EPJ Web of Conferences **37**, 03002 (2012)
DOI: 10.1051/epjconf/20123703002

$\eta' \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$: a primer analysis

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Abstract. The electromagnetic rare decays $\eta' \to \pi^0\gamma\gamma$ and $\eta' \to \pi^0\gamma\gamma$ are analysed for the first time and their predicted branching ratios given. The vector meson exchange dominant contribution is treated using Vector Meson Dominance and the scalar component is estimated by means of the Linear Sigma Model. The agreement between our calculation and the measurement of the related process $\eta \to \pi^0\gamma\gamma$ is a check of the procedure. Scalar meson effects are seen to be irrelevant for $\eta' \to \pi^0\gamma\gamma$, while a significant scalar contribution due to the $\sigma(500)$ resonance seems to emerge in the case of $\eta' \to \pi^0\gamma\gamma$. Future measurements coming from KLOE-2, Crystal Ball, WASA, and BES-III will elucidate if any of these processes carry an important scalar contribution or they are simply driven by the exchange of vector mesons.

1 Introduction

From the experimental point of view, the electromagnetic rare decays $\eta' \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$, which have not yet been observed, will be potentially measured at several experiments such as KLOE-2, Crystal Ball, WASA, and BES-III [1,2], while the related $\eta \to \pi^0\gamma\gamma$ process, for which a measurement of its branching ratio and invariant mass spectrum already exists [3], will be certainly determined with higher precision. The situation at present is the following. The branching ratio (BR) of $\eta \to \pi^0\gamma\gamma$ has been measured by GAMS-2000 [4], BR $= (7.2 \pm 1.4) \times 10^{-4}$, and CrystalBall@AGS in 2005 [5], BR $= (3.5 \pm 0.7 \pm 0.6) \times 10^{-4}$, and 2008 [3], BR $= (2.21 \pm 0.24 \pm 0.47) \times 10^{-4}$, the latter also including an invariant-mass spectrum for the two photons. The PDG 2010 fit is BR $= (2.7 \pm 0.5) \times 10^{-4}$ [6]. More recently, preliminary results from CrystalBall@MAMI [7], BR $= (2.25 \pm 0.46 \pm 0.17) \times 10^{-4}$, and KLOE [8,9], BR $= (0.84 \pm 0.27 \pm 0.14) \times 10^{-4}$, have been reported as well. For the $\eta' \to \pi^0\gamma\gamma$ decay, only an upper bound exists, BR $< 8 \times 10^{-4}$ at 90% CL, obtained by the GAMS-2000 experiment [10] 25 years ago. Finally, for $\eta' \to \eta\gamma\gamma$ there is no experimental evidence so far.

On the theory side, the $\eta \to \pi^0\gamma\gamma$ process has been studied in many different frameworks, starting from Chiral Perturbation Theory (ChPT) [11] and ending with the most recent chiral unitary approach [12,13], most of them in combination with the Vector Meson Dominance (VMD) prediction. On the contrary, there are no theoretical analyses neither for $\eta' \to \pi^0\gamma\gamma$ nor for $\eta' \to \eta\gamma\gamma$.

Here, we present a first calculation of the electromagnetic rare decays $\eta' \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$ with the aim of completing the existing calculations for $\eta \to \pi^0\gamma\gamma$. Since we are more interested in an estimate rather than a detailed calculation, we include only the two main contributions, that is, the exchange of intermediate vector mesons through the decay chain $P^0 \to V\gamma \to P^0\gamma\gamma$ plus the chiral loops. Later, the chiral-loop prediction will be substituted by a Linear Sigma Model (LσM) calculation where the effects of scalar meson resonances are taken into account explicitly.

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2 Chiral-loop prediction

In ChPT, the first non-vanishing contribution to $\eta \to \pi^0\gamma\gamma$ comes at $O(p^4)$ through a loop of charged kaons (pion loops violate G-parity and are thus suppressed). To simplify, we work in the isospin limit and neglect the second contribution. The $O(p^8)$ loop corrections from diagrams with two anomalous vertices are seen to be very small [11] and not considered here. The explicit contributions due to the exchange of intermediate vector and scalar resonances, which in the framework of ChPT are understood as tree-level contributions at $O(p^6)$ as soon as the full propagators are expanded and their lowest terms retained, are postponed to the next sections.

