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Physics with low energy radioactive beams

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Abstract

This is a review of the basic mechanisms leading to high energy breakup reactions, of the formalisms used to describe them and of some of the methods to obtain the optical potentials necessary in calculations involving exotic nuclei. Semiclassical reductions of the DWBA approximation for inclusive and exclusive breakup will be presented. The observables that can be calculated are projectile's core parallel momentum distributions, angular distributions of nucleons from the breakup, total breakup cross sections. Methods to study resonances at threshold in unbound nuclei are also introduced. In this case the exclusive observable calculated is the neutron-core relative energy spectrum. Finally, we study the two neutron transfer to the continuum reaction between two "normal" nuclei. The observable is the continuum excitation energy spectrum of the "exotic" residual nucleus created.

1 Introduction

Exotic nuclei are located away from the stability valley and have large differences in the number of neutrons and protons. Until recently only nuclei with valence particle separation energies S_n much smaller than the average 8 MeV expected in nuclear matter were studied. For light nuclei the weak binding is associated to small angular momentum values of the occupied orbitals and, due to deformation of the nuclear surface, to strong particle vibration couplings which are at the origin of shell inversion effects [1]. Thus in the extreme case of halo nuclei such as several beryllium and lithium isotopes

(^{11}Be,^{12}Be,^{14}Be,^{11}Li), S$_n$ is even less than 1 MeV and the valence orbitals are s or p. As a consequence as much as 10% to 40% of the total reaction cross section is due to just one or two channels: 1n or 2n breakup and/or transfer to target bound states depending on the most favorable matching conditions. Therefore, *out of necessity* (N. Orr), breakup has been one of the most studied reactions for low intensity beams and the one for which several new models have been developed. On the other hand the advance in technical developments makes it possible at present the creation and study of beams of heavier nuclei with very asymmetric neutron and proton numbers, in which neutrons are deeply bound while protons are weakly bound or the other way around.

In his first attempt to describe quantitatively reaction cross sections and breakup of exotic nuclei, I. Tanihata [2] used the optical limit of Glauber model and in particular the strong absorption model of the total reaction cross section. This well known geometrical model supposes that the interaction between two heavy nuclei at intermediate to high energy can be described by just one parameter, the strong absorption radius R$_s$ which is the distance at which the probability of elastic scattering between two nuclei is reduced by 50%. Within this model Kox et al. [3] were able to reproduce consistently a very large set of data. In such a model a simple form of the nucleus-nucleus S-matrix is

$$P_{NN}(b) = |S_{NN}|^2 = e^{(-\ln 2 exp[(R_s - b)/a])}. \tag{1}$$

It states that for impact parameters smaller than R$_s$ the nucleus-nucleus interaction is strongly absorptive and the elastic scattering probability is small, while for impact parameters larger than R$_s$ the elastic scattering probability goes quickly to one. The strong absorption radius can be parametrized as R$_s \approx 1.4(A_p^{1/3} + A_t^{1/3})$ fm. The values of R$_s$ thus obtained agree within a few percent with those of the Kox parameterization [3]. a is a diffuseness parameter whose typical values are 0.5-0.7fm, however Tanihata pointed out that for exotic nuclei the surface diffuseness parameter could be different. We shall see in the following that this is indeed the case.

In this short review I will present the mechanisms which lead to breakup, the relative observables that are measured and the semiclassical reduction of DWBA which is at the basis of models presented here. For each of them a description will follow of the most recent advances in the models used for theoretical calculations.

