

# Parametric excitation of surface electron cyclotron X- and O-modes by alternating electric fields

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## 1. Introduction

Propagation of different electromagnetic surface waves (SW) is recognized as one of the most possible cause of undesirable plasma periphery heating, strengthening interaction between plasma column and walls of fusion device, impurity generation. This is found out clearly in experiments with ion cyclotron heating [1]. On the other hand in order to supply periodical operation of fusion devices, one can clean the devices inner surface. One of the prospective methods that can be used for this procedure is application both ion and electron cyclotron hardwares (without dismantling the whole device) for sustaining gas discharge, which can remove different contaminations from the chamber wall. The urgency of researches into parametric instability of SW at harmonics of cyclotron frequencies is caused as well by wide application of SW in various plasma technologies [2].

The present paper is devoted to development theory of parametric instability of slow surface X- and O-modes at electron cyclotron frequency for the model of semi-bounded plasma. Surface electron cyclotron O-modes (SECOM) were found theoretically [3] to propagate along plane plasma-dielectric interface, when an external constant magnetic field is parallel to the plasma boundary and penetration depth of the modes is approximately equal to their wavelength. SECOM frequencies decrease if their wave number grows, their damping is determined by both collision (interaction between plasma particles) and kinetic (interaction between particles and plasma interface) mechanisms. Dispersion properties of surface electron cyclotron X-modes (SECXM) were studied theoretically in [4]. Unlike the SECOM they are unidirectional modes, their propagation coincides with direction of Larmor rotation of electrons on the plasma surface. They can propagate even along plasma-metal interface that makes them especially dangerous for destroying walls of fusion chambers.

Sets of equations for these X- and O-modes are obtained by application of non-linear boundary conditions for their tangential magnetic field. Non-linear boundary conditions, which describe discontinuity of tangential component of their magnetic field are formulated by the aid of conception of existence of surface electric current, which is induced by the mentioned above external alternation electric field on the plasma interface. The reduced sets of equations for the X- and O-modes wave packets are obtained in approximation that the packets are composed by the main harmonic and two nearest satellite harmonics.

## 2. The basic equations

Let's consider uniform magneto-active plasma, which occupies area  $0 \leq x$  and is bounded by vacuum. An external constant magnetic field  $\vec{B}_0$  is oriented along axis  $\vec{z}$ . The spatial dispersion of plasma is supposed to be weak, it means that the length of the investigated waves is essentially larger than Larmor radius of electrons  $|\vec{k}\rho_e| \ll 1$ . Plasma is effected by an external alternating electric field  $\vec{E}_0 \cos(\omega_0 t) \perp \vec{z}$ . Frequency of the external alternating electric field  $\omega_0$  is of the same order as electron cyclotron frequency  $\omega_e$  value. The amplitude of this alternating field  $\vec{E}_0$  is assumed to be uniform that can be realized in the case of small value of gas-kinetic pressure of plasma. It is assumed that these modes propagate strictly across the external magnetic field, so  $\vec{k} \perp \vec{z}$ .

The plasma particles motion is described by Vlasov-Boltzmann kinetic equation with Maxwellian non-perturbed plasma particles distribution function. Fields of the modes are described by Maxwell set of equations. It can be separated into two independent sets, namely the O-mode with components:  $H_x$ ,  $H_y$ ,  $E_z$ , and the X-mode with components  $E_x$ ,  $E_y$ ,  $H_z$ . Solving Maxwell equations by Fourier method, it is possible to derive sets of the equations for Fourier coefficients of their fields. Components of plasma permittivity tensor depend on amplitude and frequency of the alternating electric field in such way [5]:

$$\sigma_{ik} \propto \sum_{s,m,l=-\infty}^{+\infty} \frac{e^{-y_\alpha} I_s(y_\alpha)}{\omega_{n+m} - s\omega_\alpha} J_m(g) J_{m-l}(g), \quad (1)$$

