EPJ Web of Conferences **37**, 05002 (2012)
DOI: 10.1051/epjconf/20123705002

Hadronic light-by-light scattering in the muon $g-2$: impact of proposed measurements of the $\pi^0 \to \gamma\gamma$ decay width and the $\gamma^*\gamma \to \pi^0$ transition form factor with the KLOE-2 experiment

Andreas Nyffeler[a]

Regional Centre for Accelerator-based Particle Physics, Harish-Chandra Research Institute
Chhatnag Road, Jhusi, Allahabad - 211019, India

Abstract. We discuss, how planned measurements at KLOE-2 of the $\pi^0 \to \gamma\gamma$ decay width and the $\gamma^*\gamma \to \pi^0$ transition form factor can improve estimates for the numerically dominant pion-exchange contribution to hadronic light-by-light scattering in the muon $g-2$ and what are the limitations related to the modelling of the off-shellness of the pion.

1 Introduction

The anomalous magnetic moment of the muon a_μ provides an important test of the Standard Model (SM) and is potentially sensitive to contributions from New Physics [1]. In fact, for several years now a deviation is observed between the experimental measurement and the SM prediction, $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} \sim (250-300) \times 10^{-11}$, corresponding to about $3-3.5$ standard deviations [1,2].

Hadronic effects dominate the uncertainty in the SM prediction of a_μ and make it difficult to interpret this discrepancy as a sign of New Physics. In particular, in contrast to the hadronic vacuum polarization in the $g-2$, which can be related to data, the estimates for the hadronic light-by-light (LbyL) scattering contribution $a_\mu^{\mathrm{had.\ LbyL}} = (105 \pm 26) \times 10^{-11}$ [3] and $a_\mu^{\mathrm{had.\ LbyL}} = (116 \pm 40) \times 10^{-11}$ [4, 1] rely entirely on calculations using *hadronic models* which employ form factors for the interaction of hadrons with photons. The more recent papers [5,6] yield a larger central value and a larger error of about $(150 \pm 50) \times 10^{-11}$. For a brief review of had. LbyL scattering in a_μ see Ref. [7], which also includes a reanalysis of the charged pion loop contribution, see also Ref. [8]. To fully profit from future planned $g-2$ experiments with a precision of 15×10^{-11}, these large model uncertainties have to be reduced. Maybe lattice QCD will at some point give a reliable number, see the talk [9] for some encouraging progress recently. Meanwhile, experimental measurements and theoretical constraints of the relevant form factors can help to constrain the models and to reduce the uncertainties in $a_\mu^{\mathrm{had.\ LbyL}}$.

In most model calculations, pion-exchange gives the numerically dominant contribution[1], therefore it has received a lot of attention. In our paper [10] we studied the impact of planned measurements at KLOE-2 of the $\pi^0 \to \gamma\gamma$ decay width to 1% statistical precision and the $\gamma^*\gamma \to \pi^0$ transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(Q^2)$ for small space-like momenta, $0.01\ \mathrm{GeV}^2 \leq Q^2 \leq 0.1\ \mathrm{GeV}^2$, to 6% statistical precision in each bin, on estimates of the pion-exchange contribution $a_\mu^{\mathrm{LbyL};\pi^0}$. We would like to stress that a realistic calculation of $a_\mu^{\mathrm{LbyL};\pi^0}$ is *not* the purpose of this paper. The estimates given below are performed to demonstrate, within several models, an improvement of uncertainty, which will be possible when the KLOE-2 data appear. The simulations in Ref. [10] have been performed with the dedicated Monte-Carlo program EKHARA [11] for the process $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-P$ with $P = \pi^0, \eta, \eta'$, followed by the decay $\pi^0 \to \gamma\gamma$ and combined with a detailed detector simulation.

[a] e-mail: nyffeler@hri.res.in

[1] Apart from the recent papers [5,6], where the (dressed) quark loop gives the largest contribution.

