EPJ Web of Conferences **37**, 05008 (2012)
DOI: 10.1051/epjconf/20123705008

Vector-meson dominance revisited

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Abstract. The interaction of mesons with electromagnetism is often well described by the concept of vector-meson dominance (VMD). However, there are also examples where VMD fails. A simple chiral Lagrangian for pions, rho and omega mesons is presented which can account for the respective agreement and disagreement between VMD and phenomenology in the sector of light mesons.

1 Introduction and summary

Interactions of hadrons with electromagnetism are an important tool to learn more about QCD and in general about the fundamental forces between particles. To flash a few key words: exploring the intrinsic structure of the nucleon [1]; in-medium modifications of hadrons [2] and the quark-gluon plasma [3]; the hadronic contribution to the gyromagnetic ratio of the muon [4, 5]. Since the neutral vector mesons have the same quantum numbers as the photon they can show up as intermediate states in the corresponding cross sections. The phenomenologically rather successful concept of vector-meson dominance (VMD) [6] proposes that all interactions between hadrons and photons are mediated by the vector mesons. In the following we concentrate on mesons and on the lightest two quark flavors. In this sector the VMD assumption works very well for some reactions like the pion form factor (virtual photon coupled to two pions) [6, 7] and the single-virtual pion transition form factor (pion coupled to one real and one virtual photon) [8], but fails badly for the omega transition form factor (omega coupled to pion and virtual photon) [9, 10]. Starting from a simple chiral Lagrangian for pions, rho and omega mesons we will demonstrate in the following that one can reproduce at the same time the agreement with VMD for the pion form factors and the deviation from VMD for the omega form factor.

2 Lagrangian

Due to lack of space we only present our Lagrangian and then turn immediately to the results. Much more details about the theoretical considerations behind our Lagrangian can be found in [11] and references therein. We utilize the leading-order Lagrangian of chiral perturbation theory,

$$\mathcal{L}_{\chi\mathrm{PT}} = f^2 \,\mathrm{tr}(U^\dagger_\mu \, U^\mu) + \frac{1}{4} \, f^2 \, m_\pi^2 \,\mathrm{tr}(U^\dagger + U) \,, \tag{1}$$

the effective low-energy description of the chiral anomaly according to Wess, Zumino and Witten,[1]

$$\mathcal{L}_{\mathrm{WZW}} = \frac{3}{32\pi^2} \, \frac{e^2}{f} \, \varepsilon_{\mu\nu\alpha\beta} \, F^{\mu\nu} \, F^{\alpha\beta} \,\mathrm{tr}(Q^2 \, \Phi) + \ldots \,, \tag{2}$$

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[1] Only the relevant part is displayed explicitly.

and a Lagrangian including vector mesons in the antisymmetric tensor representation,

$$\mathcal{L}_{\rm vec} = -\frac{i}{4}\, h_P\, m_V\, {\rm tr}\Big(V_{\mu\nu}\,[U^\mu, U^\nu]\Big) - \frac{1}{16}\, e_V\, m_V\, {\rm tr}\Big(V^{\mu\nu}\,(u\,Q\,u^\dagger + u^\dagger\,Q\,u)\Big)\, F_{\mu\nu}$$
$$- \frac{1}{4}\,{\rm tr}\Big(\nabla_\mu V^{\mu\alpha}\,\nabla^\nu V_{\nu\alpha}\Big) + \frac{1}{8}\, m_V^2\,{\rm tr}\Big(V_{\mu\nu}\,V^{\mu\nu}\Big) + \frac{i}{8}\, h_A\,\varepsilon_{\mu\nu\alpha\beta}\,{\rm tr}\Big(\{V^{\mu\nu}, \nabla_\lambda V^{\lambda\alpha}\}\,U^\beta\Big)\,. \qquad (3)$$

