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Fission via compound states and $J^\pi K$ A. Bohr's channels: what we can learn from recent studies with slow neutrons

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Abstract. Last data on angular correlations of fission fragments from slow (s-wave) neutron induced binary fission of spin-aligned nuclei ^{235}U are discussed in the context of $J^\pi K$ A. Bohrs channels. Special attention is paid to $K = 0$ channel. Reasons for its suppression are specified for compound nucleus states of negative parity. A brief overview of recent data on T-odd angular correlations in ternary and binary (with emission of a third particle, a neutron or γ-quantum) fission induced by slow polarized neutrons is presented. On the basis of the developed theoretical approach it is shown that a valuable information on $J^\pi K$ fission channels at scission point can be inferred from these T-odd angular correlations.

1 Introduction

Fission of nuclei by slow neutrons is of great interest due to a possibility of studying of the process's quantum aspects related to fission channels. Really, modern high resolution time-of-flight technique makes possible an identification of quantum interference of amplitudes of fission via separated compound-states (neutron resonances) both in differential and in total cross sections of (n,f)-reaction.

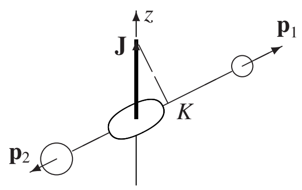

Fig. 1. Fission of a nucleus with spin J into two fragments; K is the projection of spin J on the fission axis.

More exactly, the fission amplitudes are related to channels introduced by A. Bohr [1]. Each channel is characterised not only by spin J and parity π of the corresponding compound-state, but also by a projection K of spin J onto an axis $\mathbf{n}_f$ of nuclear elongation (see Fig. 1). In fact, the channels are nuclear rotation states (transition states), and K is a good quantum number due to conservation of axial symmetry of the fissioning nucleus up to scission.

The A. Bohr's model allows to connect an angular distribution of fission fragments with a distribution over the states with different K on a fission barrier. Indeed, the wave function of the deformed nucleus is

$$\Psi_J \sim \sum_M a_M(J) \sum_K g^{JK} \Phi_K(\tau) D^J_{MK}(\mathbf{n}_f), \tag{1}$$

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where M is a projection of spin J on an axis z, and the function $\Phi_K(\tau)$ describes internal degrees of freedom of deformed nucleus. Thus, the distribution over $\mathbf{n}_f$ takes the form

$$\frac{dw(\mathbf{n}_f)}{d\Omega} \sim \int |\Psi_J|^2 d\tau \sim 1 + 3\, p_2 \langle A_2 \rangle \frac{3\cos^2\theta - 1}{2} + \ldots, \quad \langle A_2 \rangle = \sum_K |g^{JK}|^2 A_2(K), \tag{2}$$

where p_2 is the parameter of spin alignment of nuclei with respect to the axis z. The factor

$$A_2(K) \sim \frac{3K^2}{J(J+1)} - 1 \tag{3}$$

depends strongly on K, therefore the coefficient of angular anisotropy $\langle A_2 \rangle$ is very sensitive to the distribution $|g^{JK}|^2$ over K. However, the A. Bohr's model incorporates no fragment characteristics. Thus, any predictions for fragment characteristics, for example, for the fragment spin alignment, are impossible.

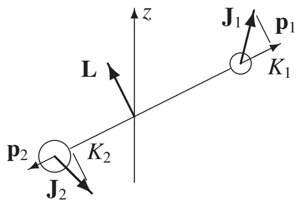

Fig. 2. Two fragments with spins J_1 and J_2, helicities K_1 and K_2 and momenta $\mathbf{p}_1$ and $\mathbf{p}_2$; L is a relative orbital momentum of two fragments.

