sigma(t)\bar{u}(p_2)h_{ab}u(p_1), \\ \Sigma^0_{ab} &= \hat{m}\ i\epsilon_{abc}\langle p_2|P_c(0)|p_1\rangle = i\Sigma(t)\bar{u}(p_2)\gamma_5 I_{ab}u(p_1), \end{aligned} \tag{9}$$

where $t = (p_2 - p_1)^2$ and we used $\partial_\mu A^\mu_b = \hat{m} P_b = \hat{m} i \bar{q}\gamma_5 \tau_b q$. Only a few experimental results for the scalar form factor, $\sigma(t)$, are available at $t = 2m_\pi^2$ in the $\pi - N$ scattering, so called Cheng-Dashen point. They were extracted from the $\pi - N$ scattering [16]. Old analysis yield 50∼70 MeV, while more recent analysis show 71∼90 MeV. Detailed references for experimental results can be found at Ref. [17].

So called $\sigma_{\pi N}$ term is defined as a value of $\sigma(t)$ at $t = 0$ *i.e.* $\sigma_{\pi N} = \sigma(t)|_{t=0}$, so that it has been investigated mostly by theoretical ways because of its problematic extrapolation to $t = 0$. For example, chiral perturbation theory and dispersive analysis [18–20] showed $\Delta\sigma = \sigma(t)|_{t=2m_\pi^2} - \sigma(t)|_{t=0} = 15\sim17$ MeV. Modern values of $\sigma_{\pi N}$, therefore, turned out to be about 10 MeV larger than old ones, and such a large value of $\sigma_{\pi N}$ gives rise to many interesting problems in the relevant fields, for instance, the strangeness content of the nucleon, searches of the Higgs boson, and so on [17].

In pion electro-production, in which the σ^0_{ab} term does not exists, the Σ^0_{ab} term leads to a contribution to the t-channel contribution because of the pion-pole structure, as discussed at our previous paper [6]. But, in pion weak-production, both σ^0_{ab} and Σ^0_{ab} terms appear. Their isospin structures are isospin-even and -odd, respectively.

In this paper, spatial parts of the Σ^ν_{ab} and the σ^ν_{ab} term are assumed to be zero by following the arguments based on the PCAC in Ref. [21–23]. Of course, beyond the PCAC on which the chiral symmetry could be realized non-linear, it is still not clear whether the relevant commutators go to zero exactly or not. More thorough investigations are necessary for the roles of these terms beyond the PCAC.

Most experiments for charged pion electro-production near threshold, although we have only a few data for this reaction near threshold [24,25], were performed to extract the axial mass M_A from E_{0^+} amplitude and the pion form factor $F_\pi(k^2)$ through the induced PS form factor $G_P(t)$ from L_{0^+} amplitude or vice versa [5,26].

However our transition amplitude at the pion threshold allows not only the $G_P(t)$ but also the PS form factor, $\Sigma(t)$. Although both form factors have a pion pole dominance, their dependence on momentum transfer t is independent of each other because of the generalized Goldberg Treiman (GT) relation among the form factors

$$2M_N G_A(t) + \frac{t}{2M_N} G_p(t) = \frac{2m_\pi^2 f_\pi}{m_\pi^2 - t} G_{\pi N}(t) = 2\Sigma(t). \tag{10}$$

Under the lowest effective lagrangian leading to the usual GT relation, $M_N g_A = f_\pi g_{\pi NN}$, the $\Sigma(t)$ can be approximated as $\frac{m_\pi^2}{4M_N} G_P(t)$. Consequently, both form factors have the same t dependence, and the t-channel pole in Born terms for pion electro-production is explained as a sum of the two pion poles at the pion threshold. The former comes from the induced PS form factor in the C^ν_{ab} term and the latter results from the PS form factor in the time component of the Σ^ν_a term.

However, the t dependence of both form factors is fully different, so that the unique extraction of the $G_P(t)$ from the experimental data, usually expected in charged pion electro-production, may have another ambiguity due to the PS form factor, $\Sigma(t)$. More careful analysis are necessary because not only the $G_P(t)$ and $F_\pi(k^2)$ but also the $\Sigma(t)$ are involved in the interpretation of the L_{0^+} amplitude from the experimental data. Of course, this PS form factor appears in the vectorial part of weak-production.