We start discussing the $\eta \to \pi^0\gamma\gamma$ case. The amplitude is written as

$$\mathcal{A}^{\chi}_{\eta\to\pi^0\gamma\gamma} = \frac{2\alpha}{\pi}\frac{1}{m^2_{K^+}}L(s_K)\{a\} \times \mathcal{A}^{\chi}_{K^+K^-\to\pi^0\eta}\,, \tag{1}$$

where $\{a\} = (\epsilon_1 \cdot \epsilon_2)(q_1 \cdot q_2) - (\epsilon_1 \cdot q_2)(\epsilon_2 \cdot q_1)$, $\epsilon_{1,2}$ and $q_{1,2}$ are the polarization and four-momentum vectors of the final photons, $s_K = s/m^2_{K^+}$, $s = (q_1 + q_2)^2 = 2q_1 \cdot q_2$ is the invariant mass of the two photons, $L(\hat{s})$ is the loop integral defined as

$$L(z) = -\frac{1}{2z} - \frac{2}{z^2}f\left(\frac{1}{z}\right)\,, \quad f(z) = \begin{cases} \frac{1}{4}\left(\log\frac{1+\sqrt{1-4z}}{1-\sqrt{1-4z}} - i\pi\right)^2 & \text{for } z < \frac{1}{4} \\ -\left[\arcsin\left(\frac{1}{2\sqrt{z}}\right)\right]^2 & \text{for } z > \frac{1}{4} \end{cases}\,, \tag{2}$$

and $\mathcal{A}^{\chi}_{K^+K^-\to\pi^0\eta}$ is the four-pseudoscalar amplitude

$$\mathcal{A}^{\chi}_{K^+K^-\to\pi^0\eta} = \frac{1}{4f_\pi^2}\left[\left(s - \frac{m_\eta^2}{3} - \frac{8m_K^2}{9} - \frac{m_\pi^2}{9}\right)(\cos\varphi_P + \sqrt{2}\sin\varphi_P) + \frac{4}{9}(2m_K^2 + m_\pi^2)\left(\cos\varphi_P - \frac{\sin\varphi_P}{\sqrt{2}}\right)\right]\,, \tag{3}$$

with φ_P the η-η' mixing angle in the quark-flavour basis, resulting from the loop computation (not to confuse it with the four-pseudoscalar scattering amplitude calculated in ChPT at lowest order). It is important to notice that in the seminal work of Ref. [11] this chiral-loop prediction was computed taking into account the η_8 contribution alone and the mixing angle was fixed to $\theta_P = \varphi_P - \arctan\sqrt{2} = \arcsin(-1/3) \simeq -19.5^\circ$. Now, the η_0 contribution is also considered (in the large-N_c limit where the pseudoscalar singlet is the ninth pseudo-Goldstone boson) and the dependence on the mixing angle is made explicit.

For the $\eta' \to \pi^0\gamma\gamma$ case, the associated amplitude is that of Eq. (1) but replacing $m_\eta \to m_{\eta'}$, $(\cos\varphi_P + \sqrt{2}\sin\varphi_P) \to (\sin\varphi_P - \sqrt{2}\cos\varphi_P)$ and $(\cos\varphi_P - \sin\varphi_P/\sqrt{2}) \to (\sin\varphi_P + \cos\varphi_P/\sqrt{2})$ in Eq. (3).

Finally, for the $\eta' \to \eta\gamma\gamma$ case, two amplitudes contribute, one through a loop of charged kaons, as in the former two cases, and the other through a loop of charged pions, which in this case is not suppressed by G-parity. Again, the corresponding amplitudes are that of Eq. (1), replacing $s_K \to s_\pi$ and $m_{K^+} \to m_{\pi^+}$ for the pion loop, with

$$\mathcal{A}^{\chi}_{K^+K^-\to\eta\eta'} = -\frac{1}{4f_\pi^2}\left[\left(s - \frac{m_\eta^2 + m_{\eta'}^2}{3} - \frac{8m_K^2}{9} - \frac{2m_\pi^2}{9}\right)\left(\sqrt{2}\cos 2\varphi_P + \frac{\sin 2\varphi_P}{2}\right) + \frac{4}{9}(2m_K^2 - m_\pi^2)\left(2\sin 2\varphi_P - \frac{\cos 2\varphi_P}{\sqrt{2}}\right)\right]\,, \tag{4}$$

and

$$\mathcal{A}^{\chi}_{\pi^+\pi^-\to\eta\eta'} = \frac{m_\pi^2}{2f_\pi^2}\sin 2\varphi_P\,. \tag{5}$$

The latter amplitude coincides with that of $\eta' \to \eta\pi^+\pi^-$ when computed in the large-N_c ChPT at lowest order [14]. Needless to say, the former amplitudes for $\eta' \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$ constitute the first chiral-loop predictions of these two processes.

