EPJ Web of Conferences 32, 01014 (2012)
DOI: 10.1051/epjconf/20123201014

Toward 3D MHD modeling of neoclassical tearing mode suppression by ECCD

J. Pratt[1,a] and E. Westerhof[1,b]

FOM Institute DIFFER - Dutch Institute for Fundamental Energy Research, Association EURATOM-FOM, Trilateral Euregio Cluster, the Netherlands, http://www.differ.nl

Abstract. We propose a framework to extend the magnetohydrodynamic (MHD) equations to include electron cyclotron current drive (ECCD) and discuss previous models proposed by Giruzzi et al.[2] and by Hegna and Callen[3]. To model neoclassical tearing mode (NTM) instabilities and study the growth of magnetic islands as NTMs evolve, we employ the nonlinear reduced-MHD simulation JOREK. We present tearing-mode growth-rate calculations from JOREK simulations.

1 Introduction

Neoclassical tearing modes (NTMs) are one of the major MHD instabilities limiting the performance of tokamak fusion reactors. The localized current drive provided by electron cyclotron current drive (ECCD) can be used to control or suppress NTMs. The growth of an NTM is currently modelled theoretically by the generalized Rutherford equation (GRE). The GRE reproduces basic experimental observations of NTM width. But theoretical and numerical works have noted deficiencies in the GRE, particularly concerning the size at which a NTM saturates [8][9][13]. Further underpinning of the GRE thus remains necessary to establish its limits of validity and what corrections are necessary. In this work we describe our progress toward designing a 3D reduced-MHD simulation of tearing mode suppression by ECCD. We first discuss extension of the reduced-MHD equations to incorporate the effect of ECCD. We critically review two older models intended to include current drive in the single- or multi-fluid MHD equations. In the following section, we describe the main features of the 3D nonlinear MHD code JOREK and show early results from simulations of tearing modes that establish the suitability of the code.

2 Incorporation of ECCD in plasma fluid equations

Most nonlinear MHD simulations of the effects of current drive on MHD modes have employed the model of Giruzzi *et al*[2]. For this model, Ohm's law is modified to include an additional current, the electron cyclotron (EC) driven current $\boldsymbol{J}_{\mathrm{ECCD}}$:

$$(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) = \frac{\eta}{\mu_0} (\boldsymbol{J} - \boldsymbol{J}_{\mathrm{ECCD}}) \ . \tag{1}$$

The MHD equations are extended with an equation to describe the evolution of $\boldsymbol{J}_{\mathrm{ECCD}}$:

$$\frac{\partial \boldsymbol{J}_{\mathrm{ECCD}}}{\partial t} = -\nabla \cdot \Gamma_{\mathrm{ECCD}} + \nu_{\mathrm{res}} (\boldsymbol{J}_0 - \boldsymbol{J}_{\mathrm{ECCD}}) \ , \tag{2}$$

$$\Gamma_{\mathrm{ECCD}} = -\chi_{\parallel} \nabla_{\parallel} \boldsymbol{J}_{\mathrm{ECCD}} - \chi_{\perp} \nabla_{\perp} \boldsymbol{J}_{\mathrm{ECCD}} \ . \tag{3}$$

[a] e-mail: J.L.Pratt@differ.nl
[b] e-mail: E.Westerhof@differ.nl

In equation 2, $\nu_{\rm res}$ is the collision frequency of the resonant electrons that contribute to the EC driven current. Although there are heuristic arguments in favor of the Giruzzi model, it lacks rigorous underpinning.

Hegna and Callen[3] propose a framework to discuss fluid closure in the presence of EC current drive. They start with the quasi-linear Fokker-Planck equation obtained by averaging the kinetic equations over the fast EC waves,

$$\frac{\mathrm{d}\langle f_{\rm s}\rangle}{\mathrm{d}t} = \langle C(f_{\rm s})\rangle + Q_{\rm ECCD}(\langle f_{\rm s}\rangle), \tag{4}$$

where $C(f_{\rm s})$ is the collision operator and the quasi-linear diffusion operator describing the effect of ECCD. The momentum and energy balance equations, produced by taking first and second moments of the quasi-linear Fokker-Planck equation 4, contain an explicit ECCD force:

$$\boldsymbol{F}_{\rm e}^{\rm ECCD} = \int d^3\boldsymbol{v}\ m_{\rm e}\boldsymbol{v}Q_{\rm ECCD}(\langle f_{\rm e}\rangle)\,, \tag{5}$$

and an explicit energy source term:

$$S_{\rm e}^{\rm ECCD} = \int d^3\boldsymbol{v}\ \frac{1}{2}m_{\rm e}v^2 Q_{\rm ECCD}(\langle f_{\rm e}\rangle)\,. \tag{6}$$