2 Optical potential and elastic scattering

In a semiclassical approximation [4, 5], the imaginary part of the nucleus-nucleus phase shift δ_I is related to the imaginary part of the optical potential by

$$\delta_I(\mathbf{b}) = -\frac{1}{2\hbar}\int_{-\infty}^{+\infty} \left(W_V(\mathbf{R}(t)) + W_S(\mathbf{R}(t))\right) dt, \tag{2}$$

$\mathbf{R}(t) = \mathbf{b} + vt$ is the classical trajectory of relative motion for the nucleus-nucleus collision. The volume potential W_V is responsible for the usual inelastic core-target interaction, while the surface term W_S takes care of the peripheral reactions like transfer and breakup of the valence particles. Thus the projectile-target elastic scattering probability is given in terms of the nucleus-nucleus S-matrix as

$$|S_{NN}(b)|^2 = e^{-4\delta_I(b)} = e^{-4(\delta_{IW}(b)+\delta_{IS}(b))}, \tag{3}$$

where we interpret $e^{-4\delta_{IW}(b)} = |S_{CT}(b)|^2$ as the core-target elastic scattering probability. According to [4]- [5] the surface optical potential $W_S(\mathbf{R}(t))$ can be related to the sum of the varius surface reaction probabilities by

$$\int_{-\infty}^{+\infty} W_S(\mathbf{R}(t))dt = -\frac{\hbar}{2}\sum_{(i,n)} P_n^{(i)} \tag{4}$$

where i means transfer and breakup and n indicates the valence neutrons. In [6] we identified W_V as the origin of the core-target interaction and W_S as the origin of the halo-target interaction, which depending on the incident energy can be either transfer dominated or breakup dominated.

As it will be shown in the following, the b-dependence of the nuclear breakup probability $p^N_{b_{up}}(b)$ will be of the exponential form $p^N_{b_{up}}(b) \approx e^{-b/\alpha}$ with $a \approx (2\gamma)^{-1}$ where γ is the decay length of the neutron initial state wave function. We now assume at large distances, where $|S_{CT}|^2 = 1$ (cf. Eq.(3)) the same exponential dependence for the absorptive potential due to nuclear breakup $W^N_S(r) = W^N_0 e^{-r/a}$. We assume also, as indicated earlier on, a straight line parameterization for the trajectory, then the LHS of Eq.(4) can be approximately evaluated as

$$\int_{-\infty}^{+\infty} W^N_S(b,z)dz = W^N_0 \int_{-\infty}^{+\infty} e^{-(b+\frac{z^2}{2b})/a} dz = W^N_0\sqrt{2\pi ba}e^{-b/a}, \tag{5}$$

where we assumed $b >> z$ in the second step. Equating the RHS of Eqs.(4) and (5) and re-naming the distance b as r gives

$$W_S^N(r) = -\frac{\hbar v}{2} p_{b_{up}}^N(r) \frac{1}{\sqrt{2\pi a r}}. \tag{6}$$

Eq.(6) shows explicitly, as already discussed in Ref. [6], that the long range nature of the nuclear breakup potential originates from the large decay length of the initial state wave function. For a typical halo separation energy of 0.5MeV, $a = (2\gamma)^{-1} = 3.2$fm, while for a 'normal' binding energy of 10MeV, $a = 0.7$fm as expected. Therefore the parameter a will depend mainly on the projectile characteristics and not on the target.

Finally using Eqs.(2) and (4) in Eq.(3) the projectile-target total reaction cross section reads:

$$\sigma_{NN} = \int d\mathbf{b} \quad (1 - |S_{NN}|^2) \approx \sigma_{CT} + \sigma_{b_{up}}, \tag{7}$$

with $1 - |S_{NN}|^2 \approx 1 - |S_{CT}|^2(1 - P_{b_{up}})$ and S_{CT} takes into account all core-target interactions while the term $e^{-P_{b_{up}}}$ which depends only on the halo neutron breakup probability has been expanded to first order because $P_{b_{up}}$ is small. Equation (7) is a well known and accepted relation obtained by separating the interaction of the core and halo nucleon with the target respectively (cf. Eq. (3.4) of Ref. [2]).

