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Dalitz plot studies of $D^0 \to K_S^0 \pi^+ \pi^-$ decays

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Abstract. The availaible data on the $D^0 \to K_S^0 \pi^+ \pi^-$ decays are analysed in the framework of the quasi two-body QCD factorization approximation. The annihilation, via W-exchange, amplitudes are added to the weak-decay tree amplitudes. The doubly Cabibbo suppressed parts of the amplitudes are also considered. The strong interactions between the kaon-pion and pion-pion pairs in the S-, P- and D-final states are described in terms of the corresponding form factors. The kaon-pion or pion-pion scalar and vector form factors are constrained by other experimental data. Unitarity, analyticity and chiral symmetry are also used to obtain their functional forms. We go through a minimization procedure to reproduce the $K_S^0 \pi^-$, $K_S^0 \pi^+$ and $\pi^+ \pi^-$ effective mass projections of the Dalitz plot distributions. The large number (27) of non-zero amplitudes leads to a large number of parameters. The resulting model distributions and branching fractions are compared to the accurate Belle Collaboration data.

1 Introduction

Dalitz-plot time dependent amplitude analyses have been recently performed [1,2] for the CP self conjugate D^0 meson decays into $K_S^0 \pi^+ \pi^-$. These studies have allowed a direct measure of the D^0-$\bar{D}^0$ mixing parameters, the knowledge of which could show the presence of new physics contribution beyond the standard model. Studies of $B^\pm \to D^{(*)} K^\pm, D \to K_S^0 \pi^+ \pi^-$ decays, in which the interference between D^0 and $\bar{D}^0$ mesons was used to measure the Cabibbo-Kobayashi-Maskawa angle γ, have also been accomplished [3,4]. A good understanding of the final state interactions between mesons in the D^0 decays into $K_S^0 \pi^+ \pi^-$ is essential in order to reduce errors of the $D^0 - \bar{D}^0$ mixing parameters and the measured values of the angle γ. A very rich resonance spectrum seen in the Dalitz plots is a direct signal of the complexity of the strong meson interactions. Using these high statistics data theoretical models of the decay amplitudes can be tested.

The experimental analyses [1,2] relied mainly on application of an isobar model. In such a model one can accomodate many resonances coupled to interacting pairs of mesons. However, one should stress that the corresponding decay amplitudes are not unitary: unitarity is not preserved in the three-body decay channels and it is also violated in the two-body subchannels. Within the isobar model it is particularly difficult to distinguish the S-wave amplitudes from the non-resonant background terms. Their interference is often very strong which means that some two-body branching fractions, extracted from the data, could be unreliable. The isobar model is flexible but it has many free parameters (at least two fitted parameters for each amplitude component). For example, the Belle Collaboration has used 40 fitted parameters in ref. [2] and the Babar Collaboration 43 free parameters in ref. [1].

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The construction of unitary three-body strong interaction amplitudes in a wide range of meson-meson effective masses is difficult. Therefore, as a first step, we incorporate two-body unitarity in our model of the D-decay amplitudes with final state interactions in the $K_S^0\pi^\pm$ S- and P-waves and in the $\pi^+\pi^-$ S-wave. The branching fraction corresponding to a sum of these amplitudes is close to 80 % of the total branching fraction of the $D \to K_S^0\pi^+\pi^-$ decay.