In a way analogous to define the sigma term $\sigma_{\pi N}$, one can define the sigma like term as $\Sigma_{\gamma N} = \Sigma(t)|_{t=0}$, which is given as $2 f_\pi G_{\pi N}(0)$. But its effects is absorbed as an half of $t-$channel contribution at the pion threshold.

4 Pion Weak-production by isovector current

Since pion weak-production by CC is much simpler than that by NC, we focus on pion weak-production by the CC. The NC weak production will be discussed elsewhere. Here we follow the approach adopted at Ref. [1]. We used the Bjorken Drell (BD) convention instead of the Pauli convention used in the Adler's work [1]. Final transition amplitudes of vectorial and axial parts for weak pion production near threshold, where we take only s wave multipoles denoted as subscripts, 0^+, are written in terms of isospin-even and -odd parts indicated as superscripts, (±), as follows

$$\frac{1}{4\pi(1+\mu)}\epsilon_\mu \mathcal{M}_V^{\mu(\pm)}|_{thr.} = i\chi_f^+[E_{0^+}^{(\pm)}\sigma\cdot\mathbf{b} + L_{0^+}^{(\pm)}\sigma\cdot\hat{\mathbf{k}}\hat{\mathbf{k}}\cdot\mathbf{a}]\chi_i \,, \tag{11}$$
$$\frac{1}{4\pi(1+\mu)}\epsilon_\mu \mathcal{M}_A^{\mu(\pm)}|_{thr.} = \chi_f^+[\mathcal{L}_{0^+}^{(\pm)} i\mathbf{k}\cdot\epsilon + \mathcal{H}_{0^+}^{(\pm)} i\epsilon\cdot k/k_0 + \mathcal{M}_{0^+}^{(\pm)}\sigma\cdot(\hat{\mathbf{k}}\times\epsilon)]\chi_i \,,$$

where $\mathbf{a} = \epsilon - \frac{\epsilon_0}{k_0}\mathbf{k}$, $\mathbf{b} = \mathbf{a} - \hat{\mathbf{k}}(\hat{\mathbf{k}}\cdot\mathbf{a}) = \epsilon - \hat{\mathbf{k}}(\hat{\mathbf{k}}\cdot\epsilon)$ and $\mu = m_\pi/M_N$.

The transition amplitudes, Eq.(11), can be decomposed into invariant amplitudes in an isobaric frame, which makes it possible to expand scattering matrix elements into multipole amplitudes. The multipole amplitudes are expressed in terms of 8 axial (6 if lepton masses terms are neglected) and 6 vectorial invariant amplitudes, A_i and V_i, which are determined from all pole contributions, non-pole parts and gauge invariance terms in the extended Born diagram approach [1,15]. For the vertex in the diagram we exploit the effective Lagrangian approach, so that higher order terms can be included implicitly in the vertex parts.

4.1 The Vectorial Parts

Since detailed discussion on the vectorial parts are summarized in our previous paper [6], here we only show final results for isospin-even and -odd $E_{0^+}^{(\pm)}$ and $L_{0^+}^{(\pm)} = (E_{0^+}^{(\pm)} + \tilde{L}_{0^+}^{(\pm)})$ amplitudes in compactly closed forms

$$E_{0^+}^{(\pm)} = \frac{g}{M_N 8\pi(1+\mu)}\sqrt{\frac{\alpha}{2}}\frac{(1+\mu)2\gamma\beta(F_1^V + 2M_N F_2^V) + \mu M_N F_2^V}{F_1^V(1-(1+\mu)2\gamma\alpha) - F_2^V M_N(1+\mu)\nu_2 - G_A/g_A} \,, \tag{12}$$
$$\tilde{L}_{0^+}^{(\pm)} = -\frac{\mu^2-\nu_2}{32\pi(1+\mu)^2}\frac{g}{M_N}\sqrt{\frac{\alpha}{2}}\frac{F_2^V(1+2\gamma\beta(1+\mu))}{4F^\pi/(\nu_2-4\nu_1) - 4(G_A - F^\pi)/\nu_2 + 2F_2^V\alpha\gamma(1+\mu)} \,,$$