Our estimate for the surface imaginary potential have already been confirmed by two experimental evidences in Refs. [7] and [8]. In the recent paper [8] the elastic scattering angular distribution of the halo ^{11}Be nucleus on a ^{64}Zn target has been studied and compared to the elastic scattering of the "normal" ^{10}Be and ^{9}Be nuclei. The data have shown a large depletion of the elastic cross section in the ^{11}Be case, while the elastic cross sections of ^{10}Be and ^{9}Be look quite similar, in spite of the fact that ^{9}Be is also weakly bound. It was also shown that transfer and breakup account for about 40% of the reaction cross section. The conclusion was that the elastic scattering depletion is due to transfer and breakup events. Di Pietro et al. [8] have been able to reproduce their data by an optical model calculation in which the real part and volume imaginary part of the potential are the same for the ^{10}Be and ^{11}Be projectiles (a part from the scaling of the radius parameter with the projectile mass). On the other hand the imaginary part in the ^{11}Be case contains a surface term besides the volume term and the surface diffuseness which fits the data has been found to be quite large (3.5fm) in perfect agreement with our previous estimate. Thus the large diffuseness in the imaginary potential was interpreted as originated in the weak binding of the halo neutron. More recently [9], the above relation (7) has been verified by analyzing data for reactions initiated by another halo nucleus,

^{6}He, at energies much lower than those discussed in Ref. [2] and it has been interpreted as an evidence for the decoupling of the halo from the core. We conclude by mentioning the importance of taking into account also the Coulomb breakup channel in the optical potential when the target is heavy. In Refs. [10] we have shown that this can be done microscopically for the imaginary part of the optical potential. Because of the form factor behavior such potential must be also of the surface type. Its large diffuseness is due to the adiabaticity parameter of Coulomb breakup.

3 Cross section and breakup mechanisms

Direct one-particle re-arrangment reactions of the peripheral type in presence of strong core-target absorption can be described by an equation like [4], [11]- [15]

$$d\sigma/d\xi_f = C^2 S \int d\mathbf{b_c} |S_{CT}(b_c)|^2 dP(b_c)/d\xi_f, \tag{8}$$

(see Eq. (2.3) of [12]) and C^2S is the spectroscopic factor for the initial single particle state. The probabilities for different processes can be represented in terms of the amplitude as $dP/d\xi = \sum |A_{fi}|^2 \delta(\xi - \xi_f)$ where ξ can be momentum, energy or any other variable for which a differential cross section is measured. The core survival probability is defined in terms of a S-matrix function of the core-target impact parameter b_c by a form like Eq.(1). The projectile-target relative motion is described as before by a semiclassical trajectory. This approximation makes the formalism applicable for incident energies above the Coulomb barrier. Along the semiclassical trajectory the amplitude for a transition from a nucleon state ψ_i bound in the projectile, to a final continuum state ψ_f, is given by [11]

$$A_{fi} = \frac{1}{i\hbar} \int_{-\infty}^{\infty} dt \langle \psi_f(\mathbf{r}, t) | V(\mathbf{r}, \mathbf{R}(t)) | \psi_i(\mathbf{r}, t) \rangle, \tag{9}$$

where V is the interaction responsible for the transition which will be specified in the following. Eqs.(8,9) can be obtained from a semiclassical reduction of the standard DWBA amplitude, following the method proposed in Ref. [16].

