

Foreword

This book proposes a written compilation of the lectures given during the thematic school “Contribution of Symmetries in Condensed Matter”, which was held in May 2009 on the Mediterranean coast at the Belambra club “La Badine”, presqu’île de Giens (Vars, France).

Natural sciences progress by formulating coherent sets of hypotheses that must withstand constant experimental verification. A set of hypotheses is generally considered superior if it offers a lower degree of complexity for the same predictive power. This is why symmetries play such an important role in the description of nature. Symmetry requirements place ad hoc constraints on the system under consideration. Some may seem intuitively evident, like the homogeneity of three-dimensional space, while others may appear extremely sophisticated and highly abstract, like the super-symmetries we find in elementary particle physics. However, what all symmetry concepts have in common is that they establish universal relations between the elements of a system irrespective of how it is actually formed.

Let us take the example of a set of interacting non-relativistic particles. The physics is fully described by the eigenfunctions of the corresponding Hamiltonian operator. If we require homogeneity in time, we can demonstrate immediately that the energy of the system has to be conserved. In the same way, the homogeneity of space leads to the conservation of the total linear momentum, and the isotropy of space to the conservation of total angular momentum. Of course, we could deduce these conservation laws explicitly for a given Hamiltonian by calculating the individual eigenfunctions or infer them by experimental observation. However, it is certainly intellectually more satisfying to deduce them in a universal manner from the postulated symmetry properties of the space in which the physical problem is formulated.

The ad-hoc symmetry requirements that we build into our theories can be described mathematically by algebraic structures that have the properties of groups. The properties of these groups are mirrored in those of matrices, which leads to the concept of representations.

Crystals give typical group structures. If we postulate long-range translational periodicity, then we know that the atomic arrangement has to be described by one of the 230 crystallographic space groups and this independently of the actual interactions between the atoms or molecules. One immediate consequence is the restriction to 2-, 3-, 4- and 6-fold rotation axes. The existence of quasi-crystals does not call the space-group concept into question. Quasi-crystals simply demonstrate that perfect order can be achieved without translational periodicity, for example by the proper tiling of space preserving rotational symmetry. It teaches us that, although symmetry requirements may seem intuitively trivial, their domain of applicability has to be judged carefully. The important distinction between long-range order and periodicity has been acknowledged by the 2011 Nobel Prize in Chemistry awarded to Dan Shechtman for his work on quasi-crystals (Shechtman, D., Blech, I., Gratias, D., and Cahn, J.W. (1984) Metallic phase with long-range orientational order and no translational symmetry, *Phys. Rev. Lett.* 53(20):1951–1954).

The school aimed to offer an introduction to the application of group theoretical concepts to condensed matter problems. Symmetry arguments are so omnipresent in solid-state physics that a one-week school could certainly not cover them all in any depth. Extremely interesting and highly relevant domains, such as the classification of electronic states, therefore had to be excluded in favor of concentrating on a selected number of areas. To consolidate the students’ understanding, the lectures were followed by tutorials and practical training sessions using modern computer tools.

Introductory courses dealing with mathematical groups and group representations set the stage by providing the necessary theoretical background. In the general physics lectures which followed, clear emphasis was given to crystallography (chemical and magnetic) and other related problems, including phase transitions, modulated structures, and phonons. Finally, topic-based lectures established the connection with current mainstream science, such as multiferroics and superconductors. In keeping

with the lectures given at the school, the written versions presented in this book reflect their different styles, which ranged from the very formal to something much closer to a kind of “cooking recipe” for solving specific problems in condensed matter physics. As this is a multi-author volume, it is inevitable that the notations used vary from one chapter to another. A few of the lectures given during the school do not appear in this written version.

The school was aimed at an audience ranging from PhD students to experienced scientists wishing to extend their knowledge of group theoretical applications in condensed matter physics. 98 people attended the school: 20 lecturers/organizers and 78 participants (including 27 PhD and post-doc students). Addressing an audience with varying degrees of prior knowledge is always a particular pedagogical challenge. From the feedback that we received from the participants, both the lectures and the tutorial/practical training sessions were highly appreciated, although the level and density of the material were apparently a little too high.

The school was held in honor of the late E.F. Levy-Bertaut. Professor Bertaut was an outstanding personality who had a tremendous influence on the application of group theory to condensed matter problems and, in particular, magnetic structure refinement.

In conclusion, we would like to thank all the voluntary reviewers: Stéphane Raymond, Frédéric Bourdarot, Pierre Rodière; as well as Brigitte Dubouloz, Christine Martinelli, Martine Giglio, Serge Claisse, Raph, and the staff of the Belambra club “La Badine” for their technical support. We would also like to acknowledge the various sponsors that supported this scientific event. They are listed on page V. Last but not least, this school would not have been possible without its well-chosen teachers, selected with the help of the scientific committee, whose members are listed on page IV.

Béatrice Grenier
Virginie Simonet
Helmut Schober

The slides of the school can be found at the following link:

<http://www.ill.eu/news-events/past-events/2009/ecole-theorie-des-groupes/transparents-cours-td-tp/>