Here, "tr" is the flavor trace, $f \approx 90\,\mathrm{MeV}$ denotes the pion decay constant and m_π and m_V are the masses of pion and rho/omega, respectively. The pion fields are encoded in

$$\Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\,\pi^+ \\ \sqrt{2}\,\pi^- & -\pi^0 \end{pmatrix}, \qquad U = u^2 = \exp(i\,\Phi/f)\,, \qquad U_\mu = \frac{1}{2}\, u^\dagger\,(D_\mu U)\, u^\dagger \qquad (4)$$

with the gauge covariant derivative $D_\mu U = \partial_\mu U + ie\,[Q, U]\,A_\mu$, the positron charge e, the quark charge matrix $Q = {\rm diag}\Big(+\frac{2}{3}, -\frac{1}{3}\Big)$, the photon field A_μ and the electromagnetic field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We have neglected isospin breaking and external fields other than the electromagnetic one. The rho and omega vector mesons are collected in

$$V_{\mu\nu} = \begin{pmatrix} \rho^0 + \omega & \sqrt{2}\,\rho^+ \\ \sqrt{2}\,\rho^- & -\rho^0 + \omega \end{pmatrix}_{\mu\nu}\,. \qquad (5)$$

Finally, the chiral derivatives are defined via $\nabla_\mu V_{\alpha\beta} = \partial_\mu V_{\alpha\beta} + [\Gamma_\mu, V_{\alpha\beta}]$ with $\Gamma_\mu = \frac{1}{2}\, u^\dagger\,(\partial_\mu + ieQA_\mu)u + \frac{1}{2}\, u\,(\partial_\mu + ieQA_\mu)u^\dagger$. The three remaining coupling constants, which appear in (3), can be determined from two-body decays: $\rho \to 2\pi$ (determines h_p), $\rho^0/\omega \to e^+e^-$ (determines $|e_V|$) and $\omega \to \pi^0\gamma$ (determines combination $|h_A\, e_V|$) [12]. One obtains

$$|e_V| \approx 0.22\,, \qquad h_P \approx 0.30\,, \qquad |h_A| \approx 2.1\,. \qquad (6)$$

3 Results

Equipped with the Lagrangian (1), (2), (3) together with the parameter values (6) we can calculate, e.g., the pion form factor [13], the pion-to-photon transition form factor [11, 14], and the omega-to-pion transition form factor [15].

The process $e^+e^- \to \pi^+\pi^-$, which gives rise to the pion form factor, can take place via a direct coupling of the pions to the photon ("direct term") or via a vector meson. In the first case, the pions couple via their electric charge. This is described by the Lagrangian (1). In the second case the Lagrangian (3) provides the coupling of the virtual photon to the rho meson $\sim e_V$ and the coupling of the rho meson to the pions $\sim h_P$. A reliable description of the reaction $e^+e^- \to \pi^+\pi^-$ and the pion form factor requires a resummation of the rescattering processes of the pions. In [13] this has been achieved by a Bethe-Salpeter equation. The corresponding scattering kernel consists of two parts: The direct contact interaction between the pions is obtained from (1). The interaction between the pions and the rho meson comes again from the h_P term of (3). The full result of this resummation is depicted on the left hand side of figure 1. The invariant mass of the virtual photon is denoted by $\sqrt{s}$. A very good agreement with the data is achieved. Note that isospin breaking leads to ρ-ω mixing. This has not been included. Therefore, the sharp omega peak seen in the data cannot be reproduced. Concerning the comparison to VMD it is illuminating to discuss also the tree-level result for the pion form factor:

$$F_\pi(s) = 1 + \frac{e_V\, h_P\, m_V^2}{16\, e\, f^2}\,\frac{s}{m_V^2 - s}\,. \qquad (7)$$