Another approach for the angular distribution of fission fragments was proposed by Strutinsky [2] in the frame of a so-called helicity representation (see Fig. 2). After the neck rupture, the spin $\mathbf{J}$ transfers into $\mathbf{J}_1+\mathbf{J}_2+\mathbf{L}$, where J_1 and J_2 are the fragment spins and $\mathbf{L}$ is the relative orbital momentum of two fragments. Therefore, the projection K transfers near-precisely into a total helicity of two fission fragments $\tilde{K}$ that is the projection of total spin $\mathbf{F} = \mathbf{J}_1 + \mathbf{J}_2$ of the fragments onto the fission axis $\mathbf{n}_f$ (notice that $\mathbf{L} \perp \mathbf{n}_f$). According Strutinsky, $\tilde{K}$ is also the good quantum number in the fission process due to insignificance of the relative orbital momentum of the fragments. As A. Bohr, Strutinsky has considered only a simplified case of fission of a nucleus with definite spin and parity J^π.

The Strutinsky's method deals not only with the fragment's helicities but with all characteristics of fragments in the exit states labeled by an index α. The asymptotic wave function in the (LF)-representation is

$$\Psi_J \to \frac{e^{ik_\alpha r}}{r} \sum_M a_M(J) \sum_{LF} (-i)^{L+1} g^\alpha(LF)\, \varphi^\alpha_{LFJM}, \quad \varphi^\alpha_{LFJM} = \sum_{\nu m} C^{JM}_{F\nu Lm} \chi^\alpha_{F\nu}\, i^L Y_{Lm}(\mathbf{n}_\alpha), \tag{4}$$

where φ^α_{LFJM} is the channel function. The fragment's functions can be expressed via the functions with definite helicities,

$$\chi^\alpha_{F\nu} = \sum_K D^F_{\nu K}(\mathbf{n}_\alpha) \chi^\alpha_{FK}. \tag{5}$$

Thus, the total wave function in the (FK)-representation takes the form (see details in [8,9])

$$\Psi_J \to \frac{e^{ik_\alpha r}}{r} \sum_M a_M(J) \sum_{FK} g^\alpha(FK)\, \varphi^\alpha_{FKJM}, \quad \varphi^\alpha_{FKJM} = \chi^\alpha_{FK} D^F_{MK}(\mathbf{n}_\alpha), \tag{6}$$

where φ^α_{FKJM} is the channel function in the helicity representation. Notice that the right-hand side of Eq. (6) looks like the right-hand side of Eq. (1). Thus, the Strutinsky's method leads to the same form (2) for the angular distribution of fission fragments. In addition, this approach allows to describe, in principle, all observable quantities after scission point. In particular, spin polarization and alignment parameters for fission fragments were expressed for the first time in [3,4] via amplitudes of fission to states with given F and K and then numerically estimated.

Later the Strutinsky's method was extended in [5–8] to consistent description of differential cross-section of (n,f)-reaction with polarized neutrons and spin-aligned target nuclei (see also [9]). We have developed a method of summation over numerous exit states and related these states to A. Bohr fission channels. The fission amplitudes

$$\gamma_\nu^\alpha(FKJ^\pi) \sim \langle\varphi^\alpha_{FKJM}|X_\nu^{J\pi M}\rangle, \tag{7}$$

where $X_\nu^{J\pi M}$ is a wave function of the compound resonance ν, can be simplified with the use of the assumption by Furman and Kliman [10]

$$X_\nu^{J\pi M} = X_{\nu c}^{J\pi M} + X_{\nu f}^{J\pi M}, \quad X_{\nu f}^{J\pi M} = \sum_K a_{\nu f}^{J\pi K} \sum_m \alpha_m^K \Psi_m^{J\pi KM}. \tag{8}$$

Here the functions $X_{\nu c}^{J\pi M}$ and $X_{\nu f}^{J\pi M}$ correspond to small and large deformations. The latter is presented as a superposition on the functions $\Psi_m^{J\pi KM} \sim \Phi_{mK}(\tau)D^J_{MK}(\mathbf{n}_f)$ (see Eq. (1)). Each of these functions depends on fission mode m and the projection K but does not depend on resonance's index ν. Only the amplitudes $a_{\nu f}^{J\pi K}$ describing the mixing over K by Coriolis interactions keep the dependence on ν. This fact leads to survival of interference terms between different compound resonances at summation over all exit states α. As a result, the fission amplitudes $\gamma_\nu(J^\pi K)$ for A. Bohr's channels arise. By this way direct connection was established between the $J^\pi K$ A.Bohr's channels and all observable quantities in the fission process.