2 Impact of KLOE-2 measurements on $a_\mu^{\mathrm{LbyL};\pi^0}$

Any experimental information on the neutral pion lifetime and the transition form factor is important in order to constrain the models used for calculating the pion-exchange contribution. However, having a good description e.g. for the transition form factor is only necessary, not sufficient, in order to uniquely determine $a_\mu^{\mathrm{LbyL};\pi^0}$. As stressed in Ref. [12], what enters in the calculation of $a_\mu^{\mathrm{LbyL};\pi^0}$ is the fully off-shell form factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)$ (vertex function), where also the pion is off-shell with 4-momentum (q_1+q_2). Such a (model dependent) form factor can for instance be defined via the QCD Green's function $\langle VVP \rangle$, see Ref. [4] for details and references to earlier work. The form factor with on-shell pions is then given by $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) \equiv \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(m_\pi^2, q_1^2, q_2^2)$. Measurements of the transition form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(Q^2) \equiv \mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(m_\pi^2, -Q^2, 0)$ are in general only sensitive to a subset of the model parameters and do not allow to reconstruct the full off-shell form factor.

For different models, the effects of the off-shell pion can vary a lot. In Ref. [4] an off-shell form factor (LMD+V) was proposed, based on large-N_C QCD matched to short-distance constraints from the operator product expansion, see also Ref. [13]. This yields the estimate $a_{\mu;\mathrm{LMD+V}}^{\mathrm{LbyL};\pi^0} = (72\pm12)\times10^{-11}$. The error estimate comes from the variation of all model parameters, where the uncertainty of the parameters related to the off-shellness of the pion completely dominates the total error and will *not* be shown in Table 1 below.

In contrast to the off-shell LMD+V model, many models, e.g. the VMD model, constituent quark models or the ansätze for the transition form factor used in Ref. [14], do not have these additional sources of uncertainty related to the off-shellness of the pion. These models often have only very few parameters, which can all be fixed by measurements of the transition form factor or from other observables. Therefore, the precision of the KLOE-2 measurement can dominate the total accuracy of $a_\mu^{\mathrm{LbyL};\pi^0}$ in such models.

It was noted in Ref. [15] that essentially all evaluations of the pion-exchange contribution use the normalization $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(m_\pi^2, 0, 0) = 1/(4\pi^2 F_\pi)$ for the form factor, as derived from the Wess-Zumino-Witten (WZW) term. Then the value $F_\pi = 92.4$ MeV is used without any error attached to it, i.e. a value close to $F_\pi = (92.2 \pm 0.14)$ MeV, obtained from $\pi^+ \to \mu^+\nu_\mu(\gamma)$ [16]. Instead, if one uses the decay width $\Gamma_{\pi^0\to\gamma\gamma}$ for the normalization of the form factor, an additional source of uncertainty enters, which has not been taken into account in most evaluations.

In our calculations we account for this normalization issue, using in the fit:

- $\Gamma^{\mathrm{PDG}}_{\pi^0\to\gamma\gamma} = 7.74 \pm 0.48$ eV from the PDG 2010 [16],
- $\Gamma^{\mathrm{PrimEx}}_{\pi^0\to\gamma\gamma} = 7.82 \pm 0.22$ eV from the PrimEx experiment [17],
- $\Gamma^{\mathrm{KLOE-2}}_{\pi^0\to\gamma\gamma} = 7.73 \pm 0.08$ eV for the KLOE-2 simulation (assuming a 1% precision).

The assumption that the KLOE-2 measurement will be consistent with the LMD+V and VMD models, allowed us in Ref. [10] to use the simulations as new "data" and evaluate the impact of such "data" on the precision of the $a_\mu^{\mathrm{LbyL};\pi^0}$ calculation. In order to do that, we fit the LMD+V and VMD models to the data sets [18] from CELLO, CLEO and BaBar for the transition form factor and the values for the decay width given above:

$$
\begin{aligned}
A0 &: \text{CELLO, CLEO, PDG} \\
A1 &: \text{CELLO, CLEO, PrimEx} \\
A2 &: \text{CELLO, CLEO, PrimEx, KLOE-2} \\
B0 &: \text{CELLO, CLEO, BaBar, PDG} \\
B1 &: \text{CELLO, CLEO, BaBar, PrimEx} \\
B2 &: \text{CELLO, CLEO, BaBar, PrimEx, KLOE-2}
\end{aligned}
$$

The BaBar measurement of the transition form factor does not show the $1/Q^2$ behavior as expected from earlier theoretical considerations by Brodsky-Lepage [19] and as seen in the CELLO and CLEO data and the recent measurements from Belle [20]. The VMD model always shows a $1/Q^2$ fall-off and therefore is not compatible with the BaBar data. The LMD+V model has another parameter, h_1, which

Table 1. KLOE-2 impact on the accuracy of $a_\mu^{\mathrm{LbyL};\pi^0}$ in case of one year of data taking (5 fb^{-1}). The values marked with asterisk (*) do not contain additional uncertainties coming from the "off-shellness" of the pion (see the text).