It is common practice to normalize the pion form factor to the direct term. In that way, the pion form factor becomes unity at the photon point. The VMD prediction for the same quantity is

$$F_\pi^{\rm VMD}(s) = \frac{m_V^2}{m_V^2 - s}\,. \qquad (8)$$

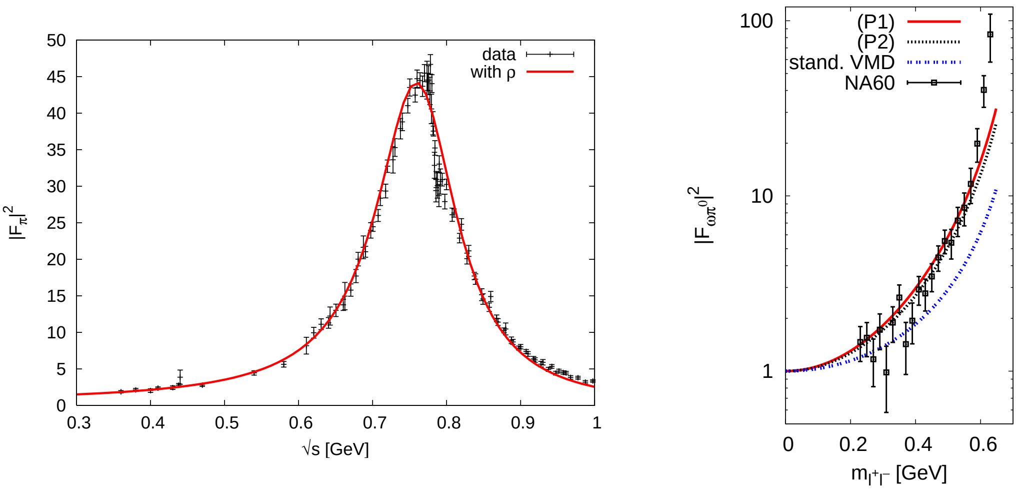

Fig. 1. *Left:* The pion form factor as compared to data [13]. *Right:* The omega-to-pion transition form factor. The present calculation is depicted by the full (red) line. Figure taken from [15].

The two formulae would analytically agree for $e_V\, h_P = 16\, e\, f^2/m_V^2 \approx 0.065$. With the values (6) one obtains $|e_V|\, h_P \approx 0.066$ which is obviously very close, provided one chooses a positive sign for e_V. Then a cancellation takes place: The two terms from the χPT Lagrangian (1) and the vector-meson Lagrangian (3) conspire such that the final result is close to VMD.

A similar cancellation takes place for the pion transition form factor ($\pi^0 \to \gamma^{(*)}\gamma^{(*)}$). In general, our approach has again two contributions, one from the Wess-Zumino-Witten term (2) and one from the vector-meson Lagrangian (3). However, the latter contribution vanishes if both photons are real. Thus, one of the much celebrated successes of the Wess-Zumino-Witten Lagrangian, the excellent description of the reaction $\pi^0 \to 2\gamma$, is not spoiled. The single-virtual pion transition form factor, normalized to the photon point, i.e. normalized to the Wess-Zumino-Witten contribution, is given by

$$F_{\pi\gamma}(s) = 1 + \frac{\pi^2\, h_A\, e_V^2}{12 e^2}\, \frac{s}{m_V^2 - s}\,. \tag{9}$$

The corresponding VMD formula is

$$F^{\rm VMD}_{\pi\gamma}(s) = \frac{m_V^2}{m_V^2 - s}\,. \tag{10}$$

Of course, there are kinematical situations where these tree-level results are not enough. For the comparison of the present approach to the VMD result, however, it is most transparent to look at the tree-level formulae (9) and (10). They would agree analytically for $h_A\, e_V^2 = 12 e^2/\pi^2 \approx 0.11$. Using the numerical values from (6) one obtains $|h_A|\, e_V^2 \approx 0.10$. Thus, the formula (9) is again numerically close to the VMD result, provided one uses a positive sign for h_A. Another cancellation has taken place, now between the Wess-Zumino-Witten term and the vector-meson contribution. As already mentioned in the introduction, the VMD prediction is close to the experimental result [8]. Therefore, the present approach agrees also with the experimental result for the single-virtual pion transition form factor. However, the picture changes for the *double-virtual* transition form factor: It is given by