where $g = g_{\pi NN}$, and dimensionless kinematical functions are defined as

$$\nu_2 = k^2/M_N^2 \,,\ \mu = m_\pi/M_N \,, \tag{13}$$
$$\alpha = ((\mu+2)^2 - \nu_2)/(2(1+\mu)) \,,\ \beta = (2\mu + \mu^2 + \nu_2)/(2(1+\mu)) \,,$$
$$\gamma = 1/[(2+\mu)(\nu_2 - (2+\mu)] \,,\ \delta = \sqrt{\mu^2 - \nu_2}/(1+\mu)^{3/2} \,.$$

The momentum transfer t is given in terms of k^2, *i.e.* $t = (k-q)^2|_{th.} = (k^2 - m_\pi^2)/(1+\mu)$ at the pion threshold. G_A and F^π are axial and pion form factors. For detailed numerical results for vectorial part, we would like to refer Ref. [27].

4.2 The Axial Parts

Our transition amplitudes for the axial parts are finally given as

$$\mathcal{M}_{0^+}^{(\pm)} = \frac{\delta}{8\pi}M_N[-\mu A_1^{(\pm)} - A_4^{(\pm)}] \,, \tag{14}$$
$$\mathcal{L}_{0^+}^{(\pm)} = \frac{\alpha}{\beta}\frac{\delta}{16\pi}M_N[-\mu\frac{\beta}{\alpha}A_1^{(\pm)} - (2+\mu)A_2^{(\pm)}$$
$$-\mu A_3^{(\pm)} - \frac{(2+\mu)}{\alpha}A_4^{(\pm)} + M_N\mu[(2+\mu)A_5^{(\pm)} + \mu A_6^{(\pm)}]] \,,$$
$$\mathcal{H}_{0^+}^{(\pm)} = \frac{\sqrt{2\alpha}M_N}{8\pi(1+\mu)}[-\alpha A_2^{(\pm)} - \mu A_3^{(\pm)} - A_4^{(\pm)} - \beta A_7^{(\pm)} + M_N\mu[\alpha A_5^{(\pm)} + \mu A_6^{(\pm)} + \beta A_8^{(\pm)}]] \,.$$

All amplitudes are evaluated at the pion threshold, $k_0 = (k_0^* = k_0^{cm}) = (W^2 - M_N^2 + k^2)/(2W), k_0^L = (W^2 - M_N^2 - k^2)/(2M_N)$. Eq.(14) is easily shown to be reduced to the results of Eq.(4) in Ref. [8], except for the $\mathcal{H}_{0^+}^{(\pm)}$ amplitude, in which some different kinematical variables seem to be used. Contributions of the nucleon poles in Born terms to the invariant amplitudes, denoted as $A_i^{B(\pm)}$, are detailed at Ref. [27]

With the results of Born and non-Born $\mathcal{M}_{0^+}^{(\pm)}$, our final amplitude for pion weak-production at the pion threshold by the isovector axial current are given as

$$\mathcal{M}_{0^+}^{(\pm)} = \frac{g}{16\pi M_N}\delta G_A \frac{-2(1+\mu)\alpha\gamma}{2(1+\mu)\beta\gamma - 1 + G_M^V/(g_A G_A)} \ . \tag{15}$$

Similarly, $\frac{k_0}{|\mathbf{k}|}\mathcal{L}_{0^+}^{(\pm)}$ amplitude is calculated as

$$\frac{k_0}{|\mathbf{k}|}\mathcal{L}_{0^+}^{(\pm)B} = \frac{-g}{8\pi M_N}\gamma\sqrt{2\alpha}G_A \frac{0}{(\beta^2-\alpha^2)/\alpha} \ , \ \frac{k_0}{|\mathbf{k}|}\mathcal{L}_{0^+}^{(+)NB} = -\frac{\sqrt{2\alpha}}{16\pi(1+\mu)}\frac{\sigma(t)}{f_\pi m_\pi} \ , \tag{16}$$

$$\frac{k_0}{|\mathbf{k}|}\mathcal{L}_{0^+}^{(-)NB} = \frac{-g}{16\pi g_A}\frac{2+\mu}{1+\mu}\sqrt{2\alpha}\,[-F_2^V + (F_1^V - g_A G_A + 2M_N F_2^V)/(\alpha M_N)] \ .$$

