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Properties of the bound nucleons

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Abstract. We present recent investigations on the properties of nucleons in nuclear matter and, in particular, nucleon electromagnetic form factors in the framework of the in-medium modified Skyrme model. Medium modifications are achieved by introducing the pion-nucleus optical potential for the pion fields in nuclear environment. In order to reproduce the bulk matter properties, the parametrization of the Skyrme term in the nuclear matter has been adjusted and the sensitivity of results to the pion-nucleus optical potential parameters are discussed.

1 Introduction

The equation of state (EOS) has been one of the most important issues in nuclear many-body problems. There is a great amount of the different theoretical approaches in trying to describe the EOS. One can mention a few representatives, e.g. microscopic many-body approaches [1,2], effective field theories [3,4], and phenomenological methods [5]. These approaches mainly concentrate on the properties of the many-body systems and, surely, provide very useful tools for understanding not only the properties of dense and hot matter but also the behavior of nuclear matter constituents under the extreme conditions.

In studying nuclear matter properties one can also start from an alternative approach: we first modify nucleon properties in nuclear environment according to some general rules and considerations, and then try to reproduce the properties of the nuclear matter itself. The in-medium modified Skyrme model [6] and its recently improved version [7] are based on a such approach and allow one to perform pertinent studies in a single and many-body levels. Initially, the in-medium modified Skyrme model [6] has been proposed to study single nucleon properties in nuclear environment not only by guessing the corresponding changes in input parameters of the Lagrangian according to the density changes but also relate those changes to the existing phenomenological data, i.e. to the pion physics in nuclear matter. These studies do not take into account the direct and obvious changes of the quark core of the nucleon but take a small influence of the surrounding environment to the nucleon core via the pion profile changes. Furthermore, to be more realistic and to utilize more phenomenological information from the many-body sector, the core part of the nucleon also was modified and that medium modification has been adjusted to reproduce the binding energy of the whole system [7]. While there are no explicit quark degrees of freedom and the core of the nucleon in the Skyrme model is effectively represented by the Skyrme term in the effective Lagrangian, core modifications are related to the change of the Skyrme parameter. In more detailed treatments Skyrme's quartic stabilizing term may be revised by explicit vector meson degrees of freedom.

In the present work, we discuss the changes in the electromagnetic properties of the bound nucleons, based on the effective Lagrangian discussed in Ref. [7]. In this context, the electromagnetic form factors of the bound nucleons can be directly related to the existing experimental indications coming from the structure studies of the matter under extreme conditions.

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2 Effective chiral Lagrangian

We start from the effective chiral Lagrangian presented in Ref. [7]. It has the form

$$\mathcal{L}^{*} = \frac{F_\pi^2}{16}\,\mathrm{Tr}\left(\frac{\partial U}{\partial t}\right)\left(\frac{\partial U^\dagger}{\partial t}\right) - \frac{F_\pi^2}{16}\,\alpha_p(\mathbf{r})\mathrm{Tr}(\nabla U)\cdot(\nabla U^\dagger) + \frac{1}{32e^2\gamma(\mathbf{r})}\,\mathrm{Tr}[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U]^2 + \frac{F_\pi^2 m_\pi^2}{16}\,\alpha_s(\mathbf{r})\mathrm{Tr}(U + U^\dagger - 2)\,, \quad (1)$$

where F_π denotes the pion decay constant, e is the Skyrme parameter, and m_π stands for the pion mass. The medium functionals, α_s, α_p and γ, are written in the following forms

$$\alpha_s = 1 - \frac{4\pi b_0\rho(\mathbf{r})f}{m_\pi^2}, \quad \alpha_p = 1 - \frac{4\pi c_0\rho(\mathbf{r})}{f + g_0' 4\pi c_0\rho(\mathbf{r})}, \quad \gamma = \exp\left(-\frac{\gamma_{\mathrm{num}}\rho(\mathbf{r})}{1+\gamma_{\mathrm{den}}\rho(\mathbf{r})}\right). \quad (2)$$

They represent the influence of the surrounding environment on the properties of the single skyrmion. The parameters α_s and α_p are related to the corresponding phenomenological S- and P-wave pion-nucleus scattering lengths and volumes, i.e. b_0 and c_0, respectively, and describe the pion physics in a baryon-rich environment [8]. The last functional γ can be parametrized in terms of an exponential form and represents the modifications of the skyrme parameter. This simple form with two variational parameters γ_{num} and γ_{den} reproduces the correct depth and location of the ordinary nuclear matter [7]. The density of the surrounding nuclear environment is given by ρ. The g_0' denotes the Lorentz-Lorenz or correlation parameter, $f = 1 + m_\pi/m_N^{\mathrm{free}}$ represents the kinematical factor, and m_N^{free} is the nucleon mass in free space.

