

Simulating rate dependent spalling with an overstress model

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Abstract. It has been known for a long time that spalling (dynamic tensile failure) is a rate dependent process. Spall strength of metals is determined from the pullback velocity of the free surface velocity history in planar impact spalling tests. Conducting these tests for different volume strain rates in the tension zone, it was established that spall strength increases with strain-rate according to a power law: $\text{Strength}_{\text{spall}} = A(\text{rate})^m$, where the power m is a small number compared to unity. Nevertheless, standard spall models in commercial and propriety hydrocodes use constant spall strength, and are not able to predict the rate dependence. We propose here a rate dependent spalling model which is based on the overstress concept. In a constant spall strength model, when the negative pressure reaches the negative spall-strength value, pressure is put to zero within a single time step. In our rate dependent model we allow the negative pressure to go above the current negative spall-strength according to specified rate coefficients (calibrated from tests), while the negative pressure is decreased proportional to the amount of overstress above the current negative strength value. We calibrate our rate coefficients according to experimental data for a Stainless Steel and demonstrate how the model works using a 1D hydrocode for planar impacts with different strain rates in the tension zone.

1 Introduction

It has been known for a long time that spalling (dynamic tensile failure) is a rate dependent process [1].

Spall strength of metals is determined from the pull back velocity of the free surface velocity history in planar impact spalling tests. Conducting these tests for different volume strain rates in the tension zone up to a strain rate of 10^7 1/sec, it was established that spall strength increases with strain rate according to a power law:

$$\begin{aligned} P_k &= A(\dot{\epsilon}_V)^m \\ \dot{\epsilon}_V &= \frac{\dot{V}}{V} \end{aligned} \quad (1)$$

Where P_k is the spall strength, V is the specific volume and A , m are material parameters. As seen from Table 5.3 of [1], the exponent m is a small number, usually in the range of 0.05–0.20 for different metals.

It should be noted that some work in the literature refers to a stronger dependence on strain rate when strain rate rises above 10^7 1/sec (eg. [2]). The power law in Eq. (1) and the model proposed here do not account for this higher range.

Accounting for strain rate dependence of spall strength has great importance when extrapolating experimental plate impact test results to practical problems involving various loading histories such as cases involving an explosive loading. A demonstration of this has recently been presented by Gray et al. [3,4], examining spall strength derived through plate impact tests, having a rectangular shape loading on the specimen and through explosively driven tests having a Taylor-wave loading. The differences in the spall strength between the two loading cases are accounted to strain-rate effects.

Modeling strain-rate dependence follows different approaches in the literature, through energy criterions, microstatistical approaches and thermal considerations. An interesting discussion on spall criteria appears in the work of Hanim and Klepaczko [4].

In what follows, we propose a model which is based on the concept of *overstress*. Such approach is the basis

for the Tuler and Butcher spall model [5], which related the spalling to the tensile overstress and time duration of the pulse at the spall plane. The model we present here, accounts for strain-rate sensitivity without addressing the time duration of the pulse in an explicit manner, setting a more simple derivation to implement in a hydrocode.

Denote the quasi-static isotropic tension strength (the negative pressure that causes spontaneous pore growth) by P_{k0} . In a dynamic situation (like in a spall test), the negative pressure P_k may rise beyond the quasi-static strength, because the pore growth process may not be fast enough. The difference $P_k - P_{k0}$ is the overstress. As time proceeds, the negative pressure relaxes back towards the quasi-static strength at a rate that increases with the amount of overstress. This rate of relaxation is the source of the rate dependence observed in spall tests.

In section 2 we present the model equations and explain how it is implemented in a hydrocode. In section 3 we show results of hydrocode runs of a typical spall test. We then present simulation results of three spall configurations with dimensions that produce volume strain-rates differing by an order of magnitude from one another. From the results we are able to calibrate the two parameters in the negative pressure relaxation equation.

2 Model equations

We implement our Rate Dependent Spalling (RDS) model in the EOS-Energy (EE) subroutine of the hydro-code. For each computational cell, upon entering EE, the initial and final volumes are known, as well as the initial pressure, the initial internal energy and the artificial viscosity. EE computes the final pressure and internal energy by solving simultaneously the equation of state and the conservation of energy equation. Usually these equations are written in algebraic form:

$$E = E(P, V) \quad (2)$$

$$E_f - E_i = -[q + (P_i + P_f)](V_f - V_i) + \Delta W_p$$

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where i stands for initial, f stands for final, q is the artificial viscosity, V is the specific volume, P is the pressure, E is the specific internal energy and ΔW_p is the specific (per unit mass) plastic work during the time step.

