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Five-body Cluster Structure of the double Λ hypernucleus $^{11}_{\Lambda\Lambda}$Be

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Abstract. Energy levels of the double Λ hypernucleus, $^{11}_{\Lambda\Lambda}$Be are calculated within the framework of an $\alpha\alpha n\Lambda\Lambda$ five-body model. By comparing our results and recent observed data at KEK-E373 experiment, the Hida event, we suceeded in interpreting as an observation of the ground state of the $^{11}_{\Lambda\Lambda}$Be.

1 Introduction

Double strangeness ($S = -2$) nucleus is an entry to multi-strangeness hadronic systems and requires a unified understanding of YN and YY interactions. The $\Lambda\Lambda$ interaction, in particular, is essential to understand the core of a neutron star, which is expected to include many hyperons. The $S = -2$ nuclei will exhibit new dynamical features which will have more variety than those in ordinary nuclear systems and $S = -1$ ones. The present experimental information on the $S = -2$ system, however, is limited and quite far from such an overview, due to the experimental difficulties of the YY scattering in free space as well as producing and hunting $S = -2$ nuclei.

In the KEK-E176/E373 hybrid emulsion experiments, there have been observed several events of double-Λ hypernuclei. Among them, especially epoch-making is the observation of the NAGARA event, being identified uniquely as $^{6}_{\Lambda\Lambda}$He in the ground state with the precise value of $B_{\Lambda\Lambda} = 7.25 \pm 0.19$ MeV [1]. Another important observation is the Demachi-Yanagi event [2] identified as $^{10}_{\Lambda\Lambda}$Be with $B_{\Lambda\Lambda} = 12.33^{+0.35}_{-0.21}$ MeV. Recently, the newly observed double-Λ event has been reported, called Hida event [3]. This event has two possible interpretations: One is $^{11}_{\Lambda\Lambda}$Be with $B_{\Lambda\Lambda} = 20.83 \pm 1.27$MeV, and the other is $^{12}_{\Lambda\Lambda}$Be with $B_{\Lambda\Lambda} = 22.48 \pm 1.21$ MeV. Regarding to these two events, it is uncertain whether these are observations for the ground state or the excited state.

In the planned experiments at J-PARC, they will produce at least dozens of emulsion events of double-Λ hypernuclei. In emulsion experiments, however, it is difficult to determine their spin-parities or even to know whether observed events are in ground or excited states. Therefore, it is vitally important to interpret the emulsion data based on theoretical analyses. In order to interpret the 'Demachi-Yanagi' event, we studied ,in Ref.[2], the $^{10}_{\Lambda\Lambda}$Be hypernucleus within the framework of the $\alpha\alpha\Lambda\Lambda$ four-body cluster model, where the $\Lambda\Lambda$ interaction was taken consistently with the NAGARA event. Our calculated value was $B_{\Lambda\Lambda} = 12.28$ MeV for the 2^+ state, which was in good agreement with the observed $B_{\Lambda\Lambda} = 12.33^{+0.35}_{-0.21}$ MeV. Then, the Demachi-Yanagi event is interpreted as an observation of the 2^+ excited state of $^{10}_{\Lambda\Lambda}$Be.

The aim here is to interpret the new Hida event by our theoretical study adapting the method used for Demachi-Yanagi event. At present, Hida event has two possible interpretations, namely, $^{11}_{\Lambda\Lambda}$Be and $^{12}_{\Lambda\Lambda}$Be. In this paper, we study this event assuming a $^{11}_{\Lambda\Lambda}$Be hypernucleus. It is reasonable to employ an $\alpha\alpha n\Lambda\Lambda$ five-body model for the study of $^{11}_{\Lambda\Lambda}$Be, since, as mentioned above, the interpretation of the

