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POWER SPECTRAL DENSITY OF VELOCITY FLUCTUATIONS ESTIMATED FROM PHASE DOPPLER DATA

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Abstract: *Laser Doppler Anemometry (LDA) and its modifications such as Phase-Doppler Particle Anemometry (P/DPA) is point-wise method for optical non-intrusive measurement of particle velocity with high data rate. Conversion of the LDA velocity data from temporal to frequency domain – calculation of power spectral density (PSD) of velocity fluctuations, is a non trivial task due to non-equidistant data sampling in time. We briefly discuss possibilities for the PSD estimation and specify limitations caused by seeding density and other factors of the flow and LDA setup. Arbitrary results of LDA measurements are compared with corresponding Hot Wire Anemometry (HWA) data in the frequency domain. Slot correlation (SC) method implemented in software program Kern by Nobach (2006) is used for the PSD estimation. Influence of several input parameters on resulting PSDs is described. Optimum setup of the software for our data of particle-laden air flow in realistic human airway model is documented. Typical character of the flow is described using PSD plots of velocity fluctuations with comments on specific properties of the flow. Some recommendations for improvements of future experiments to acquire better PSD results are given.*

1. INTRODUCTION

Gas flows are studied using various intrusive as well as non-intrusive techniques. Well established HWA is usually applied only for single-phase flows. Optical non-intrusive methods such as LDA and other methods derived from this one or Particle Image Velocimetry (PIV) based methods are more frequent today. These methods use light scattering on boundaries between two different phases to acquire velocity data. Bubbly flows in two-phase gas-liquid mixture or particle laden gas flows are typical representatives of flows suitable for these optical methods. Classical PIV offers too low repetition rate for PSD estimations, Time resolved PIV, which can run up to several kHz is promising technique for high-frequency (HF) flow studies in the case of good optical access (Uda *et al*., 2011), while LDA is useful also for experiments where the access is not as simple and for even higher data rates. Random appearance of the particles/bubbles in the space results in non equidistant sampling intervals of LDA measurements. Direct application of fast Fourier transform is not suitable for such PSD calculations and other, more advanced methods must be applied in this case.
Number of different techniques was developed to treat the problem of irregular time sampling from the dawn of LDA. It was formerly solved using analogue filters of the velocity signals or by equidistant time re-sampling (Durst *et al.,* 1976). Scott (1974)

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derived an expectation of the spectrum on the basis of a SC which was introduced by Mayo *et al*. (1974) as a means of estimating the autocorrelation function (ACF) of the flow velocity fluctuations. Fourier transform of this symmetrical function yields the power spectral density (PSD). The SC remains even today one of the most viable means for PSD and ACF estimations (Benedict *et al*., 2000). A large group of estimators is based on the concept of signal reconstruction and re-sampling at equal time intervals. The most common reconstruction is zeroth-order interpolation or sample and hold (S+H) method. Other spectral estimator for LDA data was introduced by Nobach *et al*. (1996), based on one-point reconstruction techniques, but employing a refinement which accounts in a statistical manner for the velocity change between a particle arrival and the sample instant. Spectral estimation techniques currently considered the most reliable, are explained in detail by Benedict *et al*. (2000).

We have acquired large set of velocity data using P/DPA on a realistic human airway model using monodispersed micron-size particles suspended in air flow (Jedelsky *et al*., 2011). These data contain intrinsic information on temporal fluctuations of the particle-air two-phase flow. We have employed the SC technique embedded in Kern software (Nobach, 2006) for PSD estimations of velocity fluctuations. This paper is focused on description of Kern processing setup optimization using HWA and LDA data of the same flow and using our arbitrary data acquired by means of P/DPA in the airway model.

2. INPUT PARAMETERS FOR PSD ESTIMATION

The Kern uses a number of options and variables for calculation with no guideline for optimum setup. However calculated PSD is very sensitive to these input parameters.

The program offers usage of two methods, refined reconstruction (RR) and SC techniques. RR was found to generally give worse results at HF part of the PSD spectra than SC for wide range of other parameters so it was not further tested.

Published HWA and LDA data of identical flow (Nobach, 2006) have been used to find out practically the influence of different factors on PSD results. Data segments from both the HWA and LDA data files containing 16384 samples have been used as this sample number is typical for our experiments.