Model	Data	$\chi^2/d.o.f.$		Parameters		$a_\mu^{\mathrm{LbyL};\pi^0} \times 10^{11}$
VMD	A0	6.6/19	$M_V = 0.778(18)$ GeV	$F_\pi = 0.0924(28)$ GeV		$(57.2 \pm 4.0)_{JN}$
VMD	A1	6.6/19	$M_V = 0.776(13)$ GeV	$F_\pi = 0.0919(13)$ GeV		$(57.7 \pm 2.1)_{JN}$
VMD	A2	7.5/27	$M_V = 0.778(11)$ GeV	$F_\pi = 0.0923(4)$ GeV		$(57.3 \pm 1.1)_{JN}$
LMD+V, $h_1 = 0$	A0	6.5/19	$\bar{h}_5 = 6.99(32)$ GeV^4	$\bar{h}_7 = -14.81(45)$ GeV^6		$(72.3 \pm 3.5)^*_{JN}$
						$(79.8 \pm 4.2)_{MV}$
LMD+V, $h_1 = 0$	A1	6.6/19	$\bar{h}_5 = 6.96(29)$ GeV^4	$\bar{h}_7 = -14.90(21)$ GeV^6		$(73.0 \pm 1.7)^*_{JN}$
						$(80.5 \pm 2.0)_{MV}$
LMD+V, $h_1 = 0$	A2	7.5/27	$\bar{h}_5 = 6.99(28)$ GeV^4	$\bar{h}_7 = -14.83(7)$ GeV^6		$(72.5 \pm 0.8)^*_{JN}$
						$(80.0 \pm 0.8)_{MV}$
LMD+V, $h_1 \neq 0$	A0	6.5/18	$\bar{h}_5 = 6.90(71)$ GeV^4	$\bar{h}_7 = -14.83(46)$ GeV^6	$h_1 = -0.03(18)$ GeV^2	$(72.4 \pm 3.8)^*_{JN}$
LMD+V, $h_1 \neq 0$	A1	6.5/18	$\bar{h}_5 = 6.85(67)$ GeV^4	$\bar{h}_7 = -14.91(21)$ GeV^6	$h_1 = -0.03(17)$ GeV^2	$(72.9 \pm 2.1)^*_{JN}$
LMD+V, $h_1 \neq 0$	A2	7.5/26	$\bar{h}_5 = 6.90(64)$ GeV^4	$\bar{h}_7 = -14.84(7)$ GeV^6	$h_1 = -0.02(17)$ GeV^2	$(72.4 \pm 1.5)^*_{JN}$
LMD+V, $h_1 \neq 0$	B0	18/35	$\bar{h}_5 = 6.46(24)$ GeV^4	$\bar{h}_7 = -14.86(44)$ GeV^6	$h_1 = -0.17(2)$ GeV^2	$(71.9 \pm 3.4)^*_{JN}$
LMD+V, $h_1 \neq 0$	B1	18/35	$\bar{h}_5 = 6.44(22)$ GeV^4	$\bar{h}_7 = -14.92(21)$ GeV^6	$h_1 = -0.17(2)$ GeV^2	$(72.4 \pm 1.6)^*_{JN}$
LMD+V, $h_1 \neq 0$	B2	19/43	$\bar{h}_5 = 6.47(21)$ GeV^4	$\bar{h}_7 = -14.84(7)$ GeV^6	$h_1 = -0.17(2)$ GeV^2	$(71.8 \pm 0.7)^*_{JN}$

determines the behavior of the transition form factor for large Q^2. To get the $1/Q^2$ behavior according to Brodsky-Lepage, one needs to set $h_1 = 0$. However, one can simply leave h_1 as a free parameter and fit it to the BaBar data, yielding $h_1 \neq 0$ [15]. In this case the form factor does not vanish for $Q^2 \to \infty$. Since VMD and LMD+V with $h_1 = 0$ are not compatible with the BaBar data, the corresponding fits are very bad and we will not include these results in the current paper, see Ref. [10] for details.