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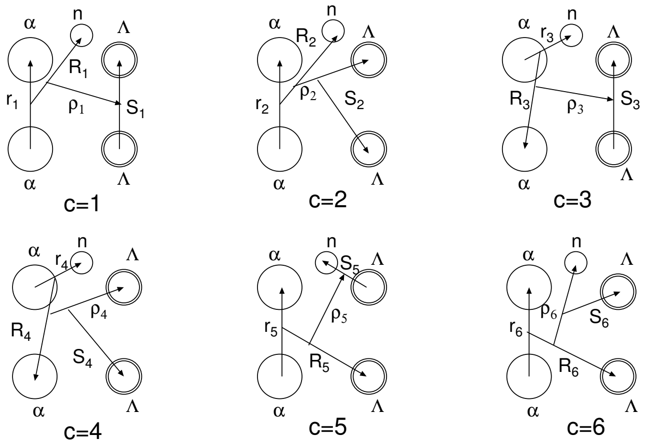

Fig. 1. Example sets ($c = 1 - 6$) of Jacobi coordinates of the $\alpha\alpha n\Lambda\Lambda$ five-body system. Antisymmetrization (symmetrization) of two Λ's (two α's) is to be made.

Demachi-Yanagi event for ${}^{10}_{\Lambda\Lambda}$Be was possible on the basis the $\alpha\alpha\Lambda\Lambda$ four-body cluster model, and ${}^{11}_{\Lambda\Lambda}$Be is composed of the ${}^{10}_{\Lambda\Lambda}$Be and one neutron. We further note that the core nucleus ^{9}Be is well described by using the $\alpha\alpha n$ three-cluster model [4], and therefore it is possible to treat the structure change of ^{9}Be due to the addition of the two Λ particles.

2 Method

In order to take into account the full five-body degrees of freedom of the $\alpha\alpha n\Lambda\Lambda$ system and the full correlations among all the constituent five particles, we describe the total wave function, $\Psi_{JM}({}^{11}_{\Lambda\Lambda}\mathrm{Be})$, as a function of the *whole* 35 sets of Jacobi coordinates $\{\mathbf{r}_c, \mathbf{r}_c, \boldsymbol{\rho}_c, \mathbf{S}_c; c = 1 - 35\}$ in which the two Λ's (two α's) have been antisymmetrized (symmetrized). Some of important coordinate sets ($c = 1 - 6$) are shown in Fig.1.

The total wave function is described as a sum of five-body basis functions $\Phi^{(c)}_{JM,\beta}$, which is an extension of our previous work using four-body basis functions [5]:

$$\Psi_{JM}({}^{11}_{\Lambda\Lambda}\mathrm{Be}) = \sum_{c=1}^{35} \sum_{\beta} C^{(c)}_{\beta} \mathcal{A}_{\Lambda\Lambda} \mathcal{S}_{\alpha\alpha}\, \Phi^{(c)}_{JM,\beta}\,, \tag{1}$$

where $\mathcal{A}_{\Lambda\Lambda}$ ($\mathcal{S}_{\alpha\alpha}$) is the antisymmetrizer (symmetrizer) for two Λ's (two α's), and

$$\begin{aligned}\Phi^{(c)}_{JM,\beta} &= \xi(\alpha_1)\xi(\alpha_2) \\ &\times\Bigg[\bigg[\Big[\big[\phi^{(c)}_{nl}(\mathbf{r}_c)\,\psi^{(c)}_{NL}(\mathbf{R}_c)\big]_I\,\varphi^{(c)}_{n'l'}(\boldsymbol{\rho}_c)\Big]_K\,\Phi^{(c)}_{N'L'}(\mathbf{S}_c)\bigg]_L \\ &\times\Big[\big[\chi_{\frac{1}{2}}(\Lambda_1)\chi_{\frac{1}{2}}(\Lambda_2)\big]_\Sigma\,\chi_{\frac{1}{2}}(n)\Big]_S\Bigg]_{JM}\,,\end{aligned} \tag{2}$$

with $\beta \equiv \{nl, NL, n'l', N'L', IKL, \Sigma S\}$ denoting a set of the quantum numbers. In Eq. (2), $\xi(\alpha)$ is the internal wave function of an α-cluster having $(0s)^4$ configuration and is used in the folding procedures for the αn, $\alpha\Lambda$, and $\alpha\alpha$ interactions. The $\chi_{\frac{1}{2}}(\Lambda)$ and $\chi_{\frac{1}{2}}(n)$ are the spin functions of the Λ and n, respectively. Following Refs. [5,6], the radial shapes of the basis function $\phi_{nlm}(\mathbf{r})(= r^l\, e^{-(r/r_n)^2} Y_{lm}(\widehat{\mathbf{r}}))$ are taken to be Gaussians with ranges postulated to lie in a geometric progression and similarly for