here  $\omega_\alpha$  is cyclotron frequency of the plasma particles, subscript  $\alpha$  is applied for designation type of plasma particles,  $g = g_0 k_\perp \rho_\alpha = \frac{k_\perp e_\alpha E_0}{m_\alpha (\omega_0^2 - \omega_\alpha^2)}$ ,  $y_\alpha = k_\perp^2 \rho_\alpha^2 / 2 \ll 1$ ,  $\rho_\alpha$  is Larmor radius of

the plasma particles,  $J_n(z)$  and  $I_n(z)$  are Bessel function of the first type, modified Bessel function, respectively, process of summarizing over subscripts  $s, m, l$  in the expression (1) can be done independently from each other in the infinite limits,  $\omega_{n+m} = \omega + (n+m)\omega_0$ .

By the aid of reverse Fourier transform one can obtain equation for  $n$ -th harmonic of the X- and O-modes tangential electric field. This transform has been carried out using Jordan lemma. Then applying the residues theory one can obtain the equation for  $n$ -th harmonic of their tangential electric field on the plasma surface.

There are two boundary conditions for these modes on the plasma-vacuum interface. The first of them is well-known linear condition for continuity of tangential electric field of the waves. The second boundary condition describes flowing of non-linear surface electric current along the plasma boundary. Discontinuity of tangential magnetic fields for the studied modes can be presented for the X- and O-modes in the following forms, respectively:

$$H_y^{(n)}(+0) + i \frac{k_2}{k} E_z^{(n)}(0) \approx \frac{i\Omega_e^2}{4\pi} \sum_{m,l=-\infty}^{\infty} f_1(y_e, m, l, g_0) E_y^{(n+l)}(0) |_{l \neq 0}, \quad (2)$$

$$H_z^{(n)}(+0) - \frac{ik_2}{k} E_y^{(n)}(0) \approx \frac{i\Omega_e^2}{4\pi} \sum_{m,l=-\infty}^{+\infty} f_2(y_e, m, l, g_0) E_y^{(n)}(+0, n+l) |_{l \neq 0}. \quad (3)$$

Detailed description of  $f_j$  is presented in [5]. Application of the conditions (2,3) is analogous to introducing a non-linear perturbations of surface charge density, which were applied in [6] for theoretical description of the process of solitary SW excitation.

### 3. Results of study of these parametric instabilities

Using boundary conditions one can obtain the following infinite set of equations for tangential component of the SECOM and SECXM electric field on the plasma-vacuum interface:

$$D^{(n)} E_{\tau}^{(n)}(0) + \sum_{l=-\infty; l \neq 0}^{+\infty} F^{(n;l)} E_{\tau}^{(n+l)}(0) = 0, \quad (4)$$

here diagonal coefficients nearby tangential components of the X- and O-modes electric field are similar to dispersion functions of these modes:  $D^{(n=0)}(\omega, k) = 0$ . Expressions for non-diagonal coefficients  $F^{(n;l)}$  are proportional to sums over amplitudes of alternating electric fields  $g_0$ .

Let's assume that for the main harmonic of the studied modes the following resonant condition:  $\omega = s|\omega_e| + \Delta T + \gamma$  is realized, here numbers of cyclotron harmonics are equal to  $s = 2$  and  $s = 1$  for SECXM and SECOM, respectively,  $\Delta T$  is their frequency shift determined by the plasma dispersion, correction  $|\gamma| \ll |\omega_e|$ . Then analytical solution of the reduced sets of equation that describe initial stage of parametric instabilities can be obtained in the following approximate forms for SECOM and SECXM, respectively:

$$\text{Im}(\gamma) \approx 2.3 |\Delta T| g_0 k_2^2 c^2 / \Omega_e^2, \quad (5)$$

$$\text{Im}(\gamma) \approx 0.3 |g_0 k_2 \rho_e \Delta T|. \quad (6)$$

Results of the numerical analysis are presented on the following Figures 1-4.