As our RDS model is rate dependent, we first convert Eq. (2) into a rate form:

$$\begin{aligned}\dot{E} &= \frac{\partial W}{\partial P} \dot{P} + \frac{\partial E}{\partial V} \dot{V} \\ \dot{E} &= -(P + q) \dot{V} + \dot{W}_p\end{aligned}\quad (3)$$

Eliminating \dot{E} and solving for \dot{P} we get:

$$\begin{aligned}\dot{P} &= -\frac{P + q + \partial E / \partial V}{\partial E / \partial P} \dot{V} + \frac{\dot{W}_p}{\partial E / \partial P} \\ \dot{P} &= A_V \dot{V} + A_W \dot{W}_p \\ A_V &= -\frac{P + q + \partial E / \partial V}{\partial E / \partial P} \\ A_W &= \frac{1}{\partial E / \partial P}\end{aligned}\quad (4)$$

In the example given in the next section we ignore the influence of material strength in Eq. (4) and use $A_W = 0$.

Equation (4) is an ODE for P with respect to time. To find P_f we integrate Eq. (4) numerically from t_i to t_f using:

$$\dot{V} \cong \frac{V_f - V_i}{t_f - t_i}\quad (5)$$

We can then compute E_f from $E(P_f, V_f)$. On the basis of Eq. (4) we can now define our RDS model equations. As long as $P > P_k$, Eq. (4) stays unchanged. Whenever $P < P_k$ we modify Eq. (4) by:

$$\dot{P} = A_V \dot{V} + A_W \dot{W}_p + R_p (P_k - P)^\alpha; \quad P < P_k \quad (6)$$

Where P_k is the current quasi-static isotropic tension strength, and R_p , α are material parameters to be calibrated from spall tests.

In a conventional constant spall strength scheme, once $P < P_k$, P and P_k are put to zero in a single time step. We choose to put P_k to zero gradually by:

$$\dot{P}_k = R_k (0 - P_k)\quad (7)$$

The rationale for putting P_k to zero is as follows: The quasi-static isotropic tension is a function of the plastic flow stress Y . When a pore grows under tension, Y decreases by damage accumulation, and P_k decreases with it towards zero. We have no information on the rate of decrease of the quasi-static tensile strength, but it is not important for a spalling event as long as it is fast enough. Finally, we solve in the EE subroutine the two simultaneous equations (6) and (3).

3 Example

A 10 mm thick stainless steel (SS) projectile impacts (1D planar impact) a 20 mm thick target of the same material with an impact velocity of 1 km/s. The mesh resolution is 5 cells/mm. We output the pressure history

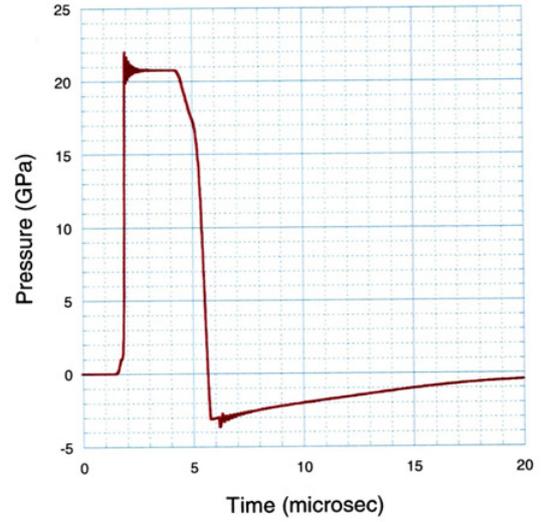


Fig. 1. Pressure history in the spall plane.

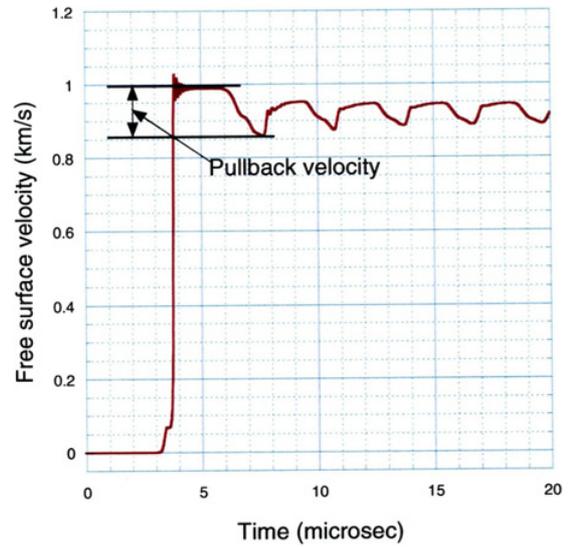


Fig. 2. Target free surface velocity history.