Hegna and Callen write a parallel Ohm's law by ignoring electron inertia and assuming $m_{\rm e} \ll m_{\rm i}$,

$$-n_e e E_{||} + R_{||} + F_{||\rm e}^{\rm ECCD} = 0. \tag{7}$$

As Hegna and Callen note, the collisional friction $R_{||}$ is modified in essential ways by the ECCD. Although Jenkins *et al* [7] base their 3D nonlinear MHD simulations of the effect of ECCD on the model as derived by Hegna and Callen, they included only the parallel force from the localized EC power deposition in the modified Ohm's law and neglect modifications to the friction term. To leading order the parallel ECCD force $\boldsymbol{F}_{\rm e}^{\rm ECCD}$ is negligible because the quasi-linear diffusion $Q_{\rm ECCD}$ dominantly operates in the perpendicular direction. The EC driven current is generated by the Fisch-Boozer effect, an asymmetric collisionality in the distribution function caused by the ECCD-generated modification of the parallel friction. With these considerations, we recover Ohm's law as formulated by Giruzzi *et al*:

$$R_{||} = e n_e \eta (J - J_{ECCD})\,. \tag{8}$$

The subtraction of J_{ECCD} from the total current J expresses the physical point that EC driven current does not contribute to the parallel friction. The problem of finding the parallel friction is thus replaced with the problem of deriving a reasonable form for the EC driven current. In quasi-linear Fokker-Planck modeling, the current drive efficiency is defined as the ratio of the total driven-current over the total absorbed-power, $\eta \equiv I_{\rm ECCD}/P_{\rm ECCD}$. We propose an equation for the driven-current

$$\frac{\partial \boldsymbol{J}_{\rm ECCD}}{\partial t} = \nu \frac{\eta p_{\rm ECCD}}{2\pi R} - \nu_{\rm res}\boldsymbol{J}_{\rm ECCD} + v_{\rm res||}\nabla_{||}\boldsymbol{J}_{\rm ECCD}. \tag{9}$$

Eq.9 consists of a driving term proportional to the local absorbed power density $p_{\rm ECCD}$, a term representing the collisional decay of the current, and a term representing parallel transport of the current. This equation differs from the Giruzzi model only in the form of the transport of the EC driven current, which is simply convected with the parallel velocity of the resonant electrons $v_{\rm res||}$. This difference from the Giruzzi model makes the MHD equations that include the effect of ECCD consistent with the closure presented by Hegna and Callen.

3 JOREK Simulation

The JOREK code [6] standardly solves the 3D nonlinear reduced-MHD equations in tokamak geometry. For reduced-MHD, the magnetic ($\boldsymbol{B}$) and velocity ($\boldsymbol{v}$) fields are represented in terms of the poloidal flux function Ψ and velocity stream-function u:

$$\boldsymbol{B} = -\hat{e}_\phi \times \frac{1}{R}\nabla\Psi + \frac{F_0}{R}\hat{e}_\phi \tag{10}$$

$$\boldsymbol{v} = \hat{e}_\phi \times R\nabla u + v_{\|}\boldsymbol{B} \tag{11}$$

In these vector equations, ϕ is the angle in the toroidal direction, and R is the major radius of the tokamak. F_0 is a constant that determines the toroidal magnetic field $B_\phi = F_0/R$. The reduced-MHD equations are solved for six variables: poloidal flux Ψ, velocity stream function u, toroidal current density j, toroidal vorticity ω, density ρ, and temperature T:

$$\frac{\partial\Psi}{\partial t} = \eta j - (v_{\mathrm{pol}} \cdot \nabla)\Psi - F_0\frac{\partial u}{\partial\phi} \tag{12}$$

$$e_\phi \cdot \nabla \times [\rho\frac{\partial \boldsymbol{v}}{\partial t} = -\rho(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} - \nabla \boldsymbol{p} + \boldsymbol{j} \times \boldsymbol{B} + \mu\triangle\boldsymbol{v}] \tag{13}$$