The most reliable source of information on A. Bohr's $J^\pi K$ channels are the angular distributions of fragments from fission of compound states with definite J^π, i.e. neutron resonances. In order that compound nuclei arising after capture of s-wave neutrons are spin aligned, an ensemble of target nuclei should possess the spin alignment. The most thorough studies were performed for the system n+^{235}U. In the late 1960s and in the early 1970s the coefficient of angular anisotropy has been measured in separated resonances [11,12], and then in 1990s in Dubna the angular anisotropy has been studied as function of neutron energy in resonances and between them [13,14]. Analysis of these data provides the most complete information on fission channels with various K. Recently we have shown [15] that the channel $K = 0$ is especially interesting because its contribution is very sensitive to the shape symmetry of the fissioning nucleus at first and second fission barriers. Below, in the second section of the report, it is explained in more details.

During the last decade new angular correlations, known as T (formal, not real, violation of Time reversal invariance) and R (Rotation) effects [16–20], have been found in fission of nuclei ^{233}U and ^{235}U by slow polarized neutrons. Formally both angular correlations are odd with respect to time reverse (T-odd), but they have nothing to do with violation of fundamental time reversal invariance. Both correlations are caused by specific interactions in the final states. The third section of the report is devoted to discussion of these T-odd correlations. Some of them give us an estimation for the accuracy of the relation $\tilde{K} = K$ crucial for the theoretical approache based on helicity representation. We argue that the observed smallness of R effect points out to high accuracy of the above equality.

2 Structural features of the fission channel $K = 0$

In processing data from experiments [13,14], special attention was paid to the $K = 0$ channel. At the first stage of data processing [13] only the $J^\pi K = 3^-0$ channel was taken into account while the $J^\pi K = 4^-0$ channel was excluded according the original simplified model [1]. Later the $J^\pi K = 4^-0$ channel was included to the analysis (see below for the reasons). The quantitative estimate of the contribution of the $K = 0$ channels has been obtained for the first time. This contribution is proved to be sizable (on the scale of 25% !). However, the $K = 0$ channel does not dominate over $K \neq 0$ channels. The suppression of the $K = 0$ channel was noted already in [12] where it was interpreted as a disagreement with A. Bohr's model [1].

In fact, we should be guided not by the original simplified model [1] but by the extended approach to deformed nuclei presented in the monography [21]. According this approach, both channels, $J^\pi K = 3^-0$ and 4^-0, are open (see details in [15]). But the wave function of the deformed nucleus with $K = 0$ is characterized by an additional quantum number, $s = \pm 1$, that is an eigenvalue of the operator $\hat{S}_i$ of inversion of the internal wave function with respect to a plane that contains the axial-symmetry axis. At the R-symmetric stage, this wave function is characterized by an additional quantum number, $r = \pm 1$, that is an eigenvalue of the operator $\hat{R}_i$ of rotation on π of the internal wave function around the axis that is orthogonal to the axial-symmetry axis and which passes through the center of the nucleus being considered. The quantum numbers r, s, and π are related by the equality

$$r = s\pi. \tag{9}$$

The nucleus ^{235}U is of negative parity, thus $\pi = -1$ for resonances excited by s-wave neutrons. One can readily see that the equality $r = -s$ always holds; that is, either r or s takes the value of -1. For the ^{235}U nucleus the first barrier appears R-symmetric, while the second one is R-asymmetric. This means that on any barrier, for any value of the spin J, the internal function changes its sign under one transformation or another, thus the corresponding internal quantum state possesses an enlarged energy. Therefore, the $K = 0$ channel is suppressed. In particular, it is the case in the fission of spin-aligned ^{235}U nuclei of negative-parity by slow (s-wave) neutrons.