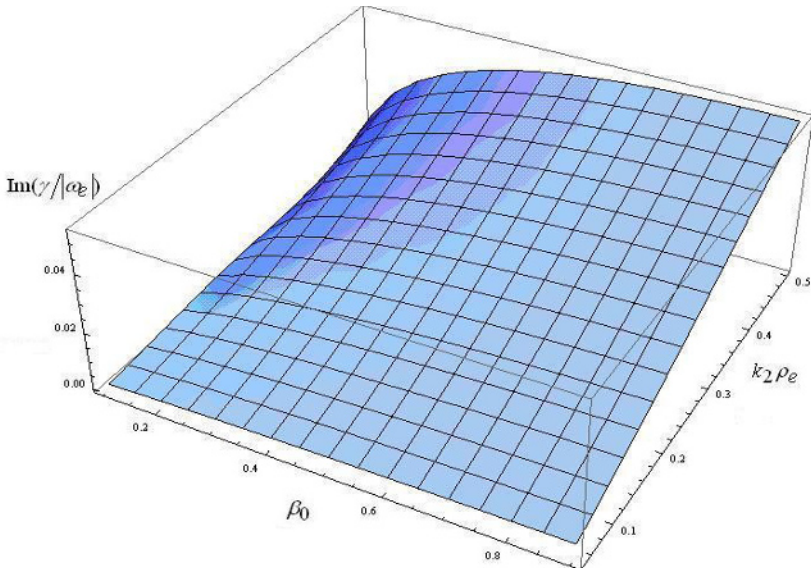
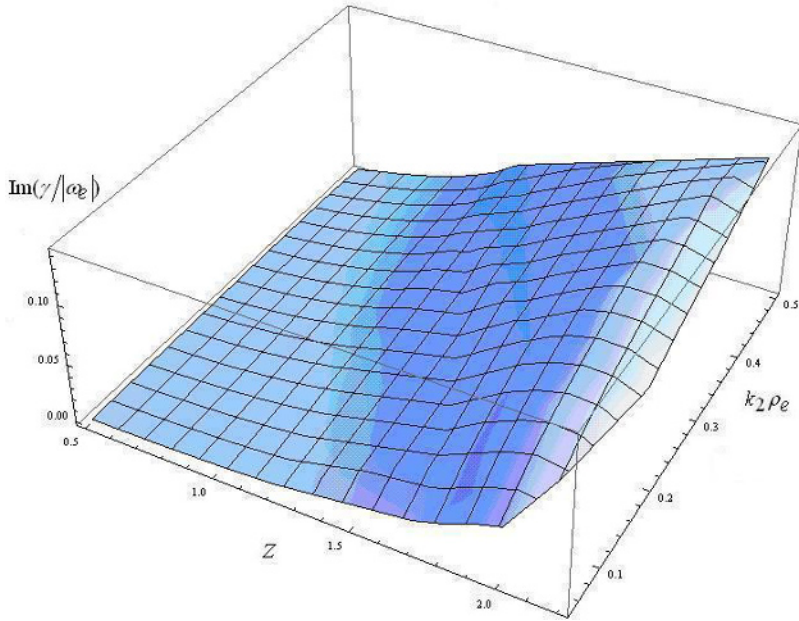
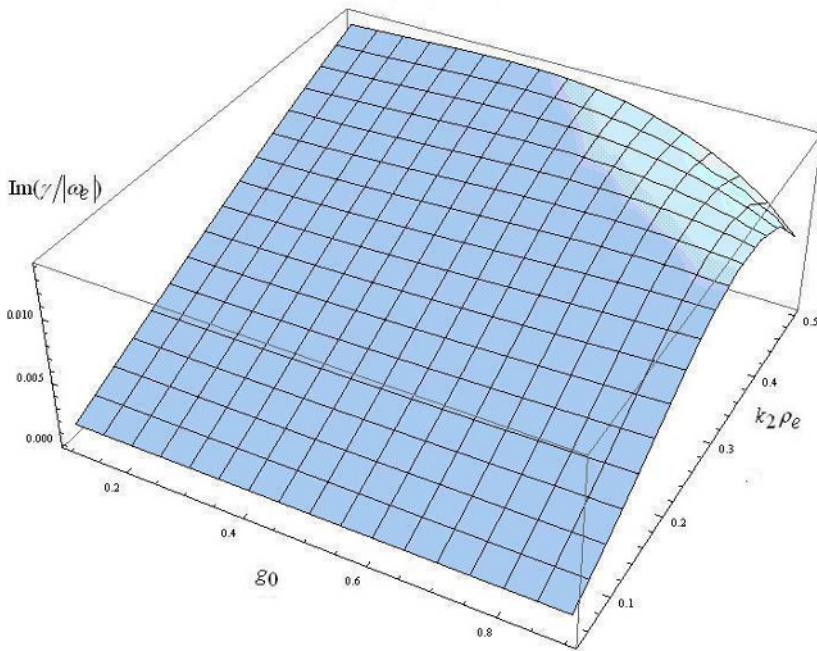


