NUMERICAL SOLUTION OF COMPRESSIBLE STEADY FLOWS IN A 2D GAMM CHANNEL AND DCA 18% PROFILE

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Abstract: The article presents results of a numerical solution of subsonic and transonic flows described by the system of Euler equations in 2D flows in a channel and around a profile. Authors used Lax-Wendroff scheme to numerically solve the flows in a GAMM channel and around half DCA 18% profile. Authors programmed the mesh generator of the type C for profile with a blunt leading edge.

1. INTRODUCTION

A numerical code has been developed for simulating transonic flow field in GAMM channel and around half DCA 18% profiles. For future simulating mesh generator type C has been developed. In this case mesh has been created for numerical solution over profile NACA 0012.

2. MATHEMATICAL MODELS

The 2D flow of an inviscid compressible fluid is described by the system of Euler equations.

$$W_t + F(W)_x + G(W)_y = 0 \quad (1)$$

where

$$W = \begin{bmatrix} \rho \\ \rho w_1 \\ \rho w_2 \\ e \end{bmatrix}, \quad F(W) = \begin{bmatrix} \rho w_1 \\ \rho w_1^2 + p \\ \rho w_1 w_2 \\ (e + p)w_1 \end{bmatrix}, \quad G(W) = \begin{bmatrix} \rho w_2 \\ \rho w_1 w_2 \\ \rho w_2^2 + p \\ (e + p)w_2 \end{bmatrix}, \quad (2)$$

$$p = (\kappa - 1) \left[e - \frac{1}{2} \rho (w_1^2 + w_2^2) \right]. \quad (3)$$

In the above equations, W is conservative variable, F , G are function of inviscid physical fluxes, ρ denotes density, w_1 , w_2 are components of velocity in the direction of axis x , y , p is pressure, e is total energy per unit volume. The parameter $\kappa = 1.4$ is the adiabatic exponent.

3. SPECIFICATION OF TEST CASES

We selected for numerical solution a structured mesh formed by quadrilateral finite volumes. Mesh for GAMM channel with selecting parameters is presented in Fig. 1. Mesh with half DCA 18% profile with selecting parameters is presented in Fig. 2. A program for generating mesh makes it possible any thickening of mesh in x direction. In y direction is controlled density only one parameter.

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Lax-Wendroff scheme in McCormack's modification (LW-MC) of the finite volume method is used on non-orthogonal structured grids of quadrilateral cells D_{ij} . In predictor part author used forward differencing and in corrector part backward differencing.

$$W_{i,j}^{n+1/2} = W_{i,j}^n - \frac{\Delta t}{\mu_{i,j}} \sum_{k=1}^4 \left(\tilde{F}_k^n \Delta y_k - \tilde{G}_k^n \Delta x_k \right) \quad (4)$$

$$W_{i,j}^{n+1/2} = W_{i,j}^n - \frac{\Delta t}{\mu_{i,j}} \sum_{k=1}^4 \left(\tilde{F}_k^n \Delta y_k - \tilde{G}_k^n \Delta x_k \right) \quad (4)$$

Corrector:

$$W_{i,j}^{n+1} = \frac{1}{2} \left(W_{i,j}^n + W_{i,j}^{n+1/2} - \frac{\Delta t}{\mu_{i,j}} \sum_{k=1}^4 \left(\tilde{F}_k^{n+1/2} \Delta y_k - \tilde{G}_k^{n+1/2} \Delta x_k \right) \right) + AD(W_{i,j}^n) \quad (5)$$

The Jameson's artificial dissipation AD damps undesirable oscillations and improves the stability of the method

$$AD(W_{i,j}^n) = k_1 \gamma_i (W_{i+1,j}^n - 2W_{i,j}^n + W_{i-1,j}^n) + k_2 \gamma_j (W_{i,j+1}^n - 2W_{i,j}^n + W_{i,j-1}^n) \quad (6)$$

$$\gamma_i = \frac{|p_{i+1,j}^n - 2p_{i,j}^n + p_{i-1,j}^n|}{|p_{i+1,j}^n| + 2|p_{i,j}^n| + |p_{i-1,j}^n|}, \quad \gamma_j = \frac{|p_{i,j+1}^n - 2p_{i,j}^n + p_{i,j-1}^n|}{|p_{i,j+1}^n| + 2|p_{i,j}^n| + |p_{i,j-1}^n|} \quad (7)$$

The convergence to the steady state is followed by log L2 residual defined by

$$\text{Rez} W_{i,j}^n = \frac{1}{N} \sqrt{\sum_{i,j} \left(\frac{W_{i,j}^{n+1} - W_{i,j}^n}{\Delta t} \right)^2} \quad (8)$$

where N is a number of all elements in the computational domain.