$$j_\phi = R^2\nabla \cdot (R^{-2}\nabla\Psi) \tag{14}$$

$$\omega = \nabla^2_{\mathrm{pol}} u \tag{15}$$

$$\frac{\partial\rho}{\partial t} = -\nabla \cdot (\rho\boldsymbol{v}) + \nabla \cdot (D_\perp\nabla_\perp\rho) + S_\rho \tag{16}$$

$$\rho\frac{\partial T}{\partial t} = -\rho\boldsymbol{v} \cdot \nabla T - (\kappa - 1)p\nabla \cdot \boldsymbol{v} + \nabla \cdot (K_\perp\nabla_\perp T + K_{\|}\nabla_{\|}T) + S_{\mathrm{T}} \tag{17}$$

In these equations, the subscript "pol" indicates the poloidal components of a vector, and perpendicular and parallel components are indicated with respect to the magnetic field. $K_{\|,\perp}$ are the parallel and perpendicular heat diffusivity, and $\kappa = 5/3$ is the ratio of specific heats. The kinematic viscosity $\nu(\mathrm{m^2s^{-1}})$ and resistivity $\eta(\Omega\mathrm{m})$ are given in SI units. There is the possibility to include a particle source S_ρ and a heat source S_{T} in JOREK. In the toroidal direction, the variables are described by Fourier decomposition; in the poloidal plane, variables are treated with a two dimensional Bezier finite element method. Time integration is accomplished with a fully-implicit Crank-Nicholson scheme. JOREK has been used successfully to simulate edge-localized modes (ELMs) and disruptions [5][4][11][10][12].

JOREK has the capacity to simulate a tokamak with a complex magnetic-field structure, including different configurations that allow for a divertor and magnetic-field x-points. For the present work, we run the JOREK simulation for a simple circular tokamak with parameters set to mimic TEXTOR ($B_{\mathrm{T}} = 1.93$, $R = 1.75$, $a = 0.47$, $\kappa = 1.0$). Figure 1 presents a Poincaré map of the magnetic field-lines in a tokamak when an $m = 2$, $n = 1$ tearing mode has developed.

Figure 2 shows the growth-rate γ of the island width while a growing tearing mode is present in the tokamak. The resistivities shown are realistic in modern tokamak experiments, and the machine set-up is the simple TEXTOR tokamak machine discussed above. For these simulation measurements, viscosity is taken to be extremely low so that it does not affect the growth-rate. The growth-rate follows the theoretically-predicted trend $\gamma \sim \eta^{3/5}$ [1] well.

4 Outlook

We have initiated a research programme to model the stabilization of tearing modes by localized ECCD in the context of 3D nonlinear reduced MHD. This research will develop a rigorous theoretical underpinning and verification of the contribution from ECCD in the generalized Rutherford equation. The work will be carried out with the help of the 3D nonlinear reduced-MHD code JOREK. Early calculations of tearing modes performed with JOREK have been presented. Inclusion of ECCD effects requires modification and extension of the reduced-MHD equations in the spirit of Hegna and Callen. This leads to a framework similar to that proposed by Giruzzi *et al*, but with an important modification. In this framework, Ohm's law is modified consistent with the frictionless nature of the EC driven