For $\mathcal{H}_{0^+}^{(\pm)}$ amplitude, we obtain

$$\mathcal{H}_{0^+}^{(\pm)B} = \frac{g}{4\pi M_N}\gamma\sqrt{2\alpha}\,[G_A\beta - \alpha - \beta\frac{G_P}{2}\frac{-\alpha}{\beta}] \ , \tag{17}$$

$$\mathcal{H}_{0^+}^{(+)NB} = \frac{g}{8\pi(1+\mu)}\beta\sqrt{2\alpha}\frac{G_P}{2M_N} - \frac{\sqrt{2\alpha}}{8\pi(1+\mu)}\frac{\sigma(t)}{f_\pi m_\pi} \ ,$$

$$\mathcal{H}_{0^+}^{(-)NB} = \frac{g\sqrt{2\alpha}}{8\pi(1+\mu)}[-\alpha\frac{F_2^V}{g_A} + (F_1^V - g_A G_A + 2M_N F_2^V)/(M_N g_A)] \ .$$

Here momentum dependence on k^2 (t) is abbreviated in all form factors. The induced PS form factor $G_P(t)$ is defined as $4M_N^2 g_A/(m_\pi^2 - t)$, so that $h_A(t)$ in Ref. [1] is given as $h_A(t) = G_P(t)/(2M_N)$. The scalar form factor $\sigma(t)$ explicitly appears at the isospin-even terms in $\frac{k_0}{|\mathbf{k}|}\mathcal{L}_{0^+}^{(+)}$ and $\mathcal{H}_{0^+}^{(+)}/k_0$ amplitudes, while the PS form factor $\Sigma(t)$ generating the Σ term appears in the vectorial part as shown in Eq.(12).

Under $\mu = 0$ limit, where the scalar form factor $\sigma(t)$ also goes to zero because of its ESB character, our multipole amplitudes are reduced to the standard form derived from the choral perturbation theory at Ref. [8].

In Figs. 1-3, we showed numerical results of the multipole amplitudes at the pion threshold in terms of momentum transfer to the nucleon. Since the momentum transfer at the pion threshold is given as a function of k^2, *i.e.* $t^{th}(k^2) = (k^2 - m_\pi^2)/(1+\mu) = M_N^2\nu_2 + O[\mu\nu_2, \mu^2]$, we present our numerical results in terms of $-k^2[GeV^2]$, whose region is restricted to $0.1(\simeq 5m_\pi^2) < -k^2 < 0.5(\simeq 25m_\pi^2)$ or $-0.5096 < t < -0.1176$ $[GeV^2]$ following the kinematical region adopted in pion electro-production at Refs. [4,25]. Since the vector form factors $F_{i=1,2}^V$ in the CC reaction are simply expressed by Pauli form factors of the nucleon, we took their values from standard Sachs form factors [3,12]. Axial and induced PS form factors are taken from the dipole form [25]. Of course, one may use the form factors yielded from the χPT [5,9]. But, in that case, all relevant form factors should be taken from those obtained by the χPT. Otherwise the consistency could be broken. For example, the generalized GT relation, Eq.(15), can be violated more or less.

For the scalar form factor, $\sigma(t)$, we choose a theoretical form factor from Ref. [28], which was calculated by the Nambu-Jona-Lasinio model. Since their model depends on the constituent quark mass, we changed the mass to adjust the form factor to reproduce the $\sigma_{\pi N} = 70$ MeV, a result from the recent analysis in Ref. [17]. Each contribution by Born (dot-dashed (sky-blue) curves) and non-Born (dotted (blue) curves) terms, and total sum of both contributions (solid (red) curves) are shown, respectively. Results of $\mu = 0$ limit are indicated as dashed (black) curves.