At inlet part set values are considered for flow at infinity. At outlet p is given and other values are extrapolated. On wall zero derivatives of velocity vector along normal is considered.

$$\frac{\delta \mathbf{w}}{\delta \mathbf{n}} = 0 \quad (9)$$

The initial conditions must agree with request of even input approaching flow. That was defined by Mach number Ma , size of density and absolute size of velocity, where α is an angle between centers neighboring cells in x direction.

$$W = \begin{bmatrix} 1 \\ Ma \cdot \cos(\alpha) \\ Ma \cdot \sin(\alpha) \\ \frac{1}{\kappa(\kappa-1)} + \frac{1}{2} Ma^2 \end{bmatrix} \quad (10)$$

5. NUMERICAL RESULTS

For a 2D numerical simulation of flows of an inviscid compressible fluid in the **GAMM channel** and around half DCA 18% profile, the authors applied **LW** numerical schemes in McCormack's modification on a structured grid with **400 x 200** cells.

In the case of GAMM channel results (Fig. 3) are in very good agreement with the results from the author Fürst, 2004 [12] (Fig. 4) with computational grid 90 x 30 cells. Positions of the shock wave (with an obvious of Zierp singularity) are the same. The maximum Mach number in our case is higher than [12] ($M_{\max} = 1.4298 \times M_{\max} = 1.33$).

In the case of half DCA 18% profile results are shown on Fig 5 and Fig.6. The result of pressure coefficient C_p (Fig. 6) is compared with G. S. Deiwert obtained by finite-difference method with computational grid 50 x 38 cells. Progress of the pressure coefficient C_p for inviscid flow from Deiwert is coarser and does not show details that are visible in our results.

Figure 3: Inviscid compressible flow in the GAMM channel: Mach number isolines (top) and Mach number on bottom and upper wall (bottom) and L_2 logarithmic residuals (right) at $Ma = 0.675$ LW-MC scheme, $M_{\max} = 1.4298$, structured grid with 400×200 elements.

Figure 4: Inviscid compressible flow in the GAMM channel: Mach number on bottom and upper wall - Fürst, 2004 [12]

Figure 5: Inviscid compressible flow around half DCA 18% airfoil: Mach number isolines (top) and Mach number on upper airfoil (bottom) and L_2 logarithmic residuals (right) at $Ma = 0.775$ LW-MC scheme, $M_{\max} = 1.5453$, structured grid with 400×200 elements.

Figure 6: Pressure distribution over DCA 18% (left) – Deiwert, 1975 [7] (right)

6. MESHES PREPARATION OF PROFILES WITH A BLUNT LEADING EDGE

The grid around profiles (wing) usually consists of a C-grid in the flow direction (Fig. 7.).

Figure 7: Structured surface and volume grid of a wing-body configuration (Courtesy O. Brodersen, DLR, Germany)

In the case of the C-topology (Fig. 7.) the aerodynamics body is enclosed by one family grid lines, which also form the wake region. The situation is sketched in Fig. 8.

Figure 8: C-grid topology in 2D (by Blazek [8])

The initial grid is generated algebraically by using the linear **TFI method** [8 - pg. 382], [9]. Afterwards, **elliptic PDE's** are employed to produce boundary-orthogonal grid with specific wall spacing. The **NACA0012** airfoil contour is approximated by a **Bezier spline** [8 - pg. 384], [10], [11].

Examples of grid created by ours program were presented in Fig. 9.

Figure 9: *Inviscid a viscid C-type mesh of NACA 0012 airfoil*

7. CONCLUSIONS

Numerical solution has been applied on structured meshes with 400x200 elements. The LW-MC scheme has been used for GAMM channel and half DCA 18% airfoil. Then results were compared with other authors [7], [12]. The mesh generator of the type C for profile with a blunt leading edge has been programmed.

ACKNOWLEDGMENTS

The work was partly supported by the grant project SGS2823 and Research Plan MSM 6840770010.

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