$$F(s_1, s_2) = 1 + \frac{\pi^2\, h_A\, e_V^2}{12 e^2}\, \frac{m_V^2\,(s_1 + s_2)}{(m_V^2 - s_1)\,(m_V^2 - s_2)}$$

$$\approx 1 + \frac{m_V^2\,(s_1+s_2)}{(m_V^2-s_1)\,(m_V^2-s_2)} = 1 \underbrace{-\frac{m_V^2}{m_V^2-s_1} - \frac{m_V^2}{m_V^2-s_2}}_{\text{“half” VMD}} + 2\underbrace{\frac{m_V^4}{(m_V^2-s_1)\,(m_V^2-s_2)}}_{\text{VMD type}} \tag{11}$$

while the VMD formula is simply

$$F^{\text{VMD}}(s_1,s_2) = \frac{m_V^4}{(m_V^2-s_1)\,(m_V^2-s_2)}\,. \tag{12}$$

The two virtualities of the photons are denoted by s_1 and s_2. Obviously, (11) is different from VMD for $s_1, s_2 \neq 0$. Future experimental data for the double-virtual transition form factor are crucial to distinguish between the present approach and the VMD scenario. Such data are also an important input for the calculation of the hadronic light-by-light-scattering contribution to the gyromagnetic ratio of the muon [4].

Finally we turn to the omega-to-pion transition form factor. Here there is only one contribution in our approach so that no cancellation can take place. Phenomenologically there is a large deviation from VMD [9,10]. For the form factor, which is normalized to 1 at the photon point, our approach provides a parameter independent prediction:

$$F_{\omega\pi}(s) = \frac{m_V^2+s}{m_V^2-s}\,. \tag{13}$$

This deviates significantly from VMD:

$$F_{\omega\pi}^{\text{VMD}}(s) = \frac{m_V^2}{m_V^2-s}\,. \tag{14}$$

Both form factors are compared to data from NA60 [10] on the right hand side of figure 1 ($\sqrt{s} = m_{l^+l^-}$). Obviously, the NA60 data are much better described by the present approach (13) than by the VMD formula (14). A follow-up experiment using dielectrons instead of dimuons is planned by the WASA at COSY collaboration. This form factor and similar ones have also been studied recently in [16,17].

References

1. H. W. Kendall, Rev. Mod. Phys. **63**, (1991) 597 (Nobel prize lecture 1990)
2. S. Leupold, V. Metag and U. Mosel, Int. J. Mod. Phys. E **19**, (2010) 147
3. B. Friman *et al.*, Lect. Notes Phys. **814**, (2011) 1
4. F. Jegerlehner and A. Nyffeler, Phys. Rept. **477**, (2009) 1
5. J. Bijnens, in these proceedings
6. J. J. Sakurai, *Currents and mesons* (University of Chicago Press, Chicago, USA, 1969)
7. F. Klingl, N. Kaiser and W. Weise, Z. Phys. A **356**, (1996) 193
8. K. Nakamura *et al.* [Particle Data Group], J. Phys. G **37**, (2010) 075021
9. L. G. Landsberg, Phys. Rept. **128**, (1985) 301
10. R. Arnaldi *et al.* [NA60 Collaboration], Phys. Lett. B **677**, (2009) 260
11. S. Leupold and C. Terschlüsen, arXiv:1206.2253 [hep-ph]
12. M. F. M. Lutz and S. Leupold, Nucl. Phys. A **813**, (2008) 96
13. S. Leupold, Phys. Rev. D **80**, (2009) 114012 [Erratum-ibid. D **83**, (2011) 079902]
14. C. Terschlüsen, B. Strandberg, S. Leupold and F. Eichstädt, in preparation
15. C. Terschlüsen and S. Leupold, Phys. Lett. B **691**, (2010) 191
16. C. Terschlüsen, S. Leupold and M. F. M. Lutz, arXiv:1204.4125 [hep-ph]
17. S. P. Schneider, B. Kubis and F. Niecknig, arXiv:1206.3098 [hep-ph]