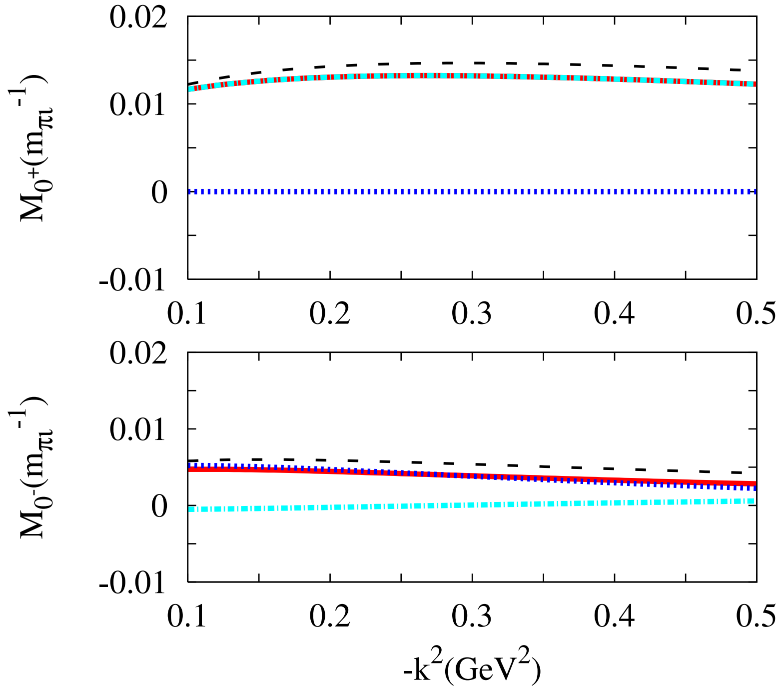

Fig. 1. (Color online) $\mathcal{M}^{(+)}_{0^+}$ (Upper) and $\mathcal{M}^{(-)}_{0^+}$ (Lower) amplitude in a unit m_π^{-1} in terms of $-k^2$[GeV^2]. Dot-dashed (sky-blue) and dotted (blue) curve are contributions by Born and non-Born term, respectively. Solid (red) curves are final results by Eq.(15). Dashed (black) curves are results with $\mu = 0$ limit of Eq. (15).

All multipole amplitudes in Figs. 1 ∼ 3 appear nearly independently of $-k^2$ near pion threshold region, contrary to those by other pion productions. Moreover, results of $\mu = 0$ limit are almost same as those of full calculations within maximally about 10 % deviation, (see the difference of solid (red) and dashed (black) curves). Therefore, the chiral $\mu = 0$ limit could be a reasonable first approximation for the threshold region.

Fig. 1 shows $\mathcal{M}^{(\pm)}_{0^+}$ amplitudes. Here non-Born term for $\mathcal{M}^{(+)}_{0^+}$ amplitude (upper) is exactly zero, while Born term is nearly zero for $\mathcal{M}^{(-)}_{0^+}$ amplitude (lower). In Fig. 2, upper figure is $\frac{k_0}{|\mathbf{k}|}\mathcal{L}^{(+)}_{0^+}$ amplitude. Role of the scalar form factor $\sigma(t)$ is explicitly shown as the difference of two sets of curves. As in Eq.(15), non-Born term entirely attributed to the scalar form factor is the only non-zero contribution to $\frac{k_0}{|\mathbf{k}|}\mathcal{L}^{(+)}_{0^+}$ amplitude. For $\frac{k_0}{|\mathbf{k}|}\mathcal{L}^{(-)}_{0^+}$ amplitude, there is a large cancelation of non-Born and Born term. In specific, $\frac{k_0}{|\mathbf{k}|}\mathcal{L}^{(\pm)}_{0^+}$ amplitudes do not show any zero points leading to singular points in the cross section, compared to the $L^{(+)}_{0^+}$ amplitudes in the vectorial part [6,26]. Result of $\frac{k_0}{|\mathbf{k}|}\mathcal{L}^{(-)}_{0^+}$ by $\mu = 0$ limit is indiscernible compared to the full calculation.