2 Decay amplitudes

The D^0 decays into $K_S^0\pi^+\pi^-$ are analysed in the framework of the quasi-two body factorization approach. The decay amplitude consists of the following 27 non-zero parts: seven *allowed* tree amplitudes, six doubly Cabibbo *suppressed* tree amplitudes, fourteen *annihilation* (W-exchange) amplitudes (7 allowed and 7 Cabibbo suppressed). The allowed amplitudes are generated by the quark $c \to sd\bar{u}$ transitions and the doubly Cabibbo suppressed amplitudes correspond to $c \to du\bar{s}$ decays. Seven partial wave amplitudes are considered. We count S, P and D waves in the $K_S^0\pi^\pm$ and the $\pi^+\pi^-$ subsystems and separately the P-wave amplitude for the G- parity violating $\omega \to \pi^+\pi^-$ transition. There are numerous resonances contributing to different partial wave amplitudes. For example, in the $K_S^0\pi^-$ S-wave subchannel one can list $K_0^*(800)^-$ or κ^- and $K_0^*(1430)^-$ resonances, in the P-wave one $K^*(892)^-$, $K_1(1410)^-$ and $K^*(1680)^-$ and in the D-wave the $K_2^*(1430)^-$ resonance. The same resonances but with the opposite charge are active in the $K_S^0\pi^+$ subchannels. In the $\pi^+\pi^-$ subchannels one can enumerate the $f_0(500)$ or σ, $f_0(980)$ and $f_0(1400)$ scalars, the $\rho(770)$, $\rho(1450)$ and $\omega(782)$ vector states and the $f_2(1270)$ tensor resonance.

The main part of the weak decay effective Hamiltonian is proportional to the following operator:

$$O = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud}\, j_1 j_2, \tag{1}$$

where G_F is the Fermi coupling constant, V_{cs} and V_{ud} are the Cabibbo-Kobayashi-Maskawa matrix elements, the two quark currents are defined as $j_1 = (\bar{s}c)_{V-A}$ and $j_2 = (\bar{u}d)_{V-A}$, the symbol $V-A$ means that the axial current is subtracted from the vector current. In the quasi-two body factorization approximation one can write:

$$\langle K_S^0\pi^-\pi^+|j_1 j_2|D^0\rangle \cong \langle K_S^0\pi^-|j_1|D^0\rangle\langle\pi^+|j_2|0\rangle + \langle\pi^-\pi^+|j_1'|D^0\rangle\langle K_S^0|j_2'|0\rangle + \langle 0|j_1'|D^0\rangle\langle K_S^0\pi^-\pi^+|j_2'|0\rangle, \tag{2}$$

where $|0\rangle$ denotes the vacuum state and $j_1' = (\bar{u}c)_{V-A}$, $j_2' = (\bar{s}d)_{V-A}$ are the quark currents obtained from the j_1, j_2 currents by the Fierz transformation. This form of the transition matrix element enables one to introduce the pion f_π, kaon f_K and D^0 f_D decay constants. The corresponding expressions are:

$$\langle\pi^+|j_2|0\rangle = i f_\pi p_\pi, \qquad \langle K_S^0|j_2'|0\rangle = i f_K p_K, \qquad \langle 0|j_1'|D^0\rangle = -i f_D p_D, \tag{3}$$

where p_π, p_K and p_D are the pion, kaon and D meson four-momenta, respectively.

2.1 Transition matrix elements

The expression of the transition matrix elements can be simplified if we assume that the two strongly interacting mesons h_2 and h_3 of momenta p_2 and p_3 form a resonance R in the final state. Then one can write:

$$\langle h_2(p_2)h_3(p_3)|j\,|D^0(p_D)\rangle = G_{Rh_2h_3}(s_{23})\langle R(p_2+p_3)|j\,|D^0(p_D)\rangle, \tag{4}$$

where $G_{Rh_2h_3}(s_{23})$ with $s_{23} = (p_2+p_3)^2$ is the vertex function describing the R decay into h_2 and h_3 mesons. For example, let us consider the reaction $D^0(p_D) \to \pi^+(p_1)K_S^0(p_2)\pi^-(p_3)$. If in the intermediate state the $K^*(892)^-$ resonance is formed and decays into a $\bar{K}^0\pi^-$ pair then the transition form factor $A_0^{DK^{*-}}(m_\pi^2)$ appears in the matrix element

$$\langle R(p_2+p_3)|j\,|D^0(p_D)\rangle = -2im_{K^*}\frac{\epsilon^*\cdot p_D}{p_1^2}\; p_1\; A_0^{DK^{*-}}(m_\pi^2)\;\; + 3\;\; other\;\; terms. \tag{5}$$