For illustration, we use the following two approaches to calculate $a_\mu^{\mathrm{LbyL};\pi^0}$:

- Jegerlehner-Nyffeler (JN) approach [4, 1] with the off-shell pion form factor;
- Melnikov-Vainshtein (MV) approach [21], where one uses the on-shell pion form factor at the internal vertex and a constant (WZW) form factor at the external vertex.

Table 1 shows the impact of the PrimEx and the future KLOE-2 measurements on the model parameters and, consequently, on the $a_\mu^{\mathrm{LbyL};\pi^0}$ uncertainty. The other parameters of the (on-shell and off-shell) LMD+V model have been chosen as in the papers [4, 1, 21]. We stress again that our estimate of the $a_\mu^{\mathrm{LbyL};\pi^0}$ uncertainty is given only by the propagation of the errors of the fitted parameters in Table 1 and therefore we may not reproduce the total uncertainties given in the original papers.

We can clearly see from Table 1 that for each given model and each approach (JN or MV), there is a trend of reduction in the error for $a_\mu^{\mathrm{LbyL};\pi^0}$ (related only to the given model parameters) by about half when going from A0 (PDG) to A1 (including PrimEx) and by about another half when going from A1 to A2 (including KLOE-2). Very roughly, we can write:

- Sets A0, B0: $\delta a_\mu^{\mathrm{LbyL};\pi^0} \approx 4 \times 10^{-11}$ (with $\Gamma^{\mathrm{PDG}}_{\pi^0\to\gamma\gamma}$)
- Sets A1, B1: $\delta a_\mu^{\mathrm{LbyL};\pi^0} \approx 2 \times 10^{-11}$ (with $\Gamma^{\mathrm{PrimEx}}_{\pi^0\to\gamma\gamma}$)
- Sets A2, B2: $\delta a_\mu^{\mathrm{LbyL};\pi^0} \approx (0.7 - 1.1) \times 10^{-11}$ (with simulated KLOE-2 data)

This is mainly due to the improvement in the normalization of the form factor, related to the decay width $\pi^0 \to \gamma\gamma$, controlled by the parameters F_π or $\bar{h}_7$, respectively, but more data also better constrain the other model parameters M_V or $\bar{h}_5$. This trend of improvement is also visible in the last part of the Table (LMD+V, $h_1 \neq 0$), when we fit the sets B0, B1 and B2 which include the BaBar data. The central values of the final results for $a_\mu^{\mathrm{LbyL};\pi^0}$ are only slightly changed, if we include the BaBar data. They shift only by about -0.5×10^{-11} compared to the corresponding data sets A0, A1 and A2. This is due to a partial compensation in $a_\mu^{\mathrm{LbyL};\pi^0}$, when the central values for $\bar{h}_5$ and h_1 are changed, see Ref. [15].

Finally, note that both VMD and LMD+V with $h_1 = 0$ can fit the data sets A0, A1 and A2 for the transition form factor very well with essentially the same χ^2 per degree of freedom for a given data set (see first and second part of the table). Nevertheless, the results for the pion-exchange contribution differ by about 20% in these two models. For VMD the result is $a_\mu^{\mathrm{LbyL};\pi^0} \sim 57.5\times10^{-11}$ and for LMD+V with $h_1 = 0$ it is 72.5×10^{-11} with the JN approach and 80×10^{-11} with the MV approach. This is due to the different behavior, in these two models, of the fully off-shell form factor $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2)$ on all momentum variables, which enters for the pion-exchange contribution [12]. The VMD model is known to have a wrong high-energy behavior with too strong damping, which underestimates the contribution. For the VMD model, measurements of the neutral pion decay width and the transition form factor completely determine the model parameters F_π and M_V and the error given in Table 1 is the total model error. Note that a smaller error, compared to the off-shell LMD+V model, does not necessarily imply that the VMD model is better, i.e. closer to reality. Maybe the model is too simplistic.