Based on the time dependent amplitude Eq.(9) and the classical projectile-target trajectory of relative motion given above, in Ref. [13] we considered the breakup of a halo nucleus like ^{11}Be consisting of a neutron bound to a ^{10}Be core in a collision with a target nucleus. The system of the halo nucleus and the target was described by Jacobi coordinates $(\mathbf{R}, \mathbf{r})$ where $\mathbf{R}(\mathbf{t})$ is the time dependent position of the center of mass of the halo nucleus relative to the target nucleus and $\mathbf{r}$ is the position of the neutron relative to the halo core. The Hamiltonian of such a system is $H = T_R + T_r + V_{nc}(\mathbf{r}) + V_{nt}(\beta_2\mathbf{r} + \mathbf{R}) + V_{ct}(\mathbf{R} - \beta_1\mathbf{r})$, with β_1 and β_2 the mass ratios of neutron and core, respectively, to that of the projectile. T_R and T_r are the kinetic energy operators associated with the coordinates $\mathbf{R}$ and $\mathbf{r}$ and V_{cn} is a real potential describing the neutron-core final state interaction. V_{cn} is neglected if the observables measured and calculated do not depend significantly on it. This happens for core energy spectra and/or parallel momentum distribution or for neutron angular distributions. In the *projectile fragmentation* case instead, discussed in the following, such an interaction dominates the measured data which are neutron-core relative energy spectra. The potential $V_2 = V_{nt} + V_{ct}$ describes the interaction between the projectile and the target. Both V_{nt} and V_{ct} are represented by complex optical potentials. The imaginary part of V_{nt} gives rise to the stripping part [11] of the halo breakup. The imaginary part of V_{ct} describes reactions of the halo core with the target. Its effect is contained in Eq.(1) and in the strong absorption limit leads to Eq.(8). The potential V_{ct} also includes the Coulomb interaction between the halo core and the target. This part of the interaction is responsible for the "recoil" part of Coulomb breakup. In fact the Coulomb force does not act directly on a valence neutron but it affects it only indirectly by causing the recoil of the charged core. For a valence proton instead, the Coulomb potential is $V(\mathbf{r}, \mathbf{R}) = \frac{Z_c Z_t e^2}{|\mathbf{R} - \beta_1 \mathbf{r}|} + \frac{Z_v Z_t e^2}{|\mathbf{R} + \beta_2 \mathbf{r}|} - \frac{(Z_v + Z_c) Z_t e^2}{R}$, with $Z_v = 1$ ($Z_v = 0$ for a neutron).

3.1 Coulomb breakup

In [13] it was shown that a way to calculate nuclear and Coulomb breakup to all-orders, is to use the sudden approximation, subtract the first order term, which diverges for large impact parameter, and then add a first order term calculated in time-dependent perturbation theory. That formalism is appropriate to calculate the coincidence cross section $A_p \to (A_p - 1) + n$ as well as partially inclusive cross section through integration of the components of $\mathbf{k}$, the neutron-core relative momentum vector in the final state. The expression for the differential cross-section is

$$\frac{d\sigma}{d\mathbf{k}} = \frac{1}{8\pi^3} \int d\mathbf{b}_c |S_{CT}(b_c)|^2 |A^{nuc}(\mathbf{k}) + A^{dir}(\mathbf{k}) + A^{rec}(\mathbf{k})|^2. \quad (10)$$

Which includes three contributions to the amplitude. A Coulomb recoil term due to the core-target interaction, a direct proton-target Coulomb term and a nuclear part. In a number of papers higher order effects and proton breakup have been discussed, among which we recall Ref. [17]. There we demonstrated that the proton breakup dynamics can be understood by taking for the wave function of a proton that of a neutron having an "effective" larger separation energy which takes into account the combined effect of the projectile-target Coulomb barrier. Finally, in Fig.14, of Ref. [14], we showed that the Coulomb breakup cross section on heavy targets would be negligible compared to the total nuclear breakup cross section for heavy, neutron rich, exotic nuclei in which the valence nucleons are expected to be in d or f orbits and with separation energies of the order of 10 MeV or more. For heavy exotic projectiles such novel type of experiments, namely breakup on a heavy target, would be therefore possible and useful.

3.2 Projectile fragmentation vs. transfer to the continuum

We call *projectile fragmentation* the elastic breakup (diffraction dissociation), when the observable studied is the exclusive neutron-core relative energy ($\varepsilon_{\mathbf{k}}$) spectrum. This kind of observable has been widely measured in Coulomb breakup reactions on heavy targets. Light target reaction data have also been presented in Ref. [18]. These data enlighten the effect of the neutron final state interaction with the core of origin, neglected in the previous section, while observables like the core energy or momentum distributions enlighten the effect of the neutron final state interaction with the target and will be described in the following with the transfer to the continuum model.