Here $j = (\bar{s}c)_{V-A}$, ϵ is the $K^*(892)^-$ polarization vector, m_{K^*} is the K^* mass and $p_D = p_1 + p_2 + p_3$. The "3 *other terms*" do not contribute to the transition amplitude in eq. (2). The vertex function $G_{K^{*-}\bar{K}^0\pi^-}$ can be expressed in terms of the kaon-pion transition vector form factor $F_1^{\bar{K}^0\pi^-}(s_{23})$

$$G_{K^{*-}\bar{K}^0\pi^-}(s_{23}) = \epsilon \cdot (p_2 - p_3)\frac{1}{m_{K^*} f_{K^*}} F_1^{\bar{K}^0\pi^-}(s_{23}). \tag{6}$$

In the above equation f_{K^*} is the K^* decay constant. Assuming isospin symmetry one can equate the transition vector form factor $F_1^{\bar{K}^0\pi^-}(s_{23})$ to the charged kaon to charged pion transition vector form factor $F_1^{K^-\pi^+}(s_{23})$ calculated in ref. [5].

The third term of eq. (2) can also be simplified if the two hadrons, for example h_2 and h_3, interact via a resonant state R. Then, similarly to eq. (5) we write

$$\langle h_1(p_1)h_2(p_2)h_3(p_3)|j'|0\rangle = G_{Rh_2h_3}(s_{23})\langle h_1(p_1)R(p_2 + p_3)|j'|0\rangle. \tag{7}$$

If, for example, $h_1 = \bar{K}^0$, $R = f_0 \to \pi^+\pi^-$ and $j' = (\bar{s}d)_{V-A}$ then the matrix element reads

$$\langle \bar{K}^0(p_1) f_0(p_2 + p_3)|j'|0\rangle = -i\frac{m^2_{K^0} - s_{23}}{p_D^2} p_D F_0^{\bar{K}^0 f_0}(m_D^2) \quad + \quad \textit{second term}, \tag{8}$$

where $F_0^{\bar{K}^0 f_0}(m_D^2)$ is the kaon to f_0 scalar transition form factor. The vertex function for the f_0 decay into $\pi^+\pi^-$ can be parametrized as

$$G_{f_0\pi^+\pi^-}(s_{23}) = \chi_2 F_0^{\pi+\pi-}(s_{23}), \tag{9}$$

where χ_2 is a constant and $F_0^{\pi+\pi-}(s_{23})$ is the pion scalar form factor. Its functional form is taken from our $B^\pm \to \pi^\pm\pi^\mp\pi^\pm$ decay study [6]. It preserves unitarity and groups together three scalar-isoscalar resonances $f_0(500)$, $f_0(980)$ and $f_0(1400)$.

2.2 Examples of allowed transition amplitudes with $K_S^0\pi^-$ final state interactions

Using the assumptions introduced in subsection 2.1 one can derive formulae for the decay amplitudes in which the $K_S^0\pi^-$ final state interactions are explicitely included. The S-wave amplitude reads:

$$A_S = -\frac{G_F}{2}\Lambda_1 a_1 f_\pi(m_D^2 - m_\pi^2)\, F_0^{DK_0^{*-}}(m_\pi^2)\, F_0^{\bar{K}^0\pi^-}(m_-^2). \tag{10}$$