$\psi_{NLM}(\mathbf{R}), \varphi_{n'l'm'}(\boldsymbol{\rho})$ and $\Phi_{N'L'M'}(\mathbf{S})$. The expansion coefficients $C^{(c)}_{\beta}$ and the eigenenergy E of the total wave function $\Psi_{JM}(^{11}_{\Lambda\Lambda}\mathrm{Be})$ are determined by solving the five-body Schrödinger equation using the Rayleigh-Ritz variational method.

In the present $\alpha\alpha n\Lambda\Lambda$ five-body model for $^{11}_{\Lambda\Lambda}$Be, it is absolutely necessary that any subcluster systems composed of the two, three, or four constituent particles are reasonably described by taking the interactions among these systems. In our previous work on double Λ hypernuclei with $A = 7-10$ within the framework of the $\alpha x\Lambda\Lambda$ four-body cluster model ($x = n, p, d, t, {}^3$He, and α) [5], the α-α, α-n, α-Λ, Λ-n, and Λ-Λ interactions were determined so as to reproduce well the following observed quantities: (i) Energies of the low-lying states and scattering phase shifts in the αn and $\alpha\alpha$ systems, (ii) Λ-binding energies B_Λ in $^5_\Lambda$He ($= \alpha\Lambda$), $^6_\Lambda$He ($= \alpha n\Lambda$) and $^9_\Lambda$Be ($= \alpha\alpha\Lambda$), (iii) double-Λ binding energies $B_{\Lambda\Lambda}$ in $^6_{\Lambda\Lambda}$He ($= \alpha\Lambda\Lambda$), the NAGARA event. We then predicted, with no more adjustable parameters, the energy level of $^{10}_{\Lambda\Lambda}$Be ($= \alpha\alpha\Lambda\Lambda$), and found that, as mentioned before, the Demachi-Yanagi event was an observation of the 2^+ excited state of $^{10}_{\Lambda\Lambda}$Be.

In the present five-body calculation, we employ the interactions of Ref. [5] so that those severe constraints are also successfully met in our two-, three-, and four-body subsystems. But, the present core nucleus ^{9}Be ($= \alpha\alpha n$) does not belong to the subsystems studied previously. Since use of the same $\alpha\alpha$ and αn interactions does not precisely reproduce the energies of the low-lying states of ^{9}Be measured from the $\alpha\alpha n$ threshold (the same difficulty is seen in another microscopic $\alpha\alpha n$ cluster-model study [4]), we introduce an additional phenomenological $\alpha\alpha n$ three-body force with a Gaussian shape, $v_0 e^{-(r_{\alpha-\alpha}/r_0)^2-(R_{\alpha\alpha-n}/R_0)^2}$, having $r_0 = 3.6$ fm, $R_0 = 2.0$ fm and $v_0 = -9.7$ MeV (+13.0 MeV) for the negative-parity (positive-parity) state so as to fit the observed energies of the $3/2^-_1$ ground state and the $5/2^-_1$, $1/2^-_1$ and $1/2^+_1$ excited states in ^{9}Be. For the latter three resonance states, the same bound-state approximation (namely, diagonalization of the Hamiltonian with the L^2-integrable basis functions) was applied. Simultaneously, we found that the calculated B_Λ of the ground state (1^-) of $^{10}_\Lambda$Be reproduced well the observed value.

In order to reproduce the observed $B_{\Lambda\Lambda} = 6.91$ MeV for $^6_{\Lambda\Lambda}$He, we tuned the $\Lambda\Lambda$ interaction {Eq.(3.6) in Ref. [5] } by multiplying the strength of the $i = 3$ part by a factor 1.244. As for $^{10}_{\Lambda\Lambda}$Be, we obtained $B^{\rm cal}_{\Lambda\Lambda}(2^+_1) = 11.88$ MeV and $B^{\rm cal}_{\Lambda\Lambda}(0^+_1) = 14.74$ MeV, which explain, respectively, the Demachi-Yanagi event for the 2^+_1 state (11.90 ± 0.13 MeV) and the data of Ref. [7] for the ground state (14.6 ± 0.4 MeV; see Table 5 of Ref.[7]). The above successful check of the energies of the subsystems encourages us to perform the five-body calculation of $^{11}_{\Lambda\Lambda}$Be with no adjustable parameter, expecting high reliability for the result.