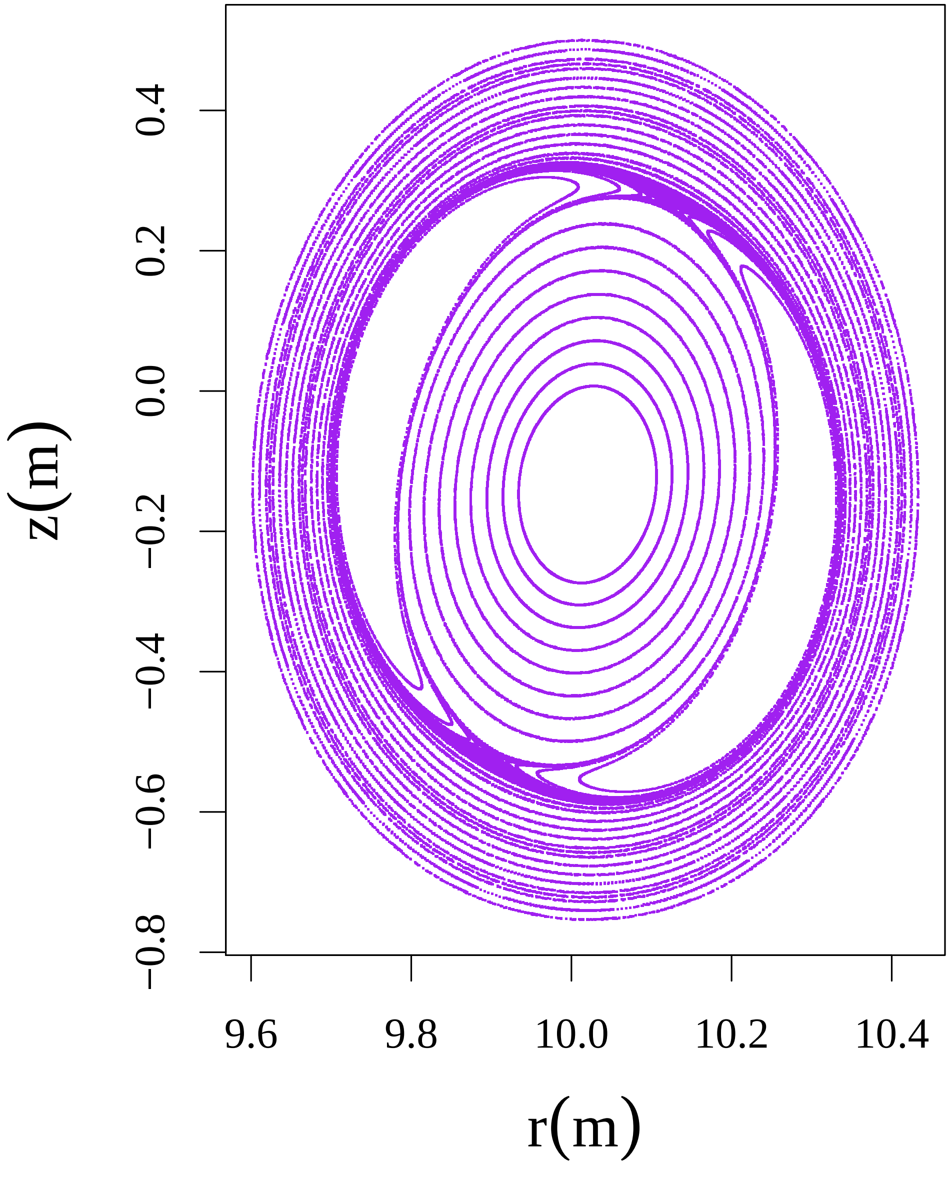

Fig. 1. Poincare map of a 2/1 tearing mode produced by JOREK simulation of a general tokamak.

current (1, and the model is extended with an additional equation describing the evolution of the EC driven-current (9). While continuing the work to design a rigorous theory, we will implement an ECCD addition in JOREK.

Acknowledgement
Many thanks to G.T.A. Huysmans, W.-C. Müller, M. Hölzl, and the participants of the ASTER project and JOREK developer group for their advice in working with JOREK.