3 VMD prediction

Next to the chiral-loop predictions, there are also contributions from the corresponding exchange of intermediate vector bosons which are calculated in the framework of VMD. The full VMD amplitude was seen to produce the dominant contribution to $\eta \to \pi^0\gamma\gamma$ [11], and the same happens, as we see below, for $\eta' \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$. Now, we review the calculation for the $\eta \to \pi^0\gamma\gamma$ case, with some improvements with respect to Ref. [11], and then calculate for the first time the full VMD amplitudes of $\eta' \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$. For $\eta \to \pi^0\gamma\gamma$, the amplitude is written as

$$\mathcal{A}^{\rm VMD}_{\eta\to\pi^0\gamma\gamma} = \sum_{V=\rho,\omega,\phi} g_{V\eta\gamma}g_{V\pi^0\gamma}\left[\frac{(P\cdot q_2 - m_\eta^2)\{a\} - \{b\}}{D_V(t)} + \left\{\begin{array}{c} q_2 \leftrightarrow q_1 \\ t \leftrightarrow u \end{array}\right\}\right] , \tag{6}$$

where $t, u = (P - q_{2,1})^2 = m_\eta^2 - 2P\cdot q_{2,1}$, $\{b\} = (\epsilon_1\cdot q_2)(\epsilon_2\cdot P)(P\cdot q_1) + (\epsilon_2\cdot q_1)(\epsilon_1\cdot P)(P\cdot q_2) - (\epsilon_1\cdot\epsilon_2)(P\cdot q_1)(P\cdot q_2) - (\epsilon_1\cdot P)(\epsilon_2\cdot P)(q_1\cdot q_2)$ and $D_V(t) = m_V^2 - t - i\, m_V \Gamma_V$ are the vector meson propagators for $V = \omega, \phi$. For the ρ we use instead an energy-dependent $\Gamma_\rho(t) = \Gamma_\rho \times [(t - 4m_\pi^2)/(m_\rho^2 - 4m_\pi^2)]^{3/2} \times \theta(t - 4m_\pi^2)$. For $\eta' \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$, the related amplitudes are Eq. (6) with the replacements $g_{V\eta\gamma}g_{V\pi^0\gamma} \to g_{V\eta'\gamma}g_{V\pi^0\gamma}$ and $g_{V\eta\gamma}g_{V\pi^0\gamma} \to g_{V\eta'\gamma}g_{V\eta\gamma}$, respectively, and $m_\eta^2 \to m_{\eta'}^2$. When the OZI-rule is applied, that is $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\phi = s\bar{s}$, the corresponding couplings are

$$\begin{aligned}
g_{\rho\eta\gamma}g_{\rho\pi^0\gamma} &= g_{\omega\eta\gamma}g_{\omega\pi^0\gamma} = \left(\frac{Ge}{\sqrt{2}g}\right)^2 \tfrac{1}{3}\cos\varphi_P\ , \qquad g_{\phi\eta\gamma}g_{\phi\pi^0\gamma} = 0\ ,\\
g_{\rho\eta'\gamma}g_{\rho\pi^0\gamma} &= g_{\omega\eta'\gamma}g_{\omega\pi^0\gamma} = \left(\frac{Ge}{\sqrt{2}g}\right)^2 \tfrac{1}{3}\sin\varphi_P\ , \qquad g_{\phi\eta'\gamma}g_{\phi\pi^0\gamma} = 0\ ,\\
g_{\rho\eta'\gamma}g_{\rho\pi^0\gamma} &= 9g_{\omega\eta'\gamma}g_{\omega\pi^0\gamma} = -\tfrac{9}{4}g_{\phi\eta'\gamma}g_{\phi\pi^0\gamma} = \left(\frac{Ge}{\sqrt{2}g}\right)^2 \cos\varphi_P\sin\varphi_P\ ,
\end{aligned} \tag{7}$$

where $G = 3g^2/(4\pi^2 f_\pi)$ and g is the vector-pseudoscalar-pseudoscalar coupling constant of VMD which can be fixed from various ρ and ω decay data.