Fig. 1. Growth rate of the SECOM vs plasma beta  $\beta_0$  and normalized wave vector  $k_2 \rho_e$ ;  $g_0 = 0.7$ .

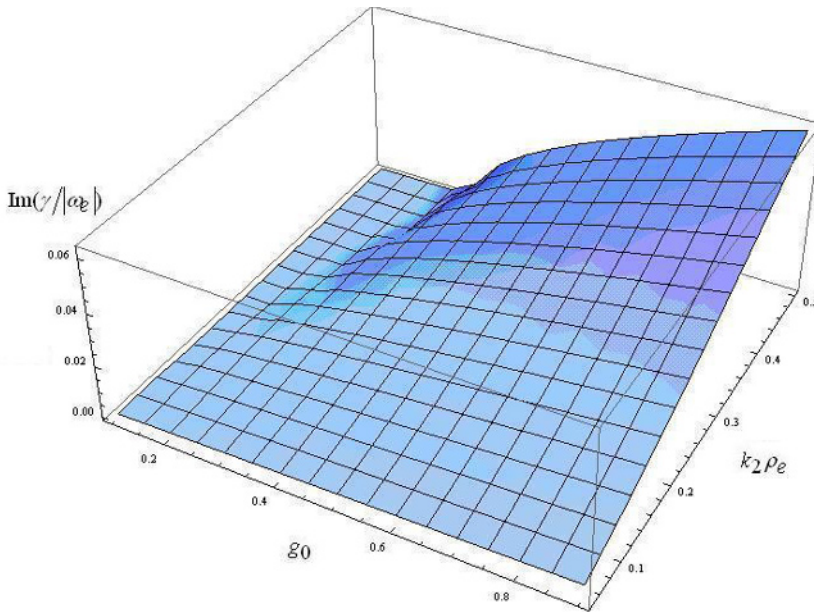


**Fig. 2.** Growth rate of the SECXM vs normalized plasma density  $z$  and wave vector  $k_2\rho_e$ ;  $g_0 = 0.5$ .



**Fig. 3.** Growth rate of the SECXM vs normalized amplitude of the external electric field  $g_0$  and wave vector

$$k_2\rho_e; z = \Omega_e^2 \omega_e^{-2} = 0.8.$$



**Fig. 4.** Growth rate of the SECOM vs amplitude of the external alternating electric field plasma beta  $g_0$  and normalized wave vector  $k_2 \rho_e$ ;  $\beta_0 = 0.5$ .

## 4. Conclusions

Obtained infinite sets of equations for harmonics of these X- and O-modes tangential electric fields allow one to describe an initial stage of their parametric instability. The obtained results can be useful at first, for development of plasma technologies based on utilization surface electron cyclotron waves propagation. Secondly it can be useful for searching the regimes of weak activity of the edge localized modes in fusion devices, because it can allow one to decrease heating of the periphery plasma and impurity generation in fusion devices.

## 5. References

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