EPJ Web of Conferences **31**, 00006 (2012)
DOI: 10.1051/epjconf/20123100006
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# Surface boiling – an "obvious" explanation for the observed limiting temperature of finite nuclei

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**Abstract**

Limits of stability of nuclear systems are explored within the framework of a finite-range interacting Fermi gas model and microcanonical thermodynamics in Thomas-Fermi approximation. It is found that with increasing excitation energy, infinite systems become unstable against volume boiling, while finite systems become subject to surface boiling, providing a natural explanation for the observed saturation-like patterns, or limiting temperature, in caloric curves. Boiling patterns of iso-asymmetric matter are discussed.

## 1 introduction

Understanding the thermodynamical limits of stability of uniform nuclear matter under various conditions is of obvious interest to nuclear scientists and has been the aim of many theoretical and experimental studies [1–3]. Somewhat surprisingly, these past studies were almost exclusively limited to *confined* systems envisioned by classical thermodynamical ensembles, microcanonical and canonical. Yet, the excited atomic nuclei cannot be subjected to an external confinement. They reside in vacuum and are free to explore all of the phase space energetically accessible to them and, notably, are guaranteed to undergo, among other things, thermal expansion so as to maximize their entropy. Obviously, the excited nuclei will eventually decay,

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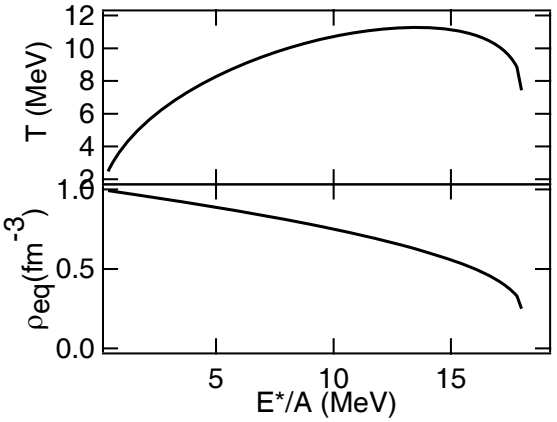


Figure 1: Caloric curve and equilibrium density for infinite self-bound Fermi liquid in Harmonic Interaction Fermi Gas Model.

which makes the exact thermodynamical treatment impossible. However, with reasonably long statistical decay time scales, the general strategy is to consider systems to be at quasi-equilibrium within the confines of a proper hyper-surface of transition states consisting of fragmentation saddle states and particle emission barriers. The frequencies at which particular transition states are arrived at are then related to respective decay times, allowing one for experimental verification of the concepts utilized in modeling of excited nuclear systems. The present study addresses the issue of stability of uniform self-bound neutral matter as a function of excitation energy by inspecting the properties of Hessian matrix (constructed of second derivatives) of the entropy function of such matter that is allowed to seek maximum entropy via adjustment of the matter density distribution. For a one-component (iso-neutral) matter, the entropy is in this case a function of solely energy (1x1 Hessian), while for a two-component (iso-active) matter, it is a function of both energy and isospin (2x2 Hessian). For the matter to be stable, the entropy function must be concave, which implies that the respective Hessian must be negative definite, and which means that all its eigenvalues must be negative. A vanishing (turning zero) eigenvalue, turning then positive, indicates the onset of instability and its associated eigenvector identifies the direction of growth of the instability in the argument space of the entropy function. This eigenvector thus contains the information on the very nature of the instability involved.

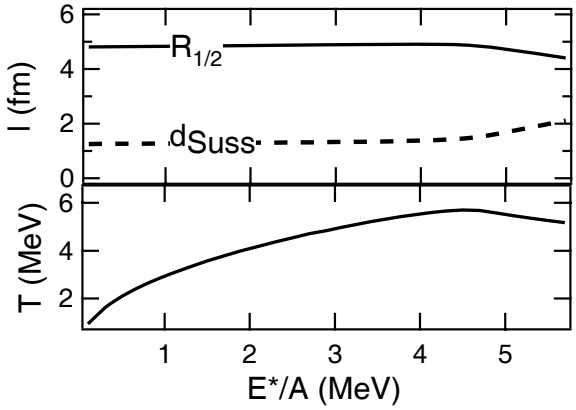


Figure 2: Evolution of finite system parameters with excitation energy (See text).

## 2 Theoretical formalism

The theoretical formalism used in the present work is the same as developed and used over an extended period of time in a series of papers. [4–10] In it, the configuration entropy is expressed in terms of a zero-temperature Fermi-gas model as

$$S_{config} = 2\sqrt{a_{config}(E - E_{config})}, \tag{1}$$

where $E$ is the system energy and $a_{config}$ and $E_{config}$ are, respectively, the level density parameter and zero-temperature energy for the given macro-configuration.