3 Results

We discuss the energy spectra of $^{11}_{\Lambda\Lambda}$Be and its relation to the Hida event. Using the same framework and interactions as those in the $3/2^-$ ground state, we calculate energies and wave functions of the $5/2^-_1$, $1/2^+_1$ and $1/2^-_1$ states of $^{11}_{\Lambda\Lambda}$Be; there is no other bound state below the lowest $^{10}_{\Lambda\Lambda}$Be $+ n$ threshold. The energy level is illustrated in Fig. 2 together with that of ^{9}Be. Interestingly enough, the order of the $1/2^+$ and $5/2^-$ states is reversed from ^{9}Be to $^{11}_{\Lambda\Lambda}$Be. This is because the energy gain due to the addition of the Λ-particle(s) is larger in the compactly coupled state ($5/2^-_1$) than in the loosely coupled state ($1/2^+_1$). Note that the similar reversion of level ordering is reported in our early work [8] for $^{13}_\Lambda$C with the $\alpha\alpha\alpha\Lambda$ structure, where the addition of the Λ particle into the compactly coupled (3^-_1) state brings about the larger energy gain than that into the loosely coupled (0^+_2) state.

As seen in Fig. 2 the calculated value of $B_{\Lambda\Lambda}(^{11}_{\Lambda\Lambda}\mathrm{Be})$ is 18.23 MeV for the $3/2^-$ ground state, while for the excited states the $B_{\Lambda\Lambda}$ values are calculated to be less than 15.5 MeV. Therefore, the observed Hida event can be interpreted to be the ground state. When our calculated binding energy is compared with the experimental value of 20.83 MeV with a large uncertainty of σ=1.27 MeV, we can say at least that our result does not contradict the data within 2σ.

Conventionally, the $\Lambda\Lambda$ interaction strength has been often estimated rather intuitively by the quantity $\Delta B_{\Lambda\Lambda}(^{\mathrm{A}}_{\Lambda\Lambda}Z)$. The observed value of $\Delta B_{\Lambda\Lambda}(^6_{\Lambda\Lambda}\mathrm{He})$ is 0.67 MeV, to which our $\Lambda\Lambda$ interaction is adjusted. The calculated value for the ground state of $^{10}_{\Lambda\Lambda}$Be is 1.32 MeV. On the other hand, that for $^{11}_{\Lambda\Lambda}$Be