The Lagrangian in Eq. (1) will be used to calculate properties of the nucleons in nuclear matter and the bulk matter properties similtaneously. The parameters of the model are fitted to be F_π = 108.78 MeV and e = 4.85 so as to reproduce the experimental values of the nucleon and Δ in free space. The pion mass is also fixed to be its experimental value, $m_\pi = m_{\pi^o}^{\mathrm{exp}}$ = 134.98 MeV. A set of values of parameters in the medium functionals (2) are taken from the analysis of phenomenological data for pion-nucleus scattering [8]. Since the environment acting on the single nucleon properties has a homogeneous and constant density, one can choose the spherically symmetric "hedgehog" form for the boson field

$$U = \exp\{i\hat{\mathbf{n}}\cdot\tau F(r)\}, \quad (3)$$

where $\mathbf{n}$ denotes the unit vector in coordinate space and τ are the usual Pauli matrices. Then the problem will be much simplified and from now on we will follow this choice.

Consequently, one has the following mass of the classical skyrmion

$$M_S^* = \pi\int_0^\infty \left\{F_\pi^2\alpha_p\left(\frac{r^2F_r^2}{2} + s^2\right) + \frac{4s^2}{e^2\gamma}\left(F_r^2 + \frac{s^2}{2r^2}\right) + F_\pi^2 m_\pi^2\alpha_s(1-\cos F)r^2\right\}\mathrm{d}r, \quad (4)$$

where "*" indicates an in-medium modified quantity. Here, we have introduced the new definitions related to the profile function $s = \sin F$ and its derivative $F_r = \partial F/\partial r$.

Furthermore, the collective quantization procedure of the rotational states of the classical skyrmion [9] yields the in-medium modified nucleon mass and the corresponding $\Delta - N$ mass splitting respectively as

$$m_N^* = M_S^* + \frac{3}{8\lambda^*}\,, \qquad m_{\Delta-N}^* = \frac{3}{2\lambda^*}\,, \quad (5)$$

where in-medium moment of inertia of the skyrmion has form

$$\lambda^* = \frac{2\pi}{3}\int_0^\infty \left\{F_\pi^2 + \frac{4}{e^2\gamma}\left(F_r^2 + \frac{s^2}{r^2}\right)\right\} s^2 r^2\, dr. \quad (6)$$

Note that due to the medium modification of the Skyrme term the moment of inertia of the skyrmion and, as a result, $\Delta - N$ mass splitting explicitly depend on the medium density.

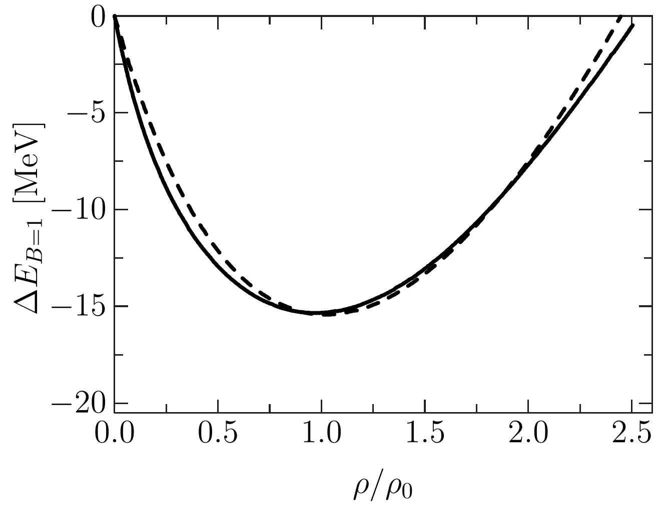

Fig. 1. The binding energy per nucleon as a function of ρ/ρ_0. The solid curve corresponds to the results with $\gamma_{\text{num}} = 2.1m_\pi^{-3}$, $\gamma_{\text{den}} = 1.45m_\pi^{-3}$ and P-wave scattering volume $c_0 = 0.21m_\pi^{-3}$. The dashed one draws the case when $\gamma_{\text{num}} = 0.8m_\pi^{-3}$, $\gamma_{\text{den}} = 0.5m_\pi^{-3}$ and P-wave scattering volume $c_0 = 0.09m_\pi^{-3}$. The S-wave scattering length has the value $b_0 = -0.024m_\pi^{-1}$. The correlation parameter is fitted near its experimental value $g_0' = 0.7$.