at the spall plane (mid section of the target) and the free surface velocity history of the target. For SS we use a Mie-Grüneisen EOS referred to the shock Hugoniot with constant $\rho\Gamma$. The EOS parameters are:

$$\begin{aligned}\rho_0 &= 7.85 \text{ gr/cc}; \quad C_0 = 4.57 \text{ km/s} \\ S &= 1.49; \quad \Gamma_0 = 2.0\end{aligned}\quad (8)$$

The RDS model parameters in the example are:

$$\begin{aligned}R_p &= 0.01 \text{ (GPa, } \mu\text{s)}, \quad \alpha = 7 \\ P_{k0} &= -1.0 \text{ GPa}, \quad R_k = 100 \text{ (}\mu\text{s}^{-1})\end{aligned}\quad (9)$$

In Fig. 1 we show the pressure history in the spall plane, and in Fig. 2 we show the target free surface history.

From Fig. 1 we see how the negative pressure goes gradually to zero. From Fig. 2 we see that the spall strength deduced from the pullback velocity is:

$$P^* = \frac{1}{2} \rho_0 C_0 \Delta u = \frac{1}{2} \times 7.85 \times 4.57 \times 0.14 = 2.5 \text{ GPa}\quad (10)$$

Where P^* is the average spall strength.

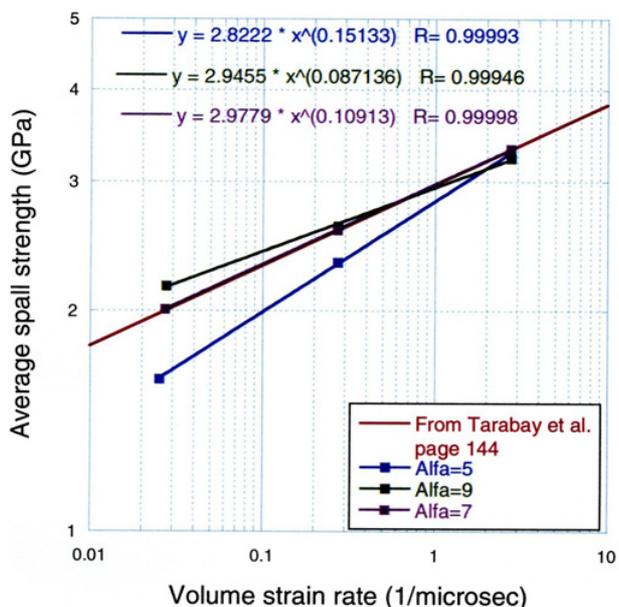


Fig. 3. Average spall strength curves for three values of the exponent α compared to the experimental curve from [1].

3.1 Calibration

The SS curve in Fig. 5.3 of [1] is given by:

$$P^* = 2.97 \times (\dot{\epsilon}_V)^{0.11} \text{ (GPa, } \mu\text{s)} \quad (11)$$

$$\dot{\epsilon}_V = \frac{\dot{V}}{V}$$

and is plotted in Fig. 3. The volume strain rate in the example above is about 3×10^5 [1/s]. The strain rate changes somewhat during the negative pressure duration, and we chose to represent it by its maximum value which occurs a little before the maximum negative pressure. To get strain rate values differing by an order of magnitude, we increased and decreased the dimensions of the SS plates by a factor of ten. We checked the three configurations with different values of the exponent α , and in each case adjusted R_p to fit the experimental data. For each computation we deduced the average spall strength from the pullback velocity obtained from the computation. We show the results in Fig. 3. We see from Fig. 3 that:

- (1) For $\alpha = 7$ and $R_p = 0.01$ we get good agreement with the experimental curve. (for the other values of α , R_p decreases by an order of magnitude for an increase by two units in α).
- (2) The value of α increases as the value of m in Eq. (1) decreases. A good first estimation for α is $1/m$.

4 Summary

Spalling tests show that spall strength, as inferred from pullback velocity signals, is rate dependent. We propose here a Rate Dependent Spalling (RDS) model and implement it in a hydrocode. Our RDS model is based on the overstress concept. This means that in a dynamic event, the state point may go beyond the quasi-static strength value, but with time it relaxes back to the quasi-static strength. The rate of this relaxation is the source of the rate dependence.

To implement the model into the hydrocode, we convert the EOS and the conservation of energy equation into rate forms, use a pressure relaxation equation with two material parameters and integrate these rate equations numerically for each cell and time step. We calibrate these parameters for stainless steel to match the experimental data.

References

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