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Anomalous decays of η' and η into four pions

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Abstract. We report about the calculation of the branching ratios of the yet unmeasured η' decays into four pions, based on a combination of chiral perturbation theory and vector-meson dominance [1]. The decays $\eta' \to 2(\pi^+\pi^-)$ and $\eta' \to \pi^+\pi^- 2\pi^0$ are P-wave dominated and can largely be thought to proceed via two ρ resonances. Branching fractions of $(1.0 \pm 0.3) \times 10^{-4}$ and $(2.4 \pm 0.7) \times 10^{-4}$, respectively, are calculated which are not much lower than the current experimental upper limits. The decays $\eta' \to 4\pi^0$ and $\eta \to 4\pi^0$, in contrast, are D-wave driven [2] as long as conservation of CP symmetry is assumed, and are significantly further suppressed. Any experimental evidence for the decay $\eta \to 4\pi^0$ could almost certainly be interpreted as a signal of CP violation. The CP-violating amplitudes for $\eta', \eta \to 4\pi^0$, induced by the QCD θ-term, are given as well.

1 Introduction

The following report about the four-pion decays of the η and η' mesons is based on a joint project with Feng-Kun Guo and Bastian Kubis (Bonn University) [1] which was initiated by Andrzej Kupść [2].

Very little is known about the four-pion decays of the η and η'. From the experimental point-of-view only upper limits on branching ratios exist [3]; this, however, may change in the near future with the advent of high-statistics η' experiments such as BES-III, WASA-at-COSY, ELSA, CB-at-MAMI-C, CLAS at Jefferson Lab, etc. On the theory side, we have found only one previous calculation, performed in the framework of a quark model [4], whose partial width predictions, however, have in the meantime been ruled out by the experimental upper limit of the channel $\eta' \to 2(\pi^+\pi^-)$ [3,5].

Note that the decays $\eta' \to 4\pi$, in contradistinction to many other η' decay channels, seem not terribly forbidden by approximate symmetries: they are neither isospin-forbidden, nor required to proceed via electromagnetic interactions. The reaction $\eta \to 4\pi$, in contrast, is essentially suppressed by tiny phase space: $M_\eta - 4M_{\pi^0} = 7.9\,\text{MeV}$, $M_\eta - 2(M_{\pi^\pm} + M_{\pi^0}) = -1.2\,\text{MeV}$, such that only the decay into $4\pi^0$ is kinematically allowed.

As an odd number of pseudoscalars, i.e. (pseudo-) Goldstone bosons, is involved in these decays, they are of odd intrinsic parity and belong to the greater class of anomalous decays. In low-energy QCD these anomalous decays are governed – in the chiral limit – by the Wess–Zumino–Witten (WZW) term [6] via chiral anomalies which are of $O(p^4)$ in the chiral counting. The so-called triangle anomaly is well-tested in processes such as $\pi^0, \eta \to \gamma\gamma$, and the box anomaly contributes e.g. to $\gamma\pi \to \pi\pi$ and $\eta \to \pi\pi\gamma$. The pentagon anomaly, however, remains more elusive; the simplest possible process for such a five-point function $PPPPP$ of pseudoscalars P is $K^+K^- \to \pi^+\pi^-\pi^0$, which has not been experimentally tested yet and which is likely to be subject to large corrections to the chiral-limit amplitude that is dictated by the WZW term.

Since any anomalous amplitude always involves the totally antisymmetric tensor $\epsilon_{\mu\nu\alpha\beta}$, it can be shown that no two pseudoscalars are allowed to be in a relative S-wave: assuming they were, this would

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reduce the five-point function $PPPPP$ effectively to a four-point function $SPPP$ (where S stands for a scalar) which doesn't any longer possess four independent vectors for the contraction of the ϵ tensor. Thus any decay with five pseudoscalars can be expected to be P-wave dominated. This also holds for decays $\eta' \to 2(\pi^+\pi^-)$ and $\eta' \to \pi^+\pi^-2\pi^0$. However, since the antisymmetric product of four SU(2) (flavor) states has to vanish, the WZW term generates only a pentangle-anomaly contribution, which is genuinely of SU(3) nature, if at least two kaons are involved. Therefore, the η' and η four-pion decays do not directly belong to the pentangle-anomaly class of $O(p^4)$.