Projectile fragmentation has also been studied for two neutron halo projectiles [19]- [25]. To first order this inelastic-like excitations can be described again by the time dependent perturbation amplitude Eq.(9) under the hypothesis that the neutron which is not detected has been stripped by the target and thus it does not modify the detected neutron-core relative energy spectrum. Here again, the potential $V(\mathbf{r}, \mathbf{R}(t))$, which is the interaction responsible for the neutron transition, moves past on a constant velocity path as described in the previous sections. The coordinate system and other details of the calculations can be found in Ref. [22]. The probability spectrum reads

$$\frac{dP_{in}}{d\varepsilon_{\mathbf{k}}} = \frac{2}{\pi}\frac{v_2^2}{\hbar^2 v^2}C_i^2\frac{m}{\hbar^2 k}\frac{1}{2l_i+1}\Sigma_{m_i,m_f}|1-\bar{S}_{m_i,m_f}|^2|I_{m_i,m_f}|^2. \tag{11}$$

Spin can be included according to Appendix B of Ref. [22] and $|I_{m_i,m_f}|^2 \approx \frac{e^{-2\gamma b_c}}{b_c^2}$. γ has been defined above, after Eq.(4) .The quantity $\bar{S} = e^{2i(\delta+\nu)}$ is an off-the-energy-shell S-matrix representing the final state interaction of the neutron with the projectile core. It depends on a phase which is the sum of δ, the free particle n-core phase shift, plus ν the phase of the *form factor* I. More examples of our calculations and comparison to recent data can be found in Refs. [26].

On the other hand the probability of transfer of a single nucleon from the orbit l_i in the projectile to a continuum state with angular momentum l_f in the target is given by [11]

$$\frac{dP}{d\varepsilon_f}(l_f, l_i) = (|1 - S_{l_f}|^2 + 1 - |S_{l_f}|^2) B(l_f, l_i) \tag{12}$$

where $B(l_f, l_i) \approx \frac{e^{-2\eta b_c}}{\eta b_c}$ can be interpreted as an elementary transition probability where η is a kinematical parameter whose minimum value is given by $\eta_{min} = \gamma$. The S-matrix represents here the final state interaction of the neutron with the target. This formalism is very flexible because by applying energy and momentum conservation the spectrum Eq.(12), obtained as a function of the neutron final energy with respect to the target, can be converted in the neutron "intrinsic" parallel momentum distribution [27], the core parallel momentum distribution [28], or in the target final excitation energy spectrum [11]. In the latter case the method has also successfully been applied recently to the study of two neutron transfer to the continuum reaction ^{13}C(^{18}O,^{16}O)^{15}C [29].

Finally for exotic beam experiments in inverse kinematics [30], the neutron transfer is from the target to the projectile. For final states well below the barrier which correspond to narrow resonances no spreading of the single particle states is expected, thus $1 - |S_{l_f}|^2 = 0$. Transfer to these states can be studied by integrating Eq.(12) over the energy region of the resonance according to Ref. [11].

3.3 Conclusions

Some reaction models for neutron and proton breakup involving exotic nuclei have been presented. We have stressed the importance of all optical potentials entering the model calculations. From the structure point of view, in the search for the drip-line position, a very important role is played by the study of nuclei unstable by neutron emission, using projectile fragmentation reactions or transfer to the continuum. We have shown that within the same time dependent semiclassical formalism it is possible to study transfer to bound states and to resonances and inelastic-like excitations. Coulomb breakup can be calculated to all orders with a *regularised* sudden approach. On the other hand increasing the mass of the exotic projectiles produced and looking for neutron deficient nuclei we are going to face the problem of envisaging new experiments to study them and new reaction models to interpret the data. These two are among the most important subjects which need to be addressed and further developed in the near future.

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