Fig. 3 shows results of the $\mathcal{H}^{(\pm)}_{0^+}$ amplitudes. One point to be noticed in the upper figure is that, if we do not consider the scalar form factor, there is a strong cancelation of Born and non-Born term in $\mathcal{H}^{(+)}_{0^+}$ amplitude, which leads to almost zero contribution. Therefore, the contribution by the scalar form factor is a main contribution leading to non-zero value for this amplitude, similarly to the roles of the $\sigma(t)$ in the $\frac{k_0}{|\mathbf{k}|}\mathcal{L}^{(+)}_{0^+}$ amplitude. But the effect by the scalar form factor may be easily evadable from the experimental data because of their small effects in their amplitudes. For $\mathcal{H}^{(-)}_{0^+}$ amplitude, there is also a large cancelation between non-Born and Born term.

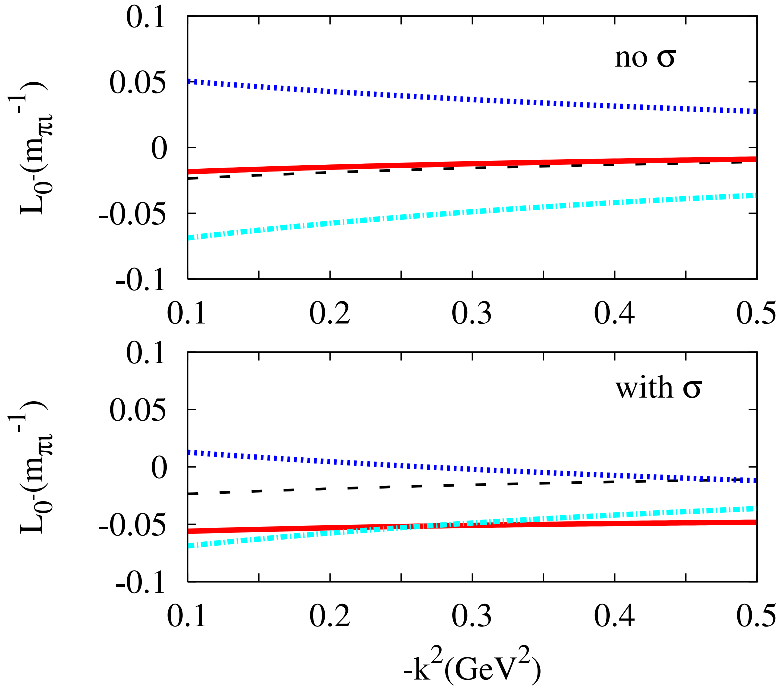

Fig. 2. (Color online) $\frac{k_0}{|\mathbf{k}|}\mathcal{L}^{(+)}_{0^+}$ (upper) and $\frac{k_0}{|\mathbf{k}|}\mathcal{L}^{(-)}_{0^+}$ (lower) amplitude in a unit m_π^{-1} in terms of $-k^2$[GeV^2]. Dot-dashed (sky-blue) and dotted (blue) curve are contributions by Born and non-Born term, respectively. Solid (red) curves are final results by Eq.(16). Dashed (black) curves are results with $\mu = 0$ limit.

5 Summary and conclusions

Here we give a brief summary and conclusions. In charged pion electro-production near threshold which corresponds to the vectorial part in pion weak-production, the extraction of the induced PS and the pion form factor, $G_P(t)$ and $F_\pi(t)$, from the E_{0^+} and the L_{0^+} amplitude may leave another ambiguity due to the PS form factor $\Sigma(t)$ hidden in the amplitude. The $\Sigma(t)$ generates the sigma-like term $\Sigma_{\gamma N} = \Sigma(t)|_{t=0}$, similarly to the scalar form factor $\sigma(t)$ and the sigma term $\sigma_{\pi N} = \sigma(t)|_{t=0}$ in $\pi - N$ scattering. The $\Sigma_{\gamma N}$ appears as an half of $t-$ channel contribution. Detailed discussions about the roles of the PS form factor can be found at our previous paper [6].

