DETERMINE THE STABILITY OF A VIBRATING TUBE

Pavel ŽITEK, Petr MILČÁK, Jiří LINHRAT*

Abstract: *The aim of this numerical study is to determine hydrodynamic force couple on tubes in a cross-flow row resulting from the water flow and to determine the stability of one vibrating tube. This work is a numerical simulation of S. S. Chen's experiment [1]. The experiment was carried out in a water channel with a row of cross-flow tubes. A part of the channel with a row of tubes is used for the numerical model. The middle tube oscillates in a direction normal to the fluid flow. The paper presents the results of the numerical simulation. The influence of the flow velocity on the excitation of hydrodynamic forces and on the stability of the vibrating tube is described. The results will be compared with the experimental data.*

1. INTRODUCTION

The numerical study concerns a vibrating tube in a tube bundle. Its purpose is to explore the influence of the velocity, the oscillation frequency and displacement amplitude on the tube's stability.

The problem was treated with the aid of CFD simulations. The influence of the turbulence model and of the mesh type used was monitored. Two techniques for evaluating the results are under preparation. Up to now, the following turbulence models have been used: $k-\omega$ SST, $k-\epsilon$ RNG and Spalart-Allmaras model. Five mesh types, each with a different size of triangular cells in the deformed zone have been used. The mesh deformation method presently used is flexible mesh smoothing. The use of local remeshing is planned. The results are evaluated using two different techniques.

2. GEOMETRY AND COMPUTATIONAL MESH

As mentioned above, the present geometry replicated the one used in Chen's experiment (see Figure 1). It is a channel with a tube row spaced at $T/D = 1.35$. Water flows through the channel. The tube row consists of a total of seventeen tubes (see Figure 2). The middle tube is in forced oscillation perpendicular to the flow direction. Hydrodynamic forces acting on the oscillating tube and its neighbours are investigated [5].

The effects on calculation stability and the results are monitored in the calculations. The impact of geometry and the computational mesh is also tracked. The currently modelled region has a total length of five metres. Up to now, five types of meshes with various numbers of elements have been designed. The deformation region is represented by triangular cells in all these meshes. The optimal one, which is used at present, has about 930,000 elements.

* Pavel Žitek, University of West Bohemia, Univerzitní 8, 306 14, Plzeň, zitek@kke.zcu.cz
Petr Milčák, University of West Bohemia, Univerzitní 8, 306 14, Plzeň, pmilcak@kke.zcu.cz
Jiří Linhart, University of West Bohemia, Univerzitní 8, 306 14, Plzeň, linhart@kke.zcu.cz

Given the size of the objects and the number of cells, the calculation is rather time-consuming. The last stage of a single calculation takes between two weeks and two months to complete. For this reason, a reduced-length geometry of 2.5 metres is under preparation.

Figure 1: Layout of experiment

Figure 2: Detailed view of the tube row

Further geometry simplifications are envisaged for the future. A comparison between results for individual geometry variants is expected to provide an optimum geometry which would be used for 3D calculations.

3. DYNAMIC MESH

FLUENT employs dynamic mesh for locations where, due to boundary movement, the shape of the computed region changes over time. The motion may be specified (through velocity or angular speed at the body's centre of gravity) or unspecified, where the resulting movement will be obtained from the real time solution (e.g. the velocity and angular speed will be obtained from the balance of forces acting on the solid body).

Updating the mesh volume is controlled automatically by FLUENT at each time increment according to the new boundary positions. The use of the dynamic mesh model requires that the initial condition of the mesh is specified and the motion of zones within the model is described. In FLUENT, it is possible to characterize the motion by means of a user-defined function (UDF).

Figure 3: Example mesh in the vicinity of an oscillating tube and one of the neighbouring tubes

FLUENT assumes that the motion will be specified either for the surface or for a zone of cells. If the model contains both moving and stationary regions, FLUENT needs to identify and classify them into relevant groups within the original volume of the generated mesh. Regions deformed by the movement must be combined into separate zones as early as within the very first mesh volume. The boundaries between the regions need not match. Unconventional or sliding interfaces can be used, which allow FLUENT to join different regions in the final model.