3 Nuclear matter properties

Before discussing the electromagnetic properties of the bound nucleons we will briefly recapitulate how to adjust the additional parameter γ represented in Eq. (2). As we mentioned above, that parametrization leads to the correct behavior of nuclear binding energy, defined as

$$\Delta E_{B=1} = m_N^*(\rho) - m_N^{\text{free}} \,, \tag{7}$$

near the equilibrium state [7].

The results of the binding energy per nucleon are shown in Fig. 1. The solid and dashed curves draw the parametrization of γ in Eq. (2). The minimization procedure has been performed in such a way that the values of the variational parameters, i.e., γ_{num} and γ_{den}, lead to the minimum of the binding energy per nucleon at normal nuclear matter density ρ_0. The difference between the two curves in Fig. 1 arise from the different values of the P-wave scattering length, i.e. $c_0 = 0.21m_\pi^{-3}$ that corresponds to the solid curve and $c_0 = 0.09m_\pi^{-3}$ for the dashed one. One can see that the dependence on the density is rather insensitive to the changes of P-wave scattering volume. The effect of the changes in b_0 is even smaller.

However, another important quantity is the compression modulus of nuclear matter defined as

$$K = 9\rho_0^2 \left. \frac{\partial^2 \Delta E_{B=1}}{\partial \rho^2} \right|_{\rho=\rho_0} \,, \tag{8}$$

which depends strongly on c_0. For example, at the empirical value of $c_0 = 0.21m_\pi^{-3}$, the compressibility turns out to be very large ($K \sim 1640$ MeV) in comparison with the results of other approaches like relativistic Dirac-Brueckner-Hartree-Fock [10,11] or the Walecka model [5]. Those approaches give much more smaller values of the compressibility. Lowering the value of P-wave scuttering volume c_0 leads to the noticeably decreasing value of K. At the values $c_0 = 0.06m_\pi^{-3}$ the result for the compressibility $K \sim 300$ MeV becomes comparable with the experimental value and with that in Dirac-Brueckner-Hartree-Fock approaches. So, one can conclude that within the present approach the smaller values of c_0 than that used in the pionic atom analysis will be preferable as far as the compressibility is concerned.

This simple analysis provides the modification of the Skyrme parameter in nuclear matter. After fitting all medium parameters one can start to scrutinize the single hadron properties according to the density changes of the surrounding environment. We will continue the discussions of the electromagnetic properties of nucleons.

4 Electromagnetic properties of the bound nucleons

Electric (E) and magnetic (M) form factors of the nucleon in the nuclear medium are defined as the following expressions

$$G_{\mathrm{E}}^{*}(q^2) = \frac{1}{2}\int \mathrm{d}^3 r\, e^{i\mathbf{q}\cdot\mathbf{r}}\, j^0(\mathbf{r})\,,$$
$$G_{\mathrm{M}}^{*}(q^2) = \frac{m_N^{\mathrm{free}}}{2}\int \mathrm{d}^3 r\, e^{i\mathbf{q}\cdot\mathbf{r}}[\mathbf{r}\times\mathbf{j}(\mathbf{r})]\,, \tag{9}$$

where j^0 and $\mathbf{j}$ are the temporal and spatial components of the properly normalized sum of the baryonic (topological) current B_μ and the third component of the isovector (Noether) current $\mathbf{V}_\mu^*$ of the in-medium modified Skyrme model. The generic form of the corresponding form factors is given by

$$G_a^{b\,*}(q^2) = \int \mathrm{d}^3 r\, e^{iqr\cos\theta}\rho_a^b(r,\theta)\,. \tag{10}$$