In pion weak-production near threshold by charged current, we derived compact forms for relevant multipole amplitudes and showed their numerical results, which show two interesting behaviors. Relevant multipole amplitudes are nearly independent of the momentum transfer to the nucleon and expansion of the amplitudes by $\mu = m_\pi/M_N$ is fully guaranteed at the pion threshold region.

Our analytical results are safely reduced to the preceding results under the chiral limit, $\mu = 0$. The scalar form factor $\sigma(t)$ appears explicitly in the $\mathcal{L}^{(+)}_{0^+}$ and the $\mathcal{H}^{(+)}_{0^+}$ amplitudes. According to our numerical calculation, non-zero contributions in the amplitudes are entirely due to the $\sigma(t)$. Therefore, theses amplitudes could give another chances to extract the scalar form factor, which has been extracted only from $\pi - N$ scattering. Moreover, in the $\mathcal{L}^{(+)}_{0^+}$ amplitude, the form factor $\sigma(t)$ is responsible for the only non-zero term. In the $\mathcal{H}^{(+)}_{0^+}$ amplitude, Born and non-Born terms are cancelled out, so that the $\sigma(t)$ turns out to be a main term. This would be the first result of realistic roles of the scalar form factor $\sigma(t)$ in pion weak-production.

Calculations by the χPT also predicted these amplitudes [8], in which low energy constants show up explicitly at each $O(q^n)$ order. But part of them could not be still pinned down from the available experimental data. Since our approach is based on the chiral Ward identity with various symmetries, detailed results beyond the chiral limit by the χPT could not be reproduced properly, but they are implicitly included in the phenomenological form factor.

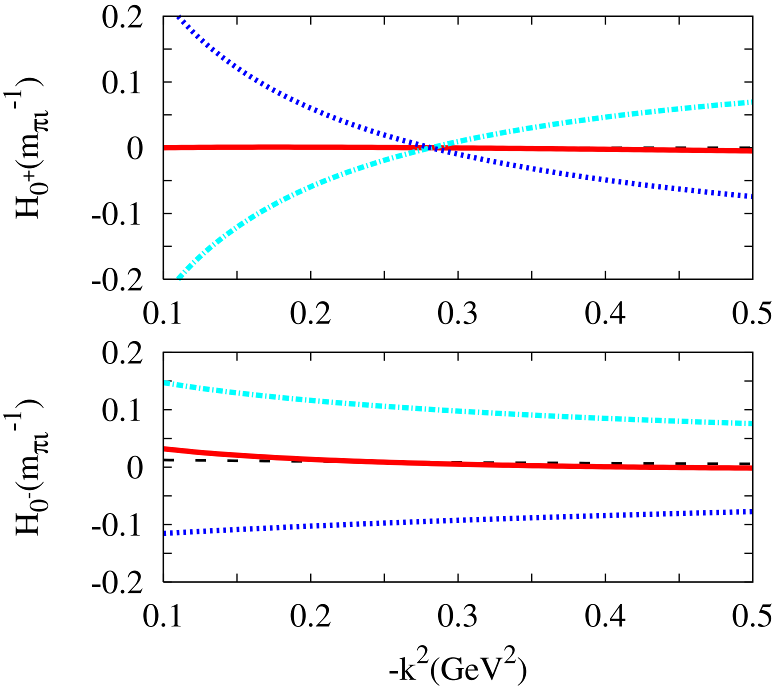

Fig. 3. (Color online) $\mathcal{H}_{0^+}^{(+)}$ (upper) and $\mathcal{H}_{0^+}^{(-)}$ (lower) amplitude in a unit m_π^{-1} in terms of $-k^2$[GeV^2]. Dot-dashed (sky-blue) and dotted (blue) curve are contributions by Born and non-Born term, respectively. Solid (red) curves are final results by Eq.(17). For $\mathcal{H}_{0^+}^{(+)}$ amplitude, non-Born term does not include the $\sigma(t)$ term. In $\mathcal{H}_{0^+}^{(-)}$ amplitude, dashed (black) curve is the result of Eq.(14) in Ref.[8]. Our result with $\mu = 0$ limit is not indicated because it is nearly indiscernible compared to the solid (red) curves.

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