3 T-odd angular correlations in fission with a third emitted particle

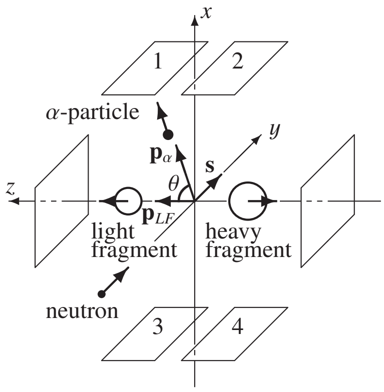

Fig. 3. Scheme of registration of T and R effects. The neutron spin **s** is directed along the axis y, the momentum of the light fragment $\mathbf{p}_{LF}$ is directed along the axis z, the asymmetry of a third particle's emission along and against to the axis x is under investigation.

Scheme of measurement of small ($\sim 10^{-3}$) asymmetries of third particle's emission in nuclear fission induced by slow (s-wave) polarized neutrons is presented on Fig. 3. The incident neutrons are longitudinally polarized. The neutrons move along the axis y, the light fragments move along the axis z, then there are asymmetries of third particle's detection, e.g. α-particle in ternary fission, by pairs of counters 1 and 3 or 2 and 4. Let us introduce asymmetry coefficients

$$D_{13}(\theta) = \frac{N_1 - N_3}{N_1 + N_3}, \quad D_{24}(\pi - \theta) = \frac{N_2 - N_4}{N_2 + N_4}, \tag{10}$$

and total asymmetry

$$D = \frac{(N_1 + N_2) - (N_3 + N_4)}{(N_1 + N_2) + (N_3 + N_4)} = \frac{D_{13} + \lambda D_{24}}{1 + \lambda}, \quad \lambda = \frac{N_2 + N_4}{N_1 + N_3}, \tag{11}$$

where λ is of the scale of unity (for ternary fission with α-particle's emission, λ is smaller than unity, because the maximum of α-particle's angular distribution falls to the angle $\theta \simeq 83^0$ due to a greater Coulomb repulsion from the heavy fragment). Obviously, $D_{13} = D_{24} = 0$ for unpolarized neutrons.

In the experiments [16,17] with target nuclei ^{233}U it was found that $D_{13} \simeq D_{24} \simeq D$. Formally, it corresponds to 3-fold correlation $(\mathbf{p}_\alpha[\mathbf{s} \times \mathbf{p}_{LF}])$, which is T-odd. Thus the asymmetry was named the T effect. In [22] a qualitative explanation of the phenomenon as the result of the interaction of the spin of residual nucleus and the orbital momentum of α-particle (spin-orbit interaction) was proposed. However, this hypothesis was met with a scepticism based on naive semiclassical interpretation of spin-orbit interaction. It was noted in [17]: "... in case of the spin-orbit interaction is at work, the correlation coefficient D should have opposite signs for angles smaller or larger that the average angle 83^0 ...", i.e. the relation $D_{13} \simeq -D_{24}$ (and $D \simeq 0$) was expected.

Surprisingly, just this result $D_{13} \simeq -D_{24}$ (and $D \simeq 0$) has been found in the middle of 2000s in ternary fission of target nuclei ^{235}U by slow polarized neutrons [18]. However, another semiclassical model for the source of this asymmetry was proposed based on the rotation of the fissioning nucleus. Thus, the corresponding asymmetry was named the R (Rotation) effect. Formally, it corresponds to 5-fold correlation $(\mathbf{p}_\alpha[\mathbf{s} \times \mathbf{p}_{LF}])(\mathbf{p}_\alpha\mathbf{p}_{LF})$ (which is also T-odd).

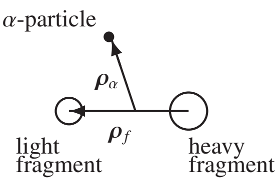

Fig. 4. Two fragments and α-particle arise after ternary fission.