Influence of particular options was investigated using the LDA data file first. Standard setup of the Kern options (SC +FIL -SELF +MBV +VW +LN +FST +FBAT +LTE +CFT) (see Nobach, 2006 for details) for F=50000, K=1024 and N_F=1000 with individual variation of only one option each time was made. Pre-filter off (-FIL) as well as model based variance estimation off (-MBV) (more about MBV in Nobach, 2000) give slightly higher values at HF and better match with the FFT of original data than standard setup. Local normalization off (-LN) as well as fuzzy slotting technique off (-FST) and forward-backward arrival time weighting off (-FBAT) do not differ from the standard setup. See van Maanen *et al.* (1999) and Tummers and Passchier, 1996 for description of LN. Local time estimation off (-LTE) is worse at HF. Selfproducts on (+SELF) gives slightly higher values at HF than standard setup and even better match with the FFT of original data than –FIL. Variable windowing off (-VW) results in fine resolution of amplitudes in the PSD spectra, while standard setup "smears" local peaks; using -VW some dropouts could however appear in the PSD plots. PSD spectra with -VW gives lower limit of minimum frequency (see Tummers and Passchier, 1996 for more information). Continuous Fourier transform off (-CFT) gives often unrealistic results in large part of the HF range of PSD plot.

Several useful setups were found, particularly: a) –FIL +SELF -MBV –VW (PSD with the lowest f_{min} and the highest f_{max}, non smoothed data, possible dropouts), b) –FIL +SELF –MBV (higher f_{min}, smoothed data, no dropouts) and c) +SELF -MBV –VW (higher f_{min}, non smoothed data, smaller dropouts compared to a). Results for these setups are shown at Fig. 1 together with FFT. The setups a) and c) are appropriate when high resolution needed, while b) should be used to eliminate dropouts in the PSD spectra.

LDA file was processed similar way as the HWA file above for F=50000, K=1024 and N_F=1000 with individual and mutual variation of particular options. Setup with –FIL +SELF -MBV –VW overestimates HF part of PSD compared to HWA PSD, –FIL +SELF -MBV also overestimates the HF part but less than the previous case and gives higher f_{min} (Fig. 2), setup with -FIL -MBV –VW reduces dropouts and generally leads to better results than all other test cases (–FIL –VW, -MBV –VW, –VW, +SELF –VW, +SELF -MBV –VW). Also common FFT without oversampling was used for the LDA data. It gives PSD with overestimated levels at HF due to added noise. This noise comes from a jitter from incorrectly used timing of the measured data for FFT. Resampling of the original data could improve the results but the resampled record still differs from real data.

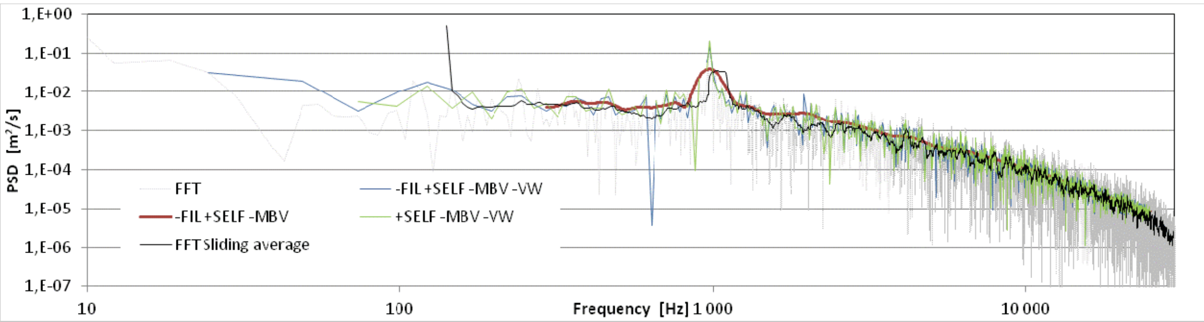

Figure 1: HWA data processed different ways.