The level density parameter $a_{config}$ is calculated via a formalism described in Ref. [4] as

$$a_{config} = a_o \rho_o^{2/3} \int\int\int \rho^{1/3}(\vec{r}) d\vec{r}, \tag{2}$$

where $a_o$ expresses the value of the level density parameter at normal matter density $\rho_o$. Note that for uniform matter distribution, eq. 2 reduces to $a_{config} = a_o(\rho/\rho_o)^{-2/3}$.

The configuration energy was evaluated in the Harmonic-Interaction Fermi-Gas Model (HIFG) with an iso-asymmetry term linear in matter density

$$E_{config} = c_V(1 - \frac{\rho}{\rho_o})^2 + c_I \frac{\rho}{\rho_o}(\frac{N-Z}{N+Z})^2 \tag{3}$$

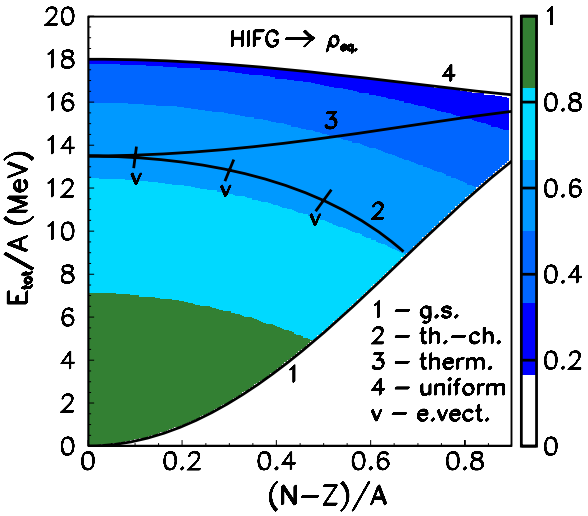


Figure 3: Contour plot of equilibrium density as a function of the total energy per nucleon and the isospin parameter. Lines represent the ground state energy (1), boundaries of thermo-chemical (2) and thermal (3) stabilities, and the limit of stability with respect to uniform expansion (4). The short bars labeled as v represent the directions of the thermo-chemical instability given by eigenvectors of Hessian matrix of the entropy function associated with null eigenvalues.

In calculations, it was assumed $c_V = -16$ MeV, $c_I = 23$ MeV, and $a_o = \mathrm{A}/15$ MeV$^{-1}$.

For the finite system, the configuration energy was calculated by folding the EOS of eq. 3 with a gaussian folding function emulating finite range of nuclear interaction. The folding width was chosen so as to result in a ground-state Süssmann width of the surface domain of 1 fm.

## 3 Results of model calculations

Fig. 1 illustrates the basic result obtained for infinite iso-neutral matter in the form of a caloric curve. In this case, Hessian of the entropy function reduces to the second derivative of entropy with respect to energy and the stability condition of this derivative being negative implies positivity of the heat capacity. As seen in this figure, the heat capacity turns in fact negative around $E^*/A \approx 13.6$ MeV and stays such up to the end-point of the curve where the matter becomes unstable against uniform expansion.

In a stark contrast to the findings made for classical confined ensembles, here the instability is not healed by a simple phase separation as there is no

container to hold the gaseous phase. Rather, here parts of the system will expand first at the expense of the heat derived from surrounding parts and cool down and thus deriving even more heat until these parts (now bubbles) reach the point of self-sustained instability against uniform expansion. At this point, they will be able to leave the system altogether in the form of vapor, eventually leaving behind a hot residue at the point of maximum temperature. The process is that of boiling, known from everyday experience, except that here it occurs at zero pressure. Its obvious signature is the presence of a limiting excitation energy per nucleon and thus, a limiting temperature - a fact that has been observed in many experiments. [11] Note that the escaping vapor here is not in equilibrium with the liquid residue - it is noticeably colder and thus implying a relatively low latent heat for boiling as compared with liquid-gas transition in a confined system.

Fig. 2 illustrates onset of instability in the case of a finite system. Here, the matter density profile was allowed to vary within the family of two-parameter error-function complement [4] so as to maximize the entropy

$$\frac{\rho}{\rho_o} = C(R_{1/2}, d)[1 - erf(\frac{r - R_{1/2}}{\sqrt{2}d})], \tag{4}$$

where $R_{1/2}$ and $d$ are the half-density radius and the Süssmann surface width, respectively, and $C(R_{1/2}, d)$ is a normalization factor assuring the desired number of nucleons in the system (here, A=100).