Generally, the isoscalar (S) and isovector (V) densities are expressed as

$$\rho_{\mathrm{E}}^{\mathrm{S}}(r,\theta) = -\frac{s^2 F_r}{4\pi^2 r^2}\,,$$
$$\rho_{\mathrm{M}}^{\mathrm{S}}(r,\theta) = -\frac{m_N^{\mathrm{free}}}{8\pi^2\lambda^*}\, s^2 F_r \sin^2\theta\,, \tag{11}$$
$$\rho_{\mathrm{E}}^{\mathrm{V}}(r,\theta) = \frac{s^2}{12\lambda^*}\left\{F_\pi^2 + \frac{4}{e^2\gamma}\left(F_r^2 + \frac{s^2}{r^2}\right)\right\}\,,$$
$$\rho_{\mathrm{M}}^{\mathrm{V}}(r,\theta) = \frac{m_N^{\mathrm{free}}}{3}\left\{F_\pi^2\alpha_p + \frac{4}{e^2\gamma}\left(F_r^2 + \frac{s^2}{r^2}\right)\right\} s^2\sin^2\theta\,.$$

In Eq. (10), a stands for E or M while b represents S or V. Due to the modification of the Skyrme term, all of the electromagnetic charge distributions, except isoscalar electric charge, depend explicitly on the medium density. There is an additional explicit medium modification α_p in the expression of the magnetic-vector density distribution.

The charges of the proton (p) and neutron (n) are defined as

$$Q^{\binom{\mathrm{p}}{\mathrm{n}}} = \frac{B}{2} + T_3^{\binom{\mathrm{p}}{\mathrm{n}}} \equiv \int \mathrm{d}^3 r\,\rho_{\mathrm{E}}^{\mathrm{S}}(r,\theta) \pm \int \mathrm{d}^3 r\,\rho_{\mathrm{E}}^{\mathrm{V}}(r,\theta)\,, \tag{12}$$

where we do not exclude the usual prefactor 1/2 in the density distributions.

Similarly, the magnetic moment of the nucleon is defined as

$$\mu^{\binom{\mathrm{p}}{\mathrm{n}}*} \equiv \int \mathrm{d}^3 r\,\rho_{\mathrm{M}}^{\mathrm{S}}(r,\theta) \pm \int \mathrm{d}^3 r\,\rho_{\mathrm{M}}^{\mathrm{V}}(r,\theta)\,. \tag{13}$$

The corresponding isoscalar and isovector mean square charge radii are given as

$$\langle r^2\rangle_{I=0} = -\frac{2}{\pi}\int_0^\infty s^2 F_r r^2 \mathrm{d}r\,, \tag{14}$$
$$\langle r^2\rangle_{I=1} = \frac{2\pi}{3\lambda^*}\int_0^\infty \left[F_\pi^2 + \frac{4}{e^2\gamma}\left(F_r^2 + \frac{s^2}{r^2}\right)\right] s^2 r^4 \mathrm{d}r\,.$$

ρ/ρ_0	μ_p^* [n.m.]	μ_n^* [n.m.]	$\langle r^2\rangle_{I=0}^{1/2*}$ [fm]	$\langle r^2\rangle_{I=1}^{1/2*}$ [fm]
		$\gamma = 1$		
0.0	1.965	-1.238	0.680	1.046
0.5	1.970	-1.226	0.708	1.050
1.0	1.968	-1.208	0.731	1.052
1.5	1.962	-1.189	0.751	1.052
2.0	1.953	-1.168	0.767	1.051
		$\gamma = \exp\{-\gamma_{\rm num}/(1+\gamma_{\rm den})\}$		
0.0	1.965	-1.238	0.680	1.046
0.5	2.134	-1.430	0.740	1.081
1.0	2.315	-1.634	0.793	1.112
1.5	2.511	-1.853	0.840	1.139
2.0	2.723	-2.087	0.881	1.163

Table 1. Magnetic moments, isoscalar and isovector charge radii of the nucleon in nuclear matter. Magnetic moments are given in units of nuclear magnetons (n.m.). The medium parameters are set at values: $g_0' = 0.7$, $b_0 = -0.024m_\pi^{-1}$, $c_0 = 0.06m_\pi^{-3}$. The Upper part of the table shows the case when the Skyrme term is intact in nuclear matter ($\gamma = 1$), while the lower part presents the results taking into account additional modifications, $\gamma_{\rm num} = 0.47m_\pi^{-3}$ and $\gamma_{\rm den} = 0.17m_\pi^{-3}$.