We conclude that the KLOE-2 data with a total integrated luminosity of 5 fb^{-1} will give a reasonable improvement in the part of the $a_\mu^{\mathrm{LbyL};\pi^0}$ error associated with the parameters accessible via the $\pi^0 \to \gamma\gamma$ decay width and the $\gamma^*\gamma \to \pi^0$ transition form factor. Depending on the modelling of the off-shellness of the pion, there might be other, potentially larger sources of uncertainty which cannot be improved by the KLOE-2 measurements.

Acknowledgements

I would like to thank my coauthors on Ref. [10] for the pleasant collaboration on this work. I am grateful to the organizers of MESON 2012 for the opportunity to present our paper and for providing a stimulating atmosphere during the meeting. This work is supported by funding from the Department of Atomic Energy, Government of India, for the Regional Centre for Accelerator-based Particle Physics (RECAPP), Harish-Chandra Research Institute.

References

1. F. Jegerlehner, A. Nyffeler, Phys. Rept. **477**, (2009) 1.
2. M. Davier, A. Hoecker, B. Malaescu, Z. Zhang, Eur. Phys. J. **C71**, (2011) 1515; Erratum-ibid. **C72** (2012) 1874; F. Jegerlehner, R. Szafron, Eur. Phys. J. **C71** (2011) 1632; K. Hagiwara *et al.*, J. Phys. **G38**, (2011) 085003; M. Benayoun, P. David, L. DelBuono, F. Jegerlehner, Eur. Phys. J. **C72** (2012) 1848; T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio, arXiv:1205.5370 [hep-ph].
3. J. Prades, E. de Rafael, A. Vainshtein, arXiv:0901.0306 [hep-ph].
4. A. Nyffeler, Phys. Rev. **D79**, (2009) 073012.
5. T. Goecke, C. S. Fischer, R. Williams, Phys. Rev. **D83**, (2011) 094006. See also talk by C. S. Fischer, Workshop on Meson Transition Form Factors, Cracow, Poland, May 2012 for revised values.
6. D. Greynat, E. de Rafael, JHEP **07** (2012) 020.
7. J. Bijnens, M. Zahiri Abyaneh, these proceedings, arXiv:1208.3548 [hep-ph].
8. K. T. Engel, H. H. Patel, M. J. Ramsey-Musolf, arXiv:1201.0809 [hep-ph]; M. Zahiri Abyaneh, arXiv:1208.2554 [hep-ph].
9. Talk by T. Blum at Lattice 2012, Cairns, Australia, June 2012.
10. D. Babusci *et al.*, Eur. Phys. J. **C72**, (2012) 1917.
11. H. Czyż, S. Ivashyn, Comput. Phys. Commun. **182**, (2011) 1338.
12. F. Jegerlehner, Acta Phys. Polon. **B38**, (2007) 3021; F. Jegerlehner, *The anomalous magnetic moment of the muon* (Springer, Berlin, Germany 2008).
13. M. Knecht, A. Nyffeler, Eur. Phys. J. **C21**, (2001) 659.
14. L. Cappiello, O. Cata, G. D'Ambrosio, Phys. Rev. **D83**, (2011) 093006.
15. A. Nyffeler, PoS **CD09**, (2009) 080.
16. K. Nakamura at al., J. Phys. **G37**, (2010) 075021.
17. I. Larin *et al.*, Phys. Rev. Lett. **106**, (2011) 162303.
18. H. J. Behrend *et al.*, Z. Phys. **C49**, (1991) 401; J. Gronberg *et al.*, Phys. Rev. **D57**, (1998) 33; B. Aubert et al., Phys. Rev. **D80**, (2009) 052002.
19. G. P. Lepage, S. J. Brodsky, Phys. Lett. **B87**, (1979) 359; Phys. Rev. **D22**, (1980) 2157; S. J. Brodsky, G. P. Lepage, Phys. Rev. **D24**, (1981) 1808.
20. S. Uehara *et al.*, arXiv:1205.3249 [hep-ex].
21. K. Melnikov, A. Vainshtein, Phys. Rev. **D70**, (2004) 113006.