As seen in Fig. 2, first both the half-density radius and the Süssmann width, as well as the temperature increase quasi-monotonically with increasing excitation energy per nucleon. Then around $E*/A \approx 4.5MeV$, the temperature reaches a maximum with a subsequent drop with increasing excitation energy - a signal of the entropy becoming a convex function of excitation energy (the sole eigenvalue of the hessian becoming positive). Clearly, the finite system became here unstable, even though the bulk matter in the interior is well within the domain of thermal stability. One is lead here to a conclusion that it is the matter density profile that becomes unstable, such that one section of the surface will increase its diffuseness at the expense of the energy derived from a neighboring section. This expansion of parts of the he surface domain must continue until "offending" parts of the surface leave the system in a process named here surface boiling. Left behind will be a quasi-stable residue at the surface-boiling temperature of approximately 5.7 MeV.

Fig. 3 illustrates the results obtained for a two-component system characterized by an isospin parameter $I = (N - Z)/(N + Z)$. In this case, the entropy per nucleon is a function of two variables, total energy per nucleon

$E_{tot}/A$ and the isospin parameter $I$. This is so, because the volume is here again allowed to change so as to maximize the entropy. Hence, the Hessian of the entropy function is here a 2x2 matrix whose both eigenvalues must be negative in order for the system to be stable.

In Fig. 3, the line labeled as 2 illustrates the limit of stability of uniform system, where one of the eigenvalues of Hessian becomes zero. The corresponding eigenvectors (short bars labeled $v$) illustrate the direction in which the instability will grow and indicates that the instability is here of thermo-chemical nature, rather than of purely thermal. The purely thermal instability (signaled by negative heat capacity) would set in at a boundary represented by the line labeled as 3. Purely chemically, the system is stable over the full range of excitation energies, up to the limit where the stability against the uniform expansion is lost (line 4). It is important to note that once the excitation energy exceeds the limit of thermo-chemical stability, parts of the system must separate in iso-rich boiling, deriving heat from the neighboring parts leaving behind a more iso-symmetric residue at the "attractor" line 2 illustrating the boundary of stable domain. The process here is that of distillation and is not identical to what is called isospin fractionation in liquid gas coexistence case of confined systems. Obviously, here the iso-rich vapor never is in equilibrium with the liquid residue.

## 4 Discussion

The present study reveals the limits of stability, with increasing excitation energy, of three types of non-confined but self-bound microcanonical systems and associates these with boiling and distillation phenomena. It is important to realize, that thermodynamics considered is valid only approximately (statistical decays being disabled via the thought concept of a confining hypersurface of transition states) and only up to the point on the energy scale where the instability sets in. The domain of instability is not accessible to thermodynamical treatment as there is no plausible way for the system ever to arrive to it. This is so because in the model, the matter considered is made on purpose uniform to conform with the aim of the study. In this respect, also the finite system considered in this study is uniform, characterized by an isotropic matter density profile. It is therefore important to note that appearance of negative heat capacity in model calculation should not be taken as an indication that such negative heat capacity should or possibly could be observed experimentally. All such appearance says is that from a certain point on, the system can no longer be uniform. Some-

times, but only in modeling of purely academic interest or value it ends up as a two-phase system of coexisting liquid and gas. In a more realistic modeling of a non-confined system, as in the present study, a "leaner" quasi-equilibrated system emerges after boiling off part of the matter and the "excess" excitation along with it. Apart from the obvious experimental determination of limiting excitation energies per nucleon, the present study contains the suggestion of identifying the boiling component in the particle spectra, which is expected to feature somewhat lower temperature than the equilibrium-evaporation one.

This work was supported by the U.S. Department of Energy grant No. DE-FG02-88ER40414.

## References

[1] H. Müller and B. Serot, Phys. Rev. C **52**, 2072 (1995).

[2] J. Lattimer and D. Ravenhall, Astr. Journ. **223**, 314 (1978).

[3] M. Barranco and J. Buchler, Phys. Rev. C **22**, 1729 (1980).

[4] J. Tõke and W. J. Swiatecki, Nucl. Phys. A **372**, 141 (1981).

[5] J. Tõke and W. U. Schröder, Phys. Rev. Lett. **82**, 5008 (1999).

[6] J. Tõke, J. Lu, and W. U. Schröder, Phys. Rev. C **67**, 034609 (2003a).

[7] J. Tõke, L. Pieńkowski, L. Sobotka, M. Houck, and W. U. Schröder, Phys. Rev. C **72**, 031601 (2005).

[8] J. Tõke, J. Lu, and W. U. Schröder, Phys. Rev. C **67**, 044307 (2003b).

[9] L. G. Sobotka, R. J. Charity, J. Tõke, and W. U. Schröder, Phys. Rev. Lett. **93**, 132702 (2004).

[10] L. G. Sobotka, and R. J. Charity, Phys. Rev. C **73**, 014609 (2006).

[11] J. Natowitz et al., Phys. Rev. C **65**, 34618 (2002).