As furthermore Bose symmetry forbids two neutral pions to be in an odd partial wave, the decays $\eta' \to 4\pi^0$ and $\eta \to 4\pi^0$ even require all π^0 to be at least in relative D-waves [2]. This, combined with the tiny phase space available, leads to the notion of $\eta \to 4\pi^0$ being CP-forbidden [3,7,8], although strictly speaking it is only S-wave CP-forbidden.

2 The P-wave dominated decays $\eta' \to 2(\pi^+\pi^-)$ and $\eta' \to \pi^+\pi^-2\pi^0$

Since the decays $\eta' \to 2(\pi^+\pi^-)$ and $\eta' \to \pi^+\pi^-2\pi^0$ cannot be described by a tree-level process at $O(p^4)$, they must be at least of $O(p^6)$ in the chiral counting. In fact, the leading order processes of these decays, given by the Feynman diagrams presented in the left and right panel of Fig. 1, do contribute at $O(p^6)$, because of the additional kaon-loop or since the contact term, which is needed to remove the divergence of the loop, results from the $O(p^6)$ anomalous chiral Lagrangian tabulated in [9].

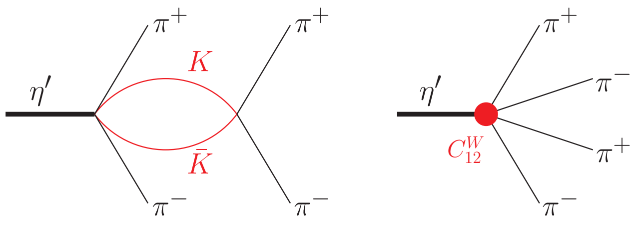

Fig. 1. Feynman diagrams contributing to $\eta' \to 2(\pi^+\pi^-)$ at leading order, $O(p^6)$. Left: The $\eta'\pi^+\pi^-K\bar{K}$ vertex (to the left of kaon loop) denotes a pentangle anomaly coupling of $O(p^4)$, the $KK \to \pi\pi$ scattering (to the right of kaon loop) is of $O(p^2)$. Right: (C^W_{12}) vertex from the $O(p^6)$ anomalous counter-term Lagrangian of [9]. The $\eta' \to \pi^+\pi^-2\pi^0$ amplitude is described by analogous amplitudes.

In order to estimate the finite contribution of the counter term proportional to the low-energy constant C^W_{12} (in the notation of [9]), the two final-state pion pairs which have to be, as mentioned above, in a relative P-wave state, can be saturated by vector mesons (here ρ mesons [2]), see Fig. 2. In

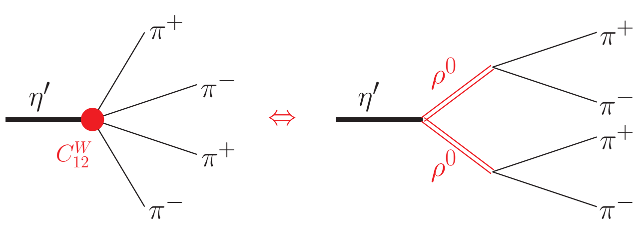

Fig. 2. Left: The vertex (red circle) denotes the (local) counter term of $O(p^6)$, see [9]. Right: $\pi^+\pi^-$ pair saturation by ρ^0 mesons. Analogous diagrams apply for the case $\eta' \to \pi^+\pi^-2\pi^0$.

[1] this estimate was performed in the framework of the original hidden local symmetry model [10–13] or modern extension [14,15] with the outcome that the vector-meson contributions totally dominate the kaon-loop ones. The corresponding results for the branching ratios are reported in Sect. 4.