FLUENT offers three mesh motion methods [2]. They allow the volume mesh to be updated in the deformed region in response to the motion specified on the boundaries:

- Spring-based smoothing
- Dynamic layering
- Local remeshing

At present, only the spring-based smoothing method is used. It is relatively time-effective but it can only be used for small displacements. The local remeshing method is being prepared for future application. It is suitable for large displacements but the calculation becomes more time-consuming. Once the spring-based smoothing method calculations are complete for all variants, the local remeshing method will be applied and the results of these two will be compared. This will reveal the influence of the mesh motion method.

Spring-Based Smoothing Method

In the spring-based smoothing method, the edges between any two mesh nodes are idealized as a network with spring connections. The initial layout of the edges before any

boundary motion constitutes the equilibrium state of the mesh. A displacement at a given boundary node will generate a force proportional to the displacement along all the springs connected to the node. Using Hooke's law, the force acting on a mesh node can be written as:

$$\vec{F}_i = \sum_j^{n_i} k_{ij} (\Delta \vec{x}_j - \Delta \vec{x}_i) \quad (1)$$

where $\Delta \vec{x}_i$ and $\Delta \vec{x}_j$ are displacements of node i and its neighbour j , n_i is the number of neighbouring nodes connected to node i and k_{ij} is the spring constant between nodes i and j . The spring constant or stiffness for the connection between nodes i and j is defined as:

$$k_{ij} = \frac{1}{\sqrt{|\vec{x}_i - \vec{x}_j|}} \quad (2)$$

At equilibrium, the resulting force on a node due to all its elastic connections must be zero. This condition requires an iterative solution of the following equation:

$$\Delta \vec{x}_i^{m+1} = \frac{\sum_j^{n_i} k_{ij} \Delta \vec{x}_j^m}{\sum_j^{n_i} k_{ij}} \quad (3)$$

As displacements at the boundaries are known (node positions have been updated), the equation (3) can be solved using Jacobi sweep on all interior nodes. At convergence, the positions are updated so that

$$\vec{x}_i^{m+1} = \vec{x}_i^m + \Delta \vec{x}_i^{m,converged} \quad (4)$$

where $m+1$ and m denote positions at the following and the current time step, respectively. The spring-based smoothing method is illustrated in Figure 4 and 5 for a cylindrical cell zone where one end of the cylinder is moving.

Application of the Spring-Based Smoothing Method

Spring-based smoothing can be used for updating any cell or zone whose boundary is moving or deforming.

For non-tetrahedral cell zones (non-triangular zones in 2D), this method is only recommended if the following conditions are fulfilled:

- The boundary of the cell zone moves predominantly in one direction (e.g. no excessive anisotropic stretching or compression of zone cells).
- The motion takes place predominantly in the direction of the normal to the zone boundary.

If these conditions are not met, the resulting cells may have a high skewness value, which might affect the quality of the calculation.

Spring-based smoothing for zones with non-tetrahedral or non-triangular cells is disabled by default. For using this method on meshes with other types of cells, the model must be activated.

Figure 4: Spring-based smoothing: start

Figure 5: Spring-based smoothing: end

4. CFD CALCULATIONS

In the present problem, the magnitude of hydrodynamic forces and the stability of the tube in the bundle were investigated. Their dependence on the flow velocity, the frequency of oscillation and amplitude of the middle tube was examined. In addition, efforts were made to identify a suitable mathematical model of turbulence for this type of problem [3].

At present, the impact of the oscillation frequency on the magnitude of the resulting hydrodynamic force is investigated. The entrance velocity of the flowing medium is 0.032 m/s. The average gap velocity is 0.125 m/s. The amplitude of the excited tube is 1.714 mm. All frequencies used are listed in the following table.

Variant no.		1	2	3	4	5	6
frequency [s^{-1}]		1.892	1.789	1.668	1.562	1.469	1.348
reduced velocity [1]		2.6	2.75	2.95	3.15	3.35	3.65
7	8	9	10	11	12	13	14
1.245	1.144	1.047	0.834	0.723	0.630	0.417	0.208
3.95	4.3	4.7	5.9	6.8	7.8	11.8	23.6

Figure 6: Graphs of drag and lift forces for all three turbulence models at 1.469 Hz

The reduced velocity is given by the equation:

$$U_R = \frac{U}{f \cdot D} \quad (5)$$

where: U_R ... reduced velocity

U gap velocity

f oscillation frequency

D tube diameter

Along with finding the frequency, the suitability of the turbulence model was examined. Results obtained using Spalart-Allmaras, $k-\varepsilon$ RNG and $k-\omega$ SST models were compared.