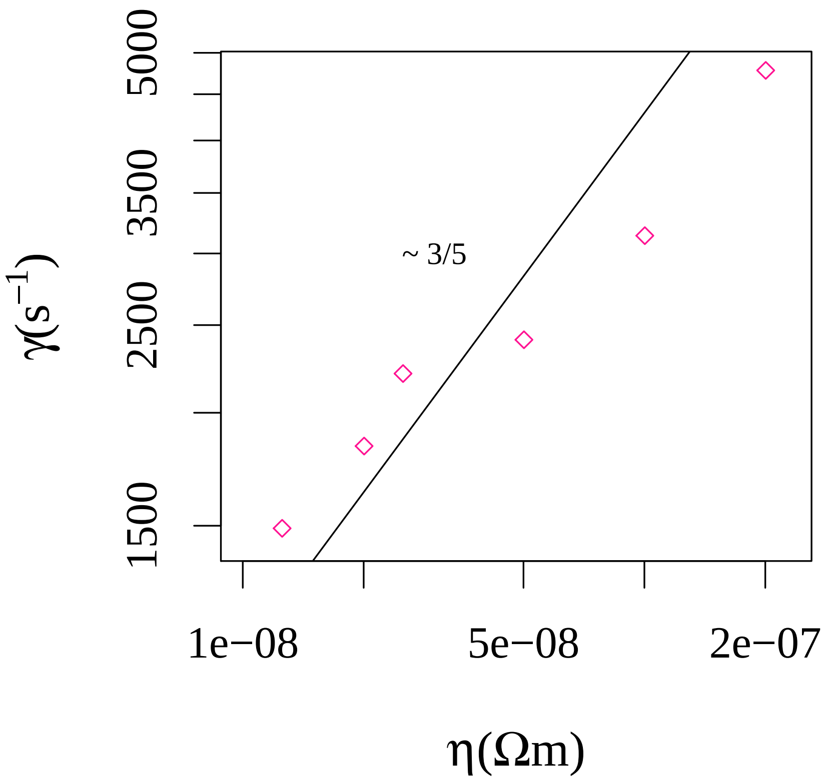

Fig. 2. Tearing-mode growth-rate vs. resistivity in JOREK simulations of a circular tokamak with TEXTOR parameters. The range of resistivities examined include those in current tokamak experiments. The viscosity in these simulations is set to be very low, in order to avoid viscosity effects.

The work in this paper has been performed within the framework of the NWO-RFBR Centre of Excellence (grant 047.018.002) on Fusion Physics and Technology. This work, supported by NOW and the European Communities under the contract of the Association EURATOM/FOM, was carried out within the framework of the European Fusion Programme. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

References

1. D. Biskamp. *Nonlinear Magnetohydrodynamics*. Cambridge University Press, Cambridge, 1993.
2. G. Giruzzi, M. Zabiégo, T. Gianakon, X. Garbet, A. Cardinali, and S. Bernabei. Dynamical modelling of tearing mode stabilization by RF current drive. *Nuc. Fus.*, 39(1):107, 1999.
3. C. C. Hegna and J. D. Callen. A closure scheme for modeling RF modifications to the fluid equations. *Phys. Plasmas*, 16(11):112501, 2009.
4. M. Hölzl, S. Günter, R. P. Wenninger, W.-C. Müller, G. Huysmans, K. Lackner, I. Krebs, and ASDEX Upgrade Team. Reduced-MHD simulations of toroidally and poloidally localized ELMs. *arXiv:1201.5765v2*, 2012.
5. G. Huysmans and O. Czarny. MHD stability in x-point geometry: simulation of ELMs. *Nuc. Fus.*, 47(7):659, 2007.
6. G. T. A. Huysmans, S. Pamela, E. van der Plas, and P. Ramet. Non-linear MHD simulations of edge localized modes (ELMs). *Plasma Physics and Controlled Fusion*, 51(12):124012, 2009.

7. T. G. Jenkins, S. E. Kruger, C. C. Hegna, D. D. Schnack, and C. R. Sovinec. Calculating electron cyclotron current drive stabilization of resistive tearing modes in a nonlinear magnetohydrodynamic model. *Phys. Plasmas*, 17(1):012502, 2010.
8. H. Lütjens and J.-F. Luciani. Saturation levels of neoclassical tearing modes in International Thermonuclear Experimental Reactor plasmas. *Phys. Plasmas*, 12(8):080703, 2005.
9. F. Militello and F. Porcelli. Simple analysis of the nonlinear saturation of the tearing mode. *Phys. Plasmas*, 11(5):L13–L16, 2004.
10. S. Pamela, G. Huysmans, and S. Benkadda. Influence of poloidal equilibrium rotation in MHD simulations of edge-localized modes. *Plasma Physics and Controlled Fusion*, 52(7):075006, 2010.
11. S. J. P. Pamela, G. T. A. Huysmans, M. N. A. Beurskens, S. Devaux, T. Eich, S. Benkadda, and J. E. contributors. Nonlinear MHD simulations of edge-localized-modes in JET. *Plasma Physics and Controlled Fusion*, 53(5):054014, 2011.
12. C. Reux, G. Huysmans, J. Bucalossi, and M. Bécoulet. Non-linear simulations of disruption mitigation using massive gas injection on tore supra. *38th EPS Conference on Plasma Physics*, page O3.117, 2011.
13. F. Waelbroeck. Theory and observations of magnetic islands. *Nuc Fus*, 49(10):104025, 2009.