5 Results and discussions

Changes in the magnetic moments and in the isoscalar and isovector charge radii of nucleons as functions of ρ/ρ_0 are shown in Table 1. For comparison, we present here the results corresponding with the intact Skyrme term (the upper part of the table) and with the Skyrme term modifications (the lower part of the table). The medium parameters are chosen in a such way that they reproduce the correct value of the binding energy per nucleon at normal nuclear matter density and the corresponding compression modulus, when the explicit changes in the Skyrme term are taken into account.

If $\gamma = \text{const} = 1$, the changes in the magnetic moments and charge radii of the nucleon do not much pronounced. However, the magnetic moments and the charge radii are much enhanced in nuclear matter if one takes into account the modification of the Skyrme term, i.e. an additional modification changes the static properties of the nucleon further. This is due to the additional density functional γ appearing explicitly in the expressions of the EM charge distributions (11).

One can note that the nucleons swell in nuclear matter, which is consistent with the results of different approaches. For example, at normal nuclear matter density the isoscalar mean square radius $\langle r^2\rangle_{I=0}^{1/2*}$ is enhanced by about 16 % while the isovector mean square radius $\langle r^2\rangle_{I=1}^{1/2*}$ is modified by about 6 %. The magnetic moments are even more drastically changed due to the additional medium factor α_p in the expression of the magnetic-vector density distribution $\rho_{\rm M}^{\rm V}$.

In Fig. 2, it is shown the proton (the left panel in the figure) and the neutron (the right panel in the figure) electric form factors $G_{\rm E}^{\rm p,n*}(q^2)$ in free space (solid curve) and in nuclear matter with the normal nuclear matter densuty ρ_0 (dashed curve). One can note that with the density increased q^2 dependence of $G_{\rm E}^{\rm p*}$ falls off faster in comparison with the case in free space. The similar qualitative results have been obtained in the framework of the quark meson coupling (QMC) model [12]. However, at a quantitative level, our results are different from those in the QMC model. For comparison, at the normal nuclear matter $\rho = \rho_0$ and at the value of $q^2 = 0.3$ GeV^2 the proton charge form factor is reduced by about 30 % from that in free space value within our approach while it is reduced only by about 8 % within the QMC model [12]. In the case of the neutron electric form factor $G_{\rm E}^{\rm n*}(q^2)$ the difference is the other way around. At $q^2 = 0.3$ GeV^2 and at the normal nuclear matter $\rho = \rho_0$ neutron charge form factor is changed only by about 1 %, whereas the QMC model brings about the change of 8 %. However, the neutron electric form factor falls off faster in nuclear matter than that in free space (e.g. at $q^2 =$ 1 GeV^2 changes are $\sim$ 30%). Otherwise our results are similar to those of the solitonic approach which includes the explicit mesonic degrees of freedom other than the pion [13]. In that approach, the

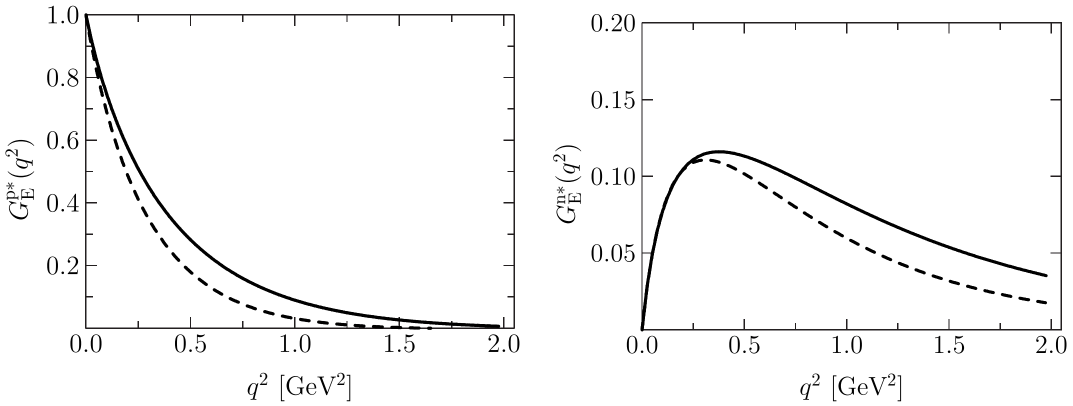

Fig. 2. The proton (left) and the neutron (right) electric form factor as a function of q^2. The solid curve corresponds to the free space case, the dashed one in nuclear matter with the density $\rho = \rho_0$. The input parameters are given as $g_0' = 0.7$, $b_0 = -0.024 m_\pi^{-1}$, $c_0 = 0.06 m_\pi^{-3}$, $\gamma_{\rm num} = 0.47 m_\pi^{-3}$ and $\gamma_{\rm den} = 0.17 m_\pi^{-3}$.