Graphs in Figure 6 show plots of results for all three turbulence models for variant no. 5 (frequency of 1.469 Hz). The tube length was 0.381 m.

5. INVESTIGATION OF TUBE OSCILLATION STABILITY

The movement of the tube is defined by the trigonometric sine function:

$$h = A_h \cdot \sin(\omega \cdot t + \phi_1) \quad (6)$$

where: h ... is the instant magnitude of the excitation signal

A_h is the excitation signal amplitude

ω is the angular speed

t is the time of oscillation

ϕ_1 is the phase shift of the excitation signal

The force on the tube consists of the mean force exerted by the water flow and of the force caused by the oscillation. In addition, the phase shift of the force acting on the tube and the phase shift of the excitation signal should be taken into account. The force acting on the tube can be written as:

$$F = F_S + A_F \cdot \sin(\omega \cdot t + \phi_2) \quad (7)$$

where: F ... is the total force on the tube

F_S is the mean force exerted by the water flow

A_F is the amplitude of the force on the tube

ϕ_2 is the phase shift of the force on the tube

The stability of oscillation is assessed on the basis of the resulting work [4], determined from the force on the tube surface. Where work performed during a single cycle is positive, the tube becomes unstable. If work performed in the course of one cycle is negative, the tube oscillation is stable.

$$W = \int_T F dh \quad (8)$$

where: W is the work performed during a cycle

F is the force acting on the tube

h is the tube displacement

$$dh = A_h \cdot \omega \cdot \cos(\omega \cdot t + \phi_1) dt \quad (9)$$

Using these in equation (8) results in:

$$W = \int_T [F_S + A_F \cdot \sin(\omega \cdot t + \phi_2)] \cdot [A_h \cdot \omega \cdot \cos(\omega \cdot t + \phi_1)] \cdot dt \quad (10)$$

After basic rearrangement and application of trigonometric formulas, the resulting equation is as follows:

$$\begin{aligned}
W = & F_S \cdot A_h \cdot \omega \cdot \cos \phi_1 \cdot \int_T \cos(\omega \cdot t) \cdot dt - F_S \cdot A_h \cdot \omega \cdot \sin \phi_1 \cdot \int_T \sin(\omega \cdot t) \cdot dt + \\
& + A_F \cdot A_h \cdot \omega \cdot \cos \phi_1 \cdot \cos \phi_2 \cdot \int_T \sin(\omega \cdot t) \cdot \cos(\omega \cdot t) \cdot dt - \\
& - A_F \cdot A_h \cdot \omega \cdot \sin \phi_1 \cdot \cos \phi_2 \cdot \int_T \sin^2(\omega \cdot t) \cdot dt + \\
& + A_F \cdot A_h \cdot \omega \cdot \cos \phi_1 \cdot \sin \phi_2 \cdot \int_T \cos^2(\omega \cdot t) \cdot dt - \\
& - A_F \cdot A_h \cdot \omega \cdot \sin \phi_1 \cdot \sin \phi_2 \cdot \int_T \sin(\omega \cdot t) \cdot \cos(\omega \cdot t) \cdot dt
\end{aligned} \tag{11}$$

Rearranging individual terms:

$$\begin{aligned}
\int_0^T \cos(\omega \cdot t) \cdot dt &= \left. \begin{array}{l} \omega \cdot t = K \\ \omega \cdot dt = dK \\ t = 0 \dots K = 0 \\ t = T \dots K = \omega \cdot t = 2\pi \cdot f \cdot T = 2\pi \cdot \frac{1}{T} \cdot T = 2\pi \end{array} \right| = \\
&= \int_0^{2\pi} \frac{1}{\omega} \cos K \cdot dK = \frac{1}{\omega} \left| \sin K \right|_0^{2\pi} = (0 - 0) = 0
\end{aligned} \tag{12}$$

$$\int_0^T \sin(\omega \cdot t) \cdot dt = \int_0^T \frac{1}{\omega} \sin K \cdot dK = \frac{1}{\omega} \left| -\cos K \right|_0^{2\pi} = (-1 - (-1)) = 0 \tag{13}$$