The approach [22] can be extended to the use of hyperspherical harmonics to describe 3 particles in the final state of ternary fission (see Fig. 4). It gives for the double differential fission cross section:

$$\frac{d\sigma}{d\Omega_f d\Omega_\alpha} = \sum_{Q,\Lambda_f,\Lambda_\alpha} \tau_{Q0}(s)\, A(Q,\Lambda_f,\Lambda_\alpha)\, \phi^{Q}_{\Lambda_f\Lambda_\alpha}(\mathbf{n}_s,\mathbf{n}_{LF},\mathbf{n}_\alpha), \tag{12}$$

where $\phi^{Q}_{\Lambda_f,\Lambda_\alpha}(\mathbf{n}_s,\mathbf{n}_{LF},\mathbf{n}_\alpha)$ are invariant spherical functions of unit vectors $\mathbf{n}_s = \mathbf{s}/s$, $\mathbf{n}_{LF} = \boldsymbol{\rho}_f/\rho_f$, $\mathbf{n}_\alpha = \boldsymbol{\rho}_\alpha/\rho_\alpha$ (see [23,9])

$$\phi^{Q}_{\Lambda_f\Lambda_\alpha}(\mathbf{n}_s,\mathbf{n}_{LF},\mathbf{n}_\alpha) = (4\pi)^{3/2} \sum_{q,\lambda_f,\lambda_\alpha} Y^*_{Qq}(\mathbf{n}_s)\, Y_{\Lambda_f\lambda_f}(\mathbf{n}_{LF})\, Y_{\Lambda_\alpha\lambda_\alpha}(\mathbf{n}_\alpha). \tag{13}$$

Here $\tau_{Q0}(s)$ $(Q = 0,1)$ is the spin-tensor of orientation for the incident neutrons, therewith $\tau_{00}(s) = 1$ and $\tau_{10}(s) = p(s)/\sqrt{3}$, where $p(s)$ is the neutron polarization; $A(Q,\Lambda_f,\Lambda_\alpha)$ is a cumbersome factor containing a bilinear combination of the S-matrix elements corresponding to transitions from the entrance neutron channel to the exit fission channels. It is easy to see that the terms in (12) for $Q = \Lambda_f = \Lambda_\alpha = 1$ and $Q = 1$, $\Lambda_f = \Lambda_\alpha = 2$ describe the observed angular correlations in ternary fission. Indeed,

$$\phi^1_{11}(\mathbf{n}_s,\mathbf{n}_{LF},\mathbf{n}_\alpha) \sim (\mathbf{n}_\alpha[\mathbf{n}_s \times \mathbf{n}_{LF}]), \quad \phi^1_{22}(\mathbf{n}_s,\mathbf{n}_{LF},\mathbf{n}_\alpha) \sim (\mathbf{n}_\alpha[\mathbf{n}_s \times \mathbf{n}_{LF}])(\mathbf{n}_\alpha\mathbf{n}_{LF}). \tag{14}$$

Thus, in the proposed approach the 3-fold and 5-fold correlations (T and R effects) in the neutron induced ternary fission are caused by interference of exit channels with different schemes of angular momenta's summation. Survival of the interference effects at summations over all exit states may be due to simplicity of the spin-orbit interaction in the system of two fragments and α-particle.

The other mechanism is responsible for formation of T-odd angular correlations in binary fission where the third particle is a γ-quantum or a neutron emitted by one of fragments (see Fig. 5). Such correlations was discovered in [19,20]. In a sequential process (first form the fragments, and then one

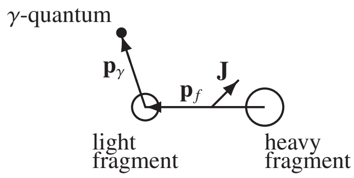

Fig. 5. Two fragment arise after binary fission of polarized nucleus with spin **J**, while a third particle, e.g. a γ-quantum, is emitted by one of the fragments.

of the fragments emits the third particle) the 3-fold T-odd asymmetry $(\mathbf{n}_{TP}[\mathbf{n}_s \times \mathbf{n}_{LF}])$ (T effect) is suppressed by a double parity prohibition (i.e. is practically zero) [24]; here $\mathbf{n}_{TP}$ is the unit vector along the direction of movement of the third particle. But the 5-fold T-odd asymmetry (R effect), $(\mathbf{n}_{TP}[\mathbf{n}_s \times \mathbf{n}_{LF}])(\mathbf{n}_{TP}\mathbf{n}_{LF})$, can exist.