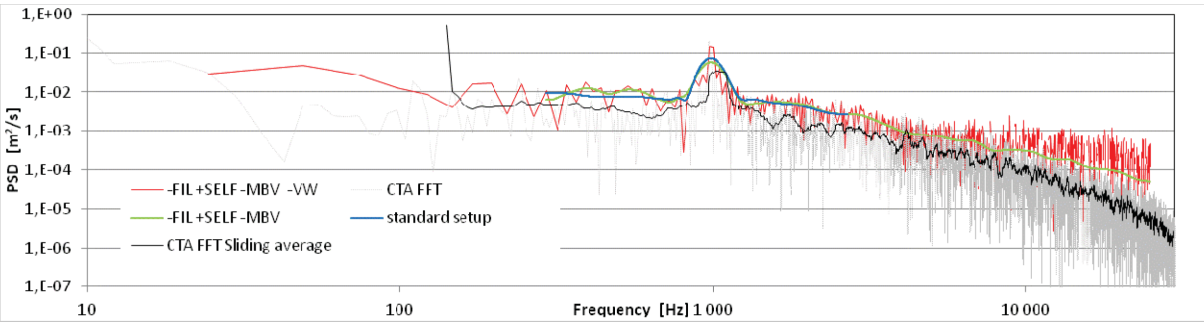

Figure 2: Comparison of HWA and LDA data.

Kern uses three input variables, F, K and N_F. F is the frequency which defines the time lag between samples in the autocorrelation function. K is the number of samples in the autocorrelation function; it corresponds to the resolution of the spectrum. N_F is the number of samples in the spectrum if the VW technique is used. In this case the spectrum is sampled logarithmic way. Without the VW this number is given through the K number and N_F can have any value (Nobach, 2006).

N_F can have any value, for number of samples n=16384 N_F =1000 is usually enough. F can be any natural number, but always $F < 1/\Delta t$, where mean interval between samples

$\overline{\Delta t} = T/n$ depends on measurement period *T* and number of samples *n*. Greater *F* means greater f_{min} and greater f_{max}, in the PSD.
The number of ACF samples, *K*, must be a natural multiple of 2, always K< N_F, preferably sufficiently lower also depending on *F*. Higher *K* means higher f_{min}, so several trials with variable *K* are recommended to find an optimum. Standard Kern setup with SC gives f_{min}=6*F*/*K* and f_{max}=*F*/2.

3. PSD ESTIMATES OF REAL DATA

After successful agreement between these HWA and LDA results we have focused on our P/DPA data. A set of three consequent data records was chosen to further optimise the PSD estimation process. The data were acquired in point A for steady flow regime 30 l/min and 1 µm particles (Fig. 3). Each file has 16384 samples and measurement period 5.8 s (average sampling frequency 2825 Hz).
Fig. 4 shows PSD of the velocity fluctuations versus frequency, *f*, using log-log plot. Average PSD curve based on the three records together with standard deviation is calculated. We have realised that any unique setup does not produce the PSD in whole frequency range provided by the measured data. So each spectra is composed of two curves; the lower frequency part (3.3 – 35 Hz) was calculated here using: SC, *F*=70, *K*=128, N_F=1000 and standard options, the upper frequency part was calculated as: SC, *F*=5000, *K*=2048, N_F =1000, standard options. Individual PSD plots (not shown here) are very consistent in the frequency range 15 – 750 Hz as indicated by the standard deviation curves. Fluctuations in amplitude that were observed in each PSD plot change record to record and therefore are not inherent property of the flow. Low frequency fluctuations in the PSD amplitude (up to about 15 Hz) are a product of natural variations of flow that cannot be fully reflected by this relatively short data record. The results differ significantly also for frequencies higher than about 1 kHz due to lack of useful data from P/DPA measurement. The differences can be partially caused also by the processing algorithm. This part of plot is not realistic and cannot be used for analysis. VW used in this calculation leads to smoothening and averaging of fluctuations in individual spectral lines. Larger fluctuations are preserved. Amplitude of velocity fluctuation in the inspected point shows only very mild decreasing tendency with frequency up to about 30 Hz, then slightly higher and relatively constant-slope decrease according $f^{-2/3}$ in range 30 – 300 Hz and more distinct decrease with constant slope $f^{-7/3}$ for frequencies higher than 300 Hz. No distinct peak in the PSD is obvious. Very similar shape of the PSD was found also at other

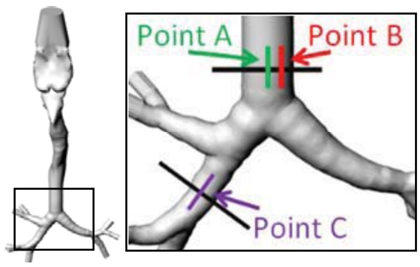

Figure 3: Airway model with measurement positions in expanded window.