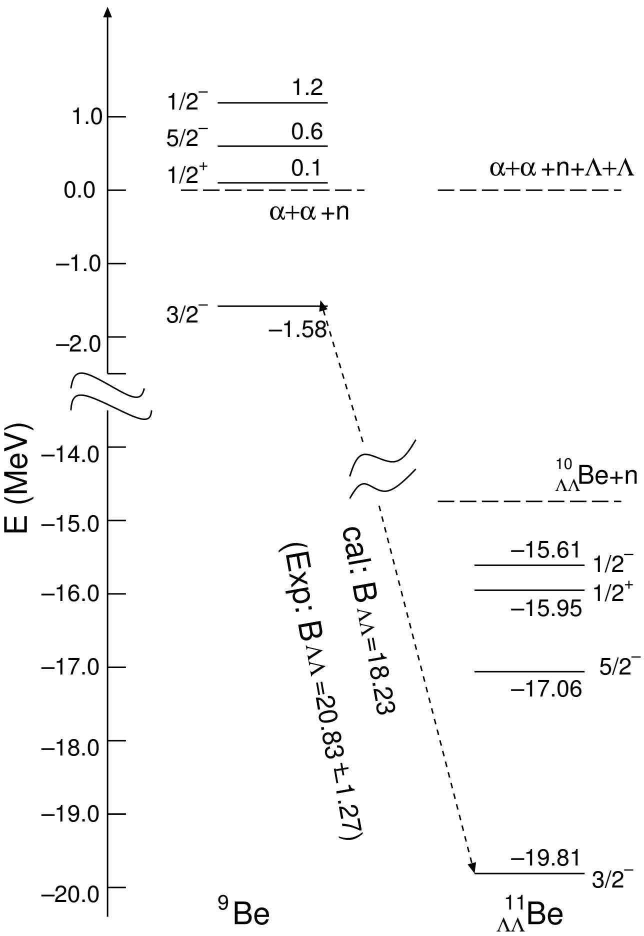

Fig. 2. Calculated energy spectra of the low-lying states of ${}^{11}_{\Lambda\Lambda}$Be together with those of the core nucleus ^{9}Be.

is obtained as only 0.29 MeV. Here, the $B_\Lambda({}^{10}_{\Lambda}\mathrm{Be})$ is given by a weighted sum of the values for ground 1^- and excited 2^- states of ${}^{10}_{\Lambda}$Be so that there is no contribution of the ΛN spin-spin interaction in the double-Λ state. One should notice here the remarkable difference between $\Delta B_{\Lambda\Lambda}$ values for ${}^{10}_{\Lambda\Lambda}$Be and ${}^{11}_{\Lambda\Lambda}$Be.

As discussed in Ref.[5], values of $\Delta B_{\Lambda\Lambda}$ are affected rather strongly by core-rearrangement effects and considered to be inadequate as indicators of $\Lambda\Lambda$ bond energies. Then, the $\mathcal{V}^{\mathrm{bond}}_{\Lambda\Lambda}$ defined by Eq.(4.1) of Ref.[5] can be used reasonably for estimating the strength of the $\Lambda\Lambda$ interaction. Table 1 lists the calculated values of $\mathcal{V}^{\mathrm{bond}}_{\Lambda\Lambda}$ for ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be and ${}^{11}_{\Lambda\Lambda}$Be which are similar to each other. Thus, the obtained $\Lambda\Lambda$ bond energy in ${}^{11}_{\Lambda\Lambda}$Be turns out to be reasonable in spite of the small value of $\Delta B_{\Lambda\Lambda}$.

Table 1. $\Lambda\Lambda$ bond energy $\mathcal{V}^{\mathrm{bond}}_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z)$ defined by Eq.(4.1) of Ref.[5].

	${}^{6}_{\Lambda\Lambda}$He	${}^{10}_{\Lambda\Lambda}$Be	${}^{11}_{\Lambda\Lambda}$Be	
$\mathcal{V}^{\mathrm{bond}}_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z)$	0.54	0.53	0.56	(MeV)

Thus, it is found that the calculated $\Lambda\Lambda$ binding energy based on the five-body $\alpha\alpha n\Lambda\Lambda$ cluster model does not contradict the interpretation that the Hida event is an observation of the ground state of $^{11}_{\Lambda\Lambda}$Be. For an alternative interpretation of the Hida event as the ground (or any excited) state of $^{12}_{\Lambda\Lambda}$Be, a corresponding six-body $\alpha\alpha nn\Lambda\Lambda$ model calculation is necessary, but such an undertaking is beyond our present consideration. Anyway, more precise data are needed in order to test our present result quantitatively. In the near future, many data for double Λ hypernuclei are expected to be found in the new emulsion experiment E07 at J-PARC. Then, our systematic predictions in the present work will be clearly tested.

4 Summary

Motivated by the recent observation of the Hida event for a new double Λ hypernucleus, we have succeeded in performing a five-body calculation of $^{11}_{\Lambda\Lambda}$Be using an $\alpha\alpha n\Lambda\Lambda$ cluster model. The calculated $\Lambda\Lambda$ binding energy does not contradict the interpretation that the Hida event is an observation of the ground state of $^{11}_{\Lambda\Lambda}$Be. In this model, we described the core nucleus ^{9}Be using the $\alpha\alpha n$ three-cluster model and found a significant the two Λ particles. For an alternative interpretation of the Hida event as the ground (or any excited) state of $^{12}_{\Lambda\Lambda}$Be, a corresponding six-body $\alpha\alpha nn\Lambda\Lambda$ model calculation is necessary, but such an undertaking is beyond our present consideration. More precise data are needed in order to test our present result quantitatively.

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