The 5-fold correlation is bilinear in $\mathbf{n}_{TP}$, therefore the fragment's spin alignment is needed to produce the correlation. In the fission process the fragments are really spin aligned. Thus, the mechanism should be only established how the polarization of the fissioning nucleus with spin **J** influences the fragment's spin alignment. This problem as noted above has been considered for the first time in [4] (see also [9]).

The differential probability of the third particle's emission, e.g. the γ-quantum, takes the form

$$\frac{dw}{d\Omega_\gamma} = \sum_{Q,\Lambda,H} \tau_{Q0}(s)\, B(Q,\Lambda,H)\, \phi^{H}_{\Lambda Q}(\mathbf{n}_\gamma, \mathbf{n}_{LF}, \mathbf{n}_J), \tag{15}$$

where $\mathbf{n}_\gamma = \mathbf{p}_\gamma/p_\gamma$, $\mathbf{n}_{LF} = \mathbf{p}_{LF}/p_{LF}$, $\mathbf{n}_J = \mathbf{J}/J$. Here $\tau_{Q0}(J)$ is the spin-tensor of orientation of the fissioning nucleus, therewith $\tau_{00}(J) = 1$ and $\tau_{10}(J) = p(J)\sqrt{J/(J+1)}$, where $p(J)$ is the polarization of compound nucleus after capture of the polarized neutron; $B(Q,\Lambda,H)$ is a cumbersome factor containing a bilinear combination of the amplitudes of fission to the exit states with two fragments. Clearly, the term in (15) corresponding to $Q = 1$, $\Lambda = H = 2$ describes the required 5-fold T-odd angular correlation

$$\phi^2_{21}(\mathbf{n}_\gamma, \mathbf{n}_{LF}, \mathbf{n}_J) \sim (\mathbf{n}_\gamma[\mathbf{n}_J \times \mathbf{n}_{LF}])(\mathbf{n}_\gamma \mathbf{n}_{LF}). \tag{16}$$

It is non-trivial that the factor $B(Q,\Lambda,H)$ comprises the following product of three Klebsh-Gordan coefficients: $C^{Qh}_{Hh\Lambda 0} C^{JK'}_{JKQh} C^{F'K'}_{FKHh}$. Here F and F' are total spins of two fragments, while K and K' are projections of these spins onto the direction $\mathbf{n}_{LF}$ of movement of the light fragment. At summation over all exit channels it would be possible to expect a disappearance of the terms caused by an interference of the exit states with $K \neq K'$. But $h = 0$ when $K = K'$, and then $C^{Q0}_{H0\Lambda 0} = 0$ at $Q = 1$, $\Lambda = H = 2$. Thus, the existence of the small ($\sim 10^{-3}$) 5-fold T-odd angular correlation in binary fission is an evidence for a regular mechanism of small mixing over K in all exit channels.

Our guess is that this mixing is connected with insignificant nonconservation of the quantum number K, caused by small (but nonzero) centrifugal barrier for the fission fragments. Therefore, the smallness of the R effect in binary fission points out the high accuracy of the matching condition $K = \tilde{K}$ at the scission point. Thus, the reliability of our treating of the A. Bohr channels in the frame of Strutinsky's approach is confirmed.

4 Conclusion

Processing the most comprehensive data on angular anisotropy of fragments from fission of spin-aligned nuclei ^{235}U by slow (s-wave) neutrons shows that the A. Bohr's channel $K = 0$ is not forbidden for any J^π but is relatively suppressed. It is due to specific symmetries of the internal wave function related to nuclear forms on first and second fission barriers. The same data for the other target nuclei are needed for consistent description of the $K = 0$ channel in connection with nuclear forms on fission barriers.

Recently discovered T- and R-effects (3- and 5-fold correlations) for the ”true” ternary fission induced by slow polarized neutrons with α-particle’s emission can be caused by specific spin-orbit interaction which mixes 3 particle’s exit states (final state interaction). On the other hand, the R-effect for γ-quanta and neutrons, emitted by a fragment in binary fission induced by slow polarized neutrons, provides a proof of correctness of the inclusion of A. Bohr’s channels into the helicity approach to the description of angular correlations in the fission process.

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