In Ref. [11], the VMD prediction for $\eta \to \pi^0\gamma\gamma$ was calculated assuming equal ρ and ω contributions and without including the decay widths in the propagators. In this case, these approximations are valid since the phase space available prevents the vector mesons to resonate. However, for $\eta' \to \pi^0\gamma\gamma$, the phase space allowed permits these vectors to be on-shell and the introduction of their decay widths is mandatory. For this reason, we include, for all the three cases, the decay widths in the vector meson propagators.

4 LσM prediction

An estimate of the scalar meson exchange effects to the processes under study can be achieved in the LσM where the complementarity between this model and ChPT can be used to include the scalar meson poles at the same time as keeping the correct low-energy behavior expected from chiral symmetry. This procedure was applied with success to the related $V \to P^0P^0\gamma$ decays [15]. The $a_0(980)$ enters into the calculation of $\eta \to \pi^0\gamma\gamma$ and $\eta' \to \pi^0\gamma\gamma$, more intensively in the latter case on account of phase space, while the $\sigma(600)$ and $f_0(980)$ do the same in $\eta' \to \eta\gamma\gamma$, although only the first contributes in a substantial way. Taking into account the scalar meson effects in an explicitly way does not provide a noticeable improvement with respect to the chiral-loop prediction, except for the case of $\eta' \to \eta\gamma\gamma$ where the σ contribution turns out to be considerable. However, the details of this calculation are involved and will be described elsewhere [16].

5 Preliminary results

The preliminary results of our analysis are shown in Table 1, where the predictions of chiral loops, the LσM, which replaces the former when scalar meson poles are incorporated, VMD, and the total decay

	chiral loops	LσM	VMD	Γ	$\mathrm{BR_{th}} \times 10^4$	$\mathrm{BR_{exp}} \times 10^4$
$\eta \to \pi^0\gamma\gamma$ (eV)	1.24×10^{-3}	4.5×10^{-4}	0.26	0.28	2.1	2.7 ± 0.5
$\eta' \to \pi^0\gamma\gamma$ (keV)	7.7×10^{-5}	1.3×10^{-4}	1.29	1.29	65	<8 (90% CL)
$\eta' \to \eta\gamma\gamma$ (eV)	1.4×10^{-2}	0.96	48.8	51.2	2.6	—

Table 1. Chiral-loop, LσM and VMD predictions for $\eta \to \pi^0\gamma\gamma$, $\eta' \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$. The total decay widths are calculated from the coherent sum of the LσM and VMD contributions. The comparison between the predicted branching ratios and the present experimental values, if available, is also performed.

width and branching ratio for the three processes are included. The comparison with the experimental results, if available, is also displayed. The total decay width is the result of adding the LσM and VMD contributions coherently. For the numerical results, we use $f_\pi = 92.2$ MeV, $|g| = 4.2$ from the present value of $\rho \to \pi\pi$, and $\varphi_P = (40.4 \pm 0.6)^\circ$ [17] for the η-η' mixing angle. For $\eta \to \pi^0\gamma\gamma$, our calculation agrees with the "all-order estimate" of Ref. [11] and the more involved analysis of Refs. [12,13], thus giving support to our approach as a starting point for the determination of the other two processes. For $\eta' \to \pi^0\gamma\gamma$, the intermediate vector meson contributions dominate and the scalar meson effects are seen to be negligible. The ω contribution prevails with a 80.2% of the total VMD signal, while the ρ contributes with a 4.6%. The predicted branching ratio appears to be one order of magnitude bigger than the old experimental upper bound. Therefore, a new measurement would be welcome. Finally, for $\eta' \to \eta\gamma\gamma$, the VMD contribution also dominates but the scalar meson effects seem to be sizable, in particular those related with the σ meson. The interference term is constructive. The ρ, ω and ϕ contribute with a 59.9%, 15.8% and 1.6%, respectively, while the LσM calculation enhances by two orders of magnitude the chiral-loop prediction. Since G-parity does not apply to this case, the loop of charged kaons is suppressed and only the charged-pion loop plays a role.

In summary, the calculated branching ratios for $\eta' \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$ give values which we consider are large enough to be measured in the near future by several experimental collaborations.

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