3 The D-wave dominated decays $\eta' \to 4\pi^0$ and $\eta \to 4\pi^0$

Since all $\pi^0\pi^0$ pairs have to emerge in relative D-waves, the chiral power counting increases from $\mathcal{O}(p^6)$ (valid for the four-pion decays with two charged pions in the final state, see Sect. 2) to $\mathcal{O}(p^{10})$, as the two charged intermediate pions have to rescatter into a D-wave $\pi^0\pi^0$ pair, a process which is of $\mathcal{O}(p^4)$, see the left panel of Fig. 3. A full calculation of the $\eta', \eta \to 4\pi^0$ amplitude at $\mathcal{O}(p^{10})$,

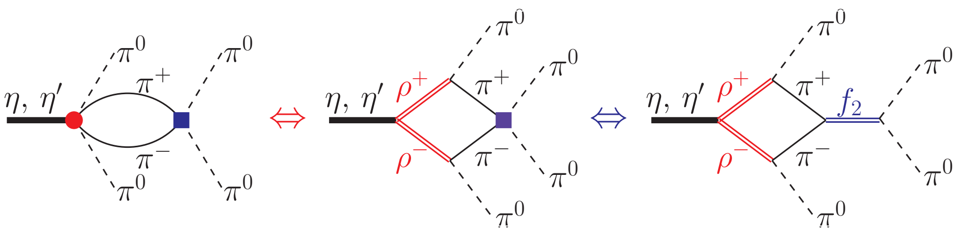

Fig. 3. Feynman diagrams contributing to $\eta', \eta \to 4\pi^0$ at $\mathcal{O}(p^{10})$. Left: Pion-loop contribution; the left vertex (red circle) denotes the effective $\eta, \eta' \to \pi^+\pi^-2\pi^0$ coupling of $\mathcal{O}(p^6)$ (see Sect. 2), the right vertex (blue square) an effective D-wave $\pi\pi$ rescattering of $\mathcal{O}(p^4)$. Middle: Saturation of the $\mathcal{O}(p^6)$ process by vector ($\rho^\pm$) meson dominance (red). Right: D-wave $\pi^+\pi^- \to \pi^0\pi^0$ scattering saturated by $f_2(1270)$ tensor meson dominance (blue).

which would be at the level of three-loops in the anomalous sector, would be a formidable task and certainly beyond the scope of an exploratory study. Instead the leading contribution to imaginary part of the decay amplitude at $\mathcal{O}(p^{10})$, represented by the charged-pion intermediate states (left panel of Fig 3), is calculated in [1] in the following approximations: given the numerical dominance of the $\mathcal{O}(p^6)$ counter term over the kaon-loop contribution, the non-linear kaon-loop terms are neglected and the $\eta, \eta' \to \pi^+\pi^-2\pi^0$ part of the amplitude is saturated by full vector-meson dominance, see the middle panel of Fig. 3. Secondly, the full D-wave final state amplitude for the rescattered $\pi^0\pi^0$ pair is reconstructed via the corresponding Omnès function (neglecting any crossed-channel effects) which at threshold is related to the isospin-zero D-wave partial wave t_2^0 as

$$\mathrm{Im}\ \Omega_2^0(s) \approx t_2^0(s)\sqrt{1 - 4M_\pi^2/s}$$

and which away from threshold can be saturated by $f_2(1270)$ tensor meson dominance

$$\Omega_2^0(s) \approx M_{f_2}^2/(M_{f_2}^2 - s)\,,$$

see the right panel of Fig. 3. The corresponding results for the branching ratios are reported in the next section, Sect. 4.