Here, $\Lambda_1 = V_{cs}^* V_{ud}$, a_1 is the effective Wilson coefficient, $F_0^{DK_0^{*-}}(m_\pi^2)$ is the scalar D^0 to K_0^{*-} transition form factor and $F_0^{\bar{K}^0\pi^-}(m_-^2)$ is the scalar $\bar{K}^0$ to π^- transition form factor which depends on the $K_S^0\pi^-$ effective mass squared m_-^2. The latter form factor can be taken equal to the K^- to π^+ transition scalar form factor $F_0^{K^-\pi^+}(m_\pi^2)$ which has been evaluated in the study of $B \to K\pi^+\pi^-$ decays [5]. In this form factor both $K_0^*(800)$ and $K_0^*(1430)$ resonances are included in a unitary way. The expression for the P-wave amplitude is:

$$A_P = -\frac{G_F}{2}\Lambda_1 a_1 \frac{f_\pi}{f_{K^*}}[m_0^2 - m_+^2 + \frac{(m_D^2 - m_\pi^2)(m_K^2 - m_\pi^2)}{m_-^2}]\, A_0^{DK^{*-}}(m_\pi^2)\, F_1^{\bar{K}^0\pi^-}(m_-^2), \tag{11}$$

where m_+^2 is the $K_S^0\pi^+$ effective mass squared and m_0^2 denotes the $\pi^+\pi^-$ effective mass squared. The D-wave decay amplitude depends on the mass $m_{K_2^*}$ and the width $\Gamma_{K_2^*}$ of the resonance $K_2^*(1430)$:

$$A_D = -\frac{G_F}{2}\Lambda_1 a_1 f_\pi F^{DK_2^*}(m_-^2)\frac{g_{K_2^*K_S^0\pi} D(m_+^2, m_-^2)}{m^2_{K_2^*} - m_-^2 - i m_{K_2^*}\Gamma_{K_2^*}}. \tag{12}$$

Here, $F^{DK_2^*}(m_-^2)$ is a combination of three types of the D to $K_2^*(1430)^-$ transition form factors, $g_{K_2^*K_S^0\pi}$ is the decay coupling constant and the $D(m_+^2, m_-^2)$ is the D-wave angular distribution function.

Derivation of other decay amplitudes proceeds in a quite similar way as shown above for the A_S, A_P and A_D amplitudes. One can notice that in our amplitude model the meson transition form factors play a very important role.

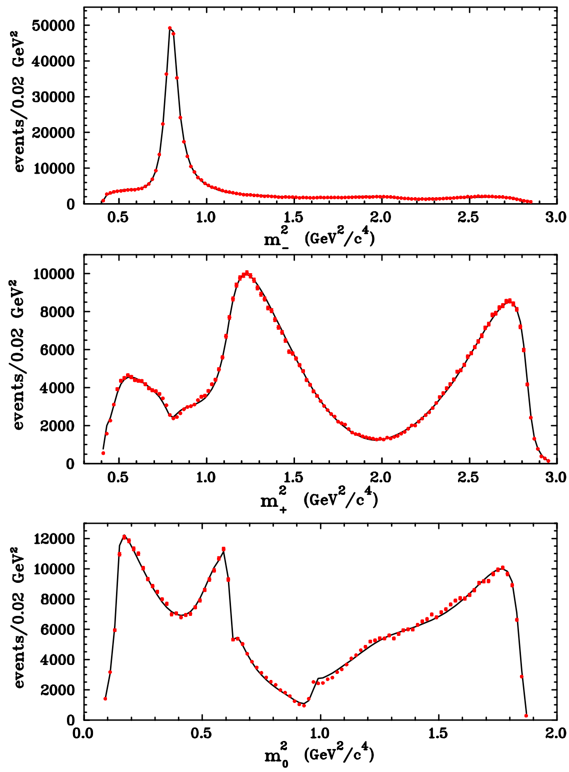

Fig. 1. Comparison of the Dalitz-plot projections for the Belle data [2] with the present model.

3 Results

The theoretical model outlined in the previous section has 28 free real parameters, most of them are unknown complex values of the meson-meson transition form factors appearing in the fourteen W- exchange amplitudes. The scalar and vector kaon-pion and scalar pion form factors are constrained using unitarity, analyticity and chiral symmetry. Fig. 1 shows that the model preliminary results are in fair agreement with the Belle Collaboration data. Also the total braching fraction is well reproduced [7].

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