changes in neutron electric form factor in nuclear matter becomes more pronounced with the density of the medium increased.

The magnetic form factors $G_M^{p*}(q^2)$ (the left panel in the figure) and $G_M^{n*}(q^2)$ (the right panel in the figure) are shown in Fig. 3. Here also the changes more pronounced within the present model in

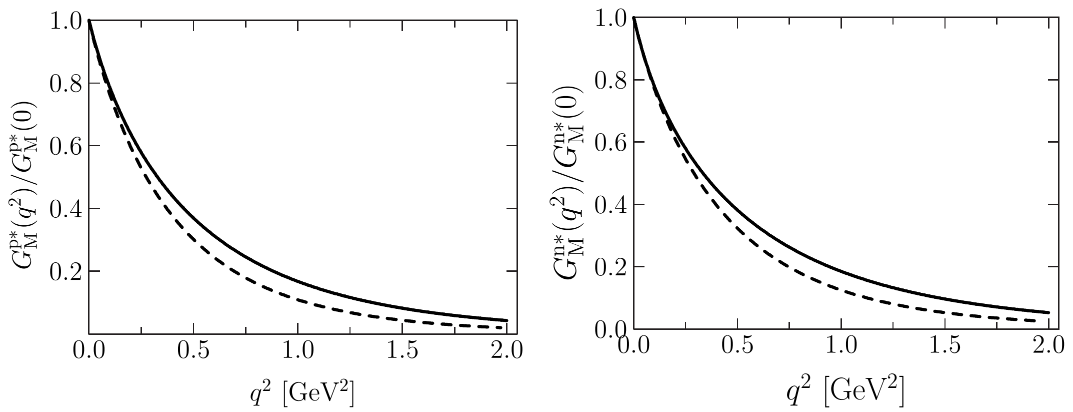

Fig. 3. The normalized proton magnetic form factor (left) and the normalized neutron magnetic form factor (right) as a function of q^2. The notations are same as in Figure 2.

comparison with QMC model. For comparison, at the normal nuclear matter $\rho = \rho_0$ and at the value of $q^2 = 0.3$ GeV^2 proton and neutron magnetic form factors reduce are by $\sim 12\%$ and $\sim 8\%$, compared to the free space value within our approach, while the QMC model yields the proton and neutron magnetic form factors reduced by $\sim 1.5\%$ and 0.9%, respectively.

As a general feature, one notes that at large values of the q^2 again the form factors fall off faster in nuclear matter than in free space.

It is known that in free space the neutron and proton form factors calculated in topological models at low momentum transfers obey the scaling relation [8]

$$\frac{G_M^p(q^2)}{G_M^p(0)} \approx \frac{G_M^n(q^2)}{G_M^n(0)} \approx G_E^p(q^2)\,.$$

We note that in the case of the medium modifications this scaling relations starts to break with increasing density, but is still partially satisfied

$$\frac{G_{\mathrm{M}}^{\mathrm{p}}(q^2)}{G_{\mathrm{M}}^{\mathrm{p}}(0)} \approx \frac{G_{\mathrm{M}}^{\mathrm{n}}(q^2)}{G_{\mathrm{M}}^{\mathrm{n}}(0)} \neq G_{\mathrm{E}}^{\mathrm{p}}(q^2)\,.$$

6 Summary

We have studied the electromagnetic properties of the nucleons in nuclear matter within the framework of the in-medium modified Skyrme model. The results of the studies are consistent with the results of other solitonic approaches which include explicit vector meson degrees of freedom [13] both qualitatively and quantitatively. Our results are also consistent with the QMC model calculations [12] at a qualitative level although there are discrepancies in detail. Ideas within the present model may be useful in constructing a more consistent theory which evaluates the many-body systems and its constituents on the same footing.

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