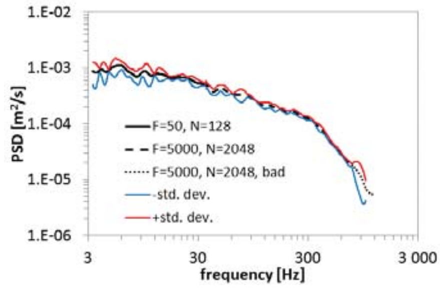

Figure 4: Average PSD with marked standard deviation limits.

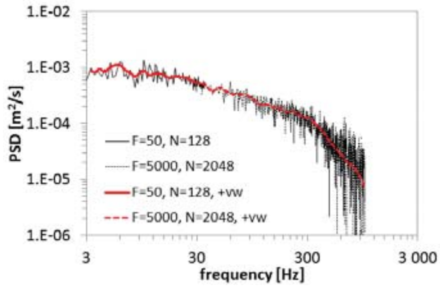

Figure 5: PSD with/without VW, average of three records.

measurement points, for other particle sizes and other flow regimes in trachea.
Cumulative turbulence kinetic energy (TKE) was calculated for given data. Amplitude of oscillations at frequencies higher than 800 Hz was approximated using the $f^{-7/3}$ rule. Low frequency oscillations up to 140 Hz participate on 50 % of the total TKE and 90 % is covered in range up to 620 Hz.
Calculation made with the same setup as previous results but without VW produced PSD plots containing strong scatter in the amplitude. No significant coherence of this "noise" in spectra among different records was found. Averaging of PSD from the three records (Fig. 5) reduces this scatter so processing of multiple data files or long measurements is important for statistically correct results. VW is even stronger instrument for the data smoothening mainly at high frequencies as indicated in the plot.
Any unique setup does not allow for PSD estimation in full possible frequency range. Combination of two spectra was found as optimum way, where the first one is produced by processing of the data using the highest possible F and the second with the lowest possible F/K and consequently with the lowest possible F; for this arbitrary case was F_1=10000, K_1=1024 and F_2=50, K_2=128.
More analysed data files in several measurement points for three different steady and cyclic breathing regimes and particles of several sizes can be found in paper Jedelsky *et al*. (2011).

4. Ways to Increase Frequency Range

As shown above PSD spectra contain important information on the velocity fluctuations, which spans over frequencies at several orders, but quality of the results and upper limit of the frequency range depends on data rate of the P/DPA measurement.
Typical concentration c of the aerosol particles in air is in range 10^3 – 10^5 particles/cm^3 for our case of aerosol generation using CMAG generator (Jedelsky *et al*. 2011). This relatively dilute aerosol (mean free path in order of hundreds particle diameters) enables measurement of the velocity v with mean data rate

$$\dot{n} = \overline{v} \cdot \overline{c} \cdot S \cdot k \,, \tag{1}$$

where S is area of the measurement volume projected in the flow direction, $\overline{v}$ is mean particle velocity and yield k depends on visibility of droplets by P/DPA and its validation rate.

Maximum reliable frequency of calculated PSD is about $\dot{n}/2\pi$ according to Adrian and Yao (1987). Presuming an equal particle distribution in space with constant concentration and mean data rate of the measurement $\dot{n}$, the intervals Δt between the particles are distributed exponentially:

$$p(\Delta t) = \dot{n} \cdot e^{-\dot{n}\Delta t} \,, \tag{2}$$

see Fig. 6, left. The most probable interparticle arrival time gaps are zero so information about very HF fluctuations is contained in the data.