4 Results and conclusions

The decays of the η' and η into four pions are processes of an odd number of pseudoscalars and are therefore anomalous. The leading contributions to the η' decay amplitudes with charged pions in the final state, where CP symmetry forbids the pions to be in relative S-waves, are of $\mathcal{O}(p^6)$ according to chiral power-counting rules. Then, in the framework of hidden local symmetry for vector mesons, the vector-meson exchange saturates the $\mathcal{O}(p^6)$ low-energy constants, such that the (P-wave) decay amplitude is entirely governed by ρ intermediate states. Hence the dominant contribution is given by the triangle anomaly via $\eta' \to \rho\rho$ (while the box term correction is numerically small), not by the pentagon anomaly. In this manner, the branching fractions for the (yet unmeasured) $\eta' \to 2(\pi^+\pi^-)$ and $\eta' \to \pi^+\pi^-2\pi^0$ decays are predicted to be [1]

$$\mathcal{B}\big(\eta' \to 2(\pi^+\pi^-)\big) = (1.0 \pm 0.3) \times 10^{-4}\,,$$
$$\mathcal{B}\big(\eta' \to \pi^+\pi^-2\pi^0\big) = (2.4 \pm 0.7) \times 10^{-4}\,, \qquad (1)$$

respectively, where the uncertainty estimates are based on the typical $1/N_c$ correction of about 30 %. The branching ratio for the first decay is only a factor of two smaller than the current experimental upper limit [3]. Thus it should be testable in the near future with the modern high-statistics facilities.

Predictions for the η' and η decays into four neutral pions are much more difficult, as Bose symmetry requires the pions to emerge in relative D-waves (assuming CP conservation), suppressing the amplitudes to $O(p^{10})$ in chiral power counting. An estimate of this decay via charged-pion-loop contribution with D-wave pion–pion charge-exchange rescattering, which dominates over an alternative, in fact, negligible mechanism through two f_2 mesons, predicts the following order-of-magnitude values of the CP-conserving branching ratios [1]

$$\begin{aligned} \mathcal{B}\Big(\eta' \to 4\pi^0\Big) &\sim 4 \times 10^{-8}\,, \\ \mathcal{B}\Big(\eta \to 4\pi^0\Big) &\sim 3 \times 10^{-30}\,. \end{aligned} \tag{2}$$

Note that the D-wave mechanism for the $\eta' \to 4\pi^0$ decay is suppressed by 3–4 orders of magnitude compared to the decays with charged-pion final states (1) which are of P-wave nature. It is also 4 orders of magnitude smaller than the current experimental bound [3, 16]. The CP-conserving decay width of $\eta \to 4\pi^0$ on the other hand is so small that any signal to be observed would indicate CP-violating physics (the current bound is 6.9×10^{-7} [3, 7], see e.g. [17] for future plans). Following the effective-Lagrangian treatment of [18] the CP-violating decays induced by the so-called θ term, an additional term in the QCD Lagrangian necessitated for the solution of the $U(1)_A$ problem, are estimated [1]

$$\begin{aligned} \mathcal{B}(\eta' \xrightarrow{\text{CPV}} 4\pi^0) &= 9 \times 10^{-2} \times \bar{\theta}_0^2\,, \\ \mathcal{B}(\eta \xrightarrow{\text{CPV}} 4\pi^0) &= 5 \times 10^{-5} \times \bar{\theta}_0^2\,. \end{aligned} \tag{3}$$

In fact, were $\bar{\theta}_0$ a quantity of natural size, the above branching ratios would demonstrate the enhancement of the CP-violating S-wave mechanism compared to the CP-conserving D-wave one. Note, however, that the current limits on the QCD vacuum angle derived from neutron electric dipole moment measurements, $\bar{\theta}_0 \lesssim 10^{-11}$ [19], bound these branching fractions beyond anything measurable. Still, for $\eta \to 4\pi^0$, the suppression of the CP-conserving D-wave mechanism, see Eq. (2), is so strong that it is even smaller than the CP-violating (S-wave) one in Eq. (3) if the current bounds are inserted for $\bar{\theta}_0$.

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