$$\begin{aligned}
\int_0^T \sin(\omega \cdot t) \cdot \cos(\omega \cdot t) \cdot dt &= \int_0^{2\pi} \frac{1}{\omega} \sin K \cdot \cos K \cdot dK = \left| \sin x \cdot \cos x = \frac{\sin 2x}{2} \right| = \\
&= \frac{1}{\omega} \int_0^{2\pi} \frac{\sin 2K}{2} dK = \frac{1}{2\omega} \int_0^{2\pi} \sin 2K \cdot dK = \left| \begin{array}{l} 2K = y \quad K = 0 \dots y = 0 \\ 2dK = dy \quad K = 2\pi \dots y = 4\pi \\ dK = \frac{1}{2} dy \end{array} \right| = \\
&= \frac{1}{4\omega} \int_0^{4\pi} \sin y \cdot dy = \frac{1}{4\omega} \left| -\cos y \right|_0^{4\pi} = -\frac{1}{4\omega} (1 - 1) = 0
\end{aligned} \tag{14}$$

$$\begin{aligned}
\int_0^T \sin^2(\omega t) \cdot dt &= \left| \begin{array}{l} \cos^2 x + \sin^2 x = 1 \\ \cos^2 x - \sin^2 x = \cos 2x \\ 2 \cdot \sin^2 x = 1 - \cos 2x \\ \sin^2 x = \frac{1 - \cos 2x}{2} \end{array} \right| = \int_0^T \frac{1 - \cos 2(\omega t)}{2} \cdot dt = \\
&= \left| \begin{array}{l} 2 \cdot \omega \cdot t = K \quad t = 0 \dots K = 0 \\ 2 \cdot \omega \cdot dt = dK \quad t = T \dots K = 2 \cdot 2\pi \cdot \frac{1}{T} \cdot T = 4\pi \\ dt = \frac{1}{2\omega} dK \end{array} \right| = \frac{1}{2\omega} \int_0^{4\pi} \frac{1 - \cos K}{2} \cdot dK = \\
\frac{1}{2\omega} \int_0^{4\pi} \frac{1}{2} \cdot dK - \frac{1}{2\omega} \int_0^{4\pi} \frac{1}{2} \cos K \cdot dK &= \frac{1}{4\omega} \cdot 4\pi - \frac{1}{4\omega} \cdot \left| \sin K \right|_0^{4\pi} = \frac{\pi}{\omega}
\end{aligned} \tag{15}$$

$$\begin{aligned}
\int_0^T \cos^2(\omega t) \cdot dt &= \int_0^T \frac{1 + \cos 2(\omega t)}{2} \cdot dt = \\
&= \left| \begin{array}{l} 2 \cdot \omega \cdot t = K \quad t = 0 \dots K = 0 \\ dt = \frac{1}{2\omega} dK \quad t = T = 4\pi \end{array} \right| = \frac{1}{2\omega} \int_0^{4\pi} \frac{1 + \cos K}{2} \cdot dK = \\
\frac{1}{4\omega} \int_0^{4\pi} dK + \frac{1}{4\omega} \int_0^{4\pi} \frac{1}{2} \cos K \cdot dK &= \frac{\pi}{\omega}
\end{aligned} \tag{16}$$

Using the adapted terms back in equation (11) results in:

$$\begin{aligned}
W &= -A_F \cdot A_h \cdot \omega \cdot \sin \phi_1 \cdot \cos \phi_2 \cdot \frac{\pi}{\omega} + A_F \cdot A_h \cdot \omega \cdot \cos \phi_1 \cdot \sin \phi_2 \cdot \frac{\pi}{\omega} = \\
&= A_F \cdot A_h \cdot \pi \cdot (\cos \phi_1 \cdot \sin \phi_2 - \sin \phi_1 \cdot \cos \phi_2) = \\
&= A_F \cdot A_h \cdot \pi \cdot \sin(\phi_2 - \phi_1) = W_{CYKLU}
\end{aligned} \tag{17}$$

6. CONCLUSIONS

Comparison of calculations for five variants of computational meshes reveal the impact of cell size. The greatest difficulties with the mesh occurred in the area of the deformed mesh where deterioration of cells is significant even with at small amplitudes. This can be adjusted by suitable setting of coefficients used for deformation.

Of the turbulence models used, k-epsilon RNG and Spalart-Allmaras proved suitable. The values obtained with these models match well, apart from a few exceptions (see Figure 7). The k-omega SST model yields different values whose scatter is considerable.

The present geometry is rather large. The computation is therefore very time-consuming. Duration of one calculation ranges between 2 weeks and 3 months, depending on the selected oscillation frequency. For future work, the size is expected to be reduced and 3D calculations to be included.

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