LDV or P/DPA is able to measure only one particle at given time; if the measurement system detects a particle, than other particles appearing in the measurement volume at the same time or flowing in after the first one will be rejected from evaluation. Maximum measurement frequency would be (for equidistant sampling):

$$\dot{n}_{max} = \frac{1}{\Delta T} = \frac{v}{a} \tag{3}$$

where a is dimension of the measurement volume along the flow direction. This dimension is equal to diameter of incident laser beam, D.

Measurement volume given by intersection of two Gaussian beams has a shape of ellipsoid; this volume is seen by receiving optics through a slit which reduces length of the ellipsoid. For the simplest arrangement case when receiver axis is perpendicular to transmitter axis resulting volume has approximately shape of a cylinder with diameter D and length s. Size and shape of the measurement volume influences resulting data rate $\dot{n}$. Increase in slit size, s, will directly enlarge the area of measurement volume, which is approximately $S = s \cdot D$. It will lead to increase in data rate according to Eq. (1). This enlargement in S will however also reduce size resolution of the measurement.

Another relatively simple way to modify the measurement volume size is to change the laser beam diameter. Enlargement of the measurement volume by increase in D will increase the size a and consequently reduce maximum frequency (Eq. (3)) but also increase the measurement cross section S and therefore increase number of detected particles with interparticle arrival time gaps that are larger than ΔT. Simultaneously this particle number will appear with shorter interparticle arrival time as seen at Fig. 6 left. For example doubling D will also double up $p(\Delta t)$ (Eq. (2)) and simultaneously will twice reduce corresponding Δt. Results of this change can be seen at Fig. 6, right, where optimum beam diameter found has about 130 % of the original beam size (0.3 mm). The data rate near its maximum drops slowly so small beam diameter variation is not critical but too large D would significantly reduce the data rate.

Particle concentration is limited by the generator used and also by the desired character of the flow. Larger concentration is generally better, but the two-phase mixture would change from dilute to dense with increase in the concentration which could affect the particle flow.

The yield k is given by the measurement system setup and its technical condition. Also monodispersity of the seeding particles could have an effect. Detector sensitivity of measurement system measuring polydisperse aerosol must be set to a value at which large particles will not cause the detector saturation yet and in such case small particles could not obey detection limit.

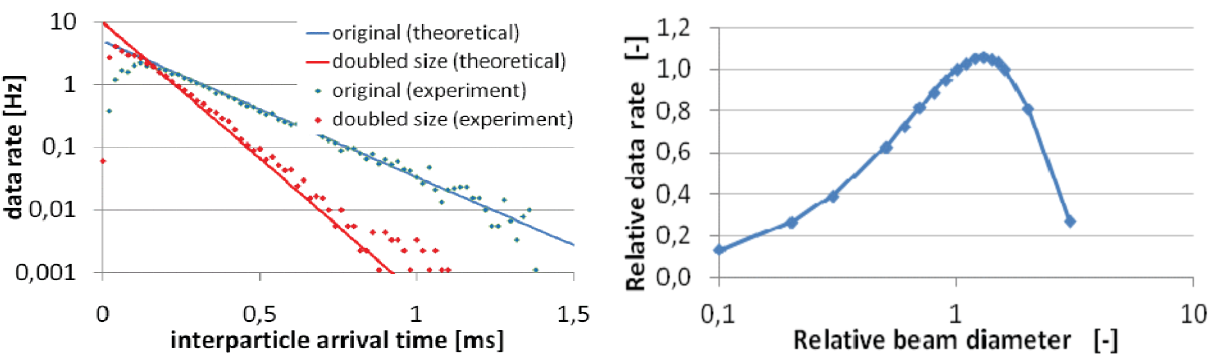

Figure 6: Frequency plot of arrival time between two particles for 1 µm particles and flow rate 30l/min in point A (left), influence of the beam size on the data rate (right).

5. Conclusions

Kern software with SC technique was used for PSD estimations of velocity fluctuations of P/DPA data. At first optimum setup of different options of the program was found using HWA and LDA data of the same flow. The setup optimization was finished using our arbitrary data acquired using P/DPA in human airway model. Specific character of the flow in airways is documented using PSD of velocity fluctuations. Possibilities to increase frequency range of the PSD results were analysed and some recommendations given.

6. Acknowledgements

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