

Precise determination of the parameters of resonances $f_0(500)$ and $f_0(980)$ by fitting the data and dispersion relations

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Abstract. The long-standing puzzle in the parameters of the $f_0(500)$, as well as the $f_0(980)$, is finally being settled [1] thanks to precise dispersive analyses carried out during the last years. Here we report on our very recent dispersive data analysis which allowed for a precise and model independent determination of the amplitudes for the S, P, D and F waves [2–4]. The analytic continuation of once subtracted dispersion relations for the $S0$ wave to the complex energy plane leads to very precise results for the $f_0(500)$ pole: $\sqrt{s}_{pole} = 457_{-13}^{+14} - i279_{-7}^{+11}$ MeV and for the $f_0(980)$ pole: $\sqrt{s}_{pole} = 996 \pm 7 - i25_{-6}^{+10}$ MeV.

1 Introduction

Our latest dispersive analysis of $\pi\pi$ scattering data including very recent K_{l4} experimental results, have led to the construction of the $\pi\pi$ amplitudes in many partial waves (S, P, D and F) [2–4] and in an energy range from threshold to about 1400 MeV. The initial unconstrained fit to the data has been supplemented by theoretical constraints from Roy-type dispersion relations, forward dispersion relations (FDR) and sum rules for threshold parameters and crossing [2]. Mutual consistency of those amplitudes expressed by the fulfillment of the conditions imposed by the dispersion relations ensures that the analytical continuation on the complex plane of the S -wave amplitude provides reliable and precise information on the $f_0(500)$ and $f_0(980)$ resonances.

2 Method and results

Among the relations mentioned above, the most accurate, given the same experimental input, and thus the most demanding are the once subtracted dispersion relations for partial waves implementing crossing symmetry (so called GKPY equations). These Roy-type equations can be expressed by mutual relations of the real and imaginary parts of the output and input amplitudes $t_\ell^I(s = m_{\pi\pi}^2)$ respectively

$$\text{Re } t_\ell^{I(OUT)}(s) = \sum_{I'=0}^2 C_{st}^{II'} a_0^{II'} + \sum_{I'=0}^2 \sum_{\ell'=0}^4 \int_{4m_\pi^2}^{\infty} ds' K_{\ell\ell'}^{II'}(s, s') \text{Im } t_{\ell'}^{I'(IN)}(s') \quad (1)$$

where the first term on the right are subtraction terms, which are combinations of the S wave scattering lengths $a_0^{II'}$. The $K_{\ell\ell'}^{II'}(s, s')$ are known kernels derived by imposing crossing symmetry conditions on the $\pi\pi \rightarrow \pi\pi$ amplitudes. The smaller the difference $|\text{Re } t_\ell^{I(OUT)}(s) - \text{Re } t_\ell^{I(IN)}(s)|$ is, the better crossing symmetry for a given amplitude is satisfied.

Consistency checks of the fit with all theoretical constraints has been performed by minimization of $\chi_{tot}^2 = \chi_{data}^2 + \bar{d}_{Roy}^2 + \bar{d}_{GKPY}^2 + \bar{d}_{FDR}^2 + \bar{d}_{SR}^2$ where $\bar{d}_i^2 = \frac{1}{np} \sum_j^{np} \left(\frac{A_i(s_j)}{\delta A_i(s_j)} \right)^2$. Note that the OUT-IN difference

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	$\sqrt{s_{\text{pole}}} \text{ (MeV)}$	$ g $
$f_0(500)^{\text{GKPY}}$	$(457^{+14}_{-13}) - i(279^{+11}_{-7})$	$3.59^{+0.11}_{-0.13} \text{ GeV}$
$f_0(500)^{\text{Roy}}$	$(445 \pm 25) - i(278^{+22}_{-18})$	$3.4 \pm 0.5 \text{ GeV}$
$f_0(980)^{\text{GKPY}}$	$(996 \pm 7) - i(25^{+10}_{-6})$	$2.3 \pm 0.2 \text{ GeV}$
$f_0(980)^{\text{Roy}}$	$(1003^{+5}_{-27}) - i(21^{+10}_{-8})$	$2.5^{+0.2}_{-0.6} \text{ GeV}$
$\rho(770)^{\text{GKPY}}$	$(763.7^{+1.7}_{-1.5}) - i(73.2^{+1.0}_{-1.1})$	$6.01^{+0.04}_{-0.07}$
$\rho(770)^{\text{Roy}}$	$(761^{+4}_{-3}) - i(71.7^{+1.9}_{-2.3})$	$5.95^{+0.12}_{-0.08}$

Table 1. Poles and residues from Roy and GKPY equations.

for a given dispersion relation i at a given energy squared s_j is denoted by $\Delta_i(s_j)$ and its uncertainty by $\delta\Delta_i(s_j)$. Final values of these averaged differences for the Roy and GKPY equations and FDR were: $\bar{d}_{\text{Roy}}^2 = 0.14$, $\bar{d}_{\text{GKPY}}^2 = 0.32$ and $\bar{d}_{\text{FDR}}^2 = 0.4$, which is a remarkably good fulfillment of the relations.

The curves on Fig. 1 present the input and output amplitudes for the GKPY equations. One can see that the $\Delta_i(s_j)$ differences remain everywhere within $\delta\Delta_i(s_j)$.

On Fig. 2 the phase shifts and inelasticities for the $S0$ wave amplitude are presented.

Analytic continuation to the energy complex plane of the output amplitudes from Roy and GKPY equations allows for a precise determination of the poles related to the $f_0(500)$, $f_0(980)$ and $\rho(770)$ resonances and the calculation of their couplings to the $\pi\pi$ channel. The parameters of these resonance poles and their couplings are presented in Table 1. The latter are obtained from the pole residues as:

$$g^2 = -16\pi \lim_{s \rightarrow s_{\text{pole}}} (s - s_{\text{pole}}) t_\ell(s) (2\ell + 1) / (2p)^{2\ell} \quad (2)$$

where $p^2 = s/4 - m_\pi^2$. One can see that the parameters and couplings for the Roy and GKPY equations differ only slightly and always agree within the estimated uncertainties.

On Fig. 3 we show the positions of the $f_0(600)$ poles used in the Particle Data Group (PDG) edition 2010 [5] and of the pole found in this analysis. For comparison, we also show the mass and width ranges estimated both in the PDG 2010 and 2012 editions [1]. Let us remark the significant difference in these parameters in the new edition: $M_{f_0(600)} = 400 - 1200 \text{ MeV}$ to $M_{f_0(500)} = 400 - 550 \text{ MeV}$ and the width from $\Gamma_{f_0(600)} = 2 \times (250 - 500) \text{ MeV}$ to $\Gamma_{f_0(500)} = 2 \times (200 - 350) \text{ MeV}$ what makes this state lighter and narrower but mostly much more accurately than previously known. Let us remark that even the name of the particle has changed from $f_0(600)$ to $f_0(500)$. The estimated values in PDG 2012 for the next light scalar-isoscalar $f_0(980)$ are $M_{f_0(980)} = 990 \pm 20 \text{ MeV}$ and $\Gamma_{f_0(980)} = 40 - 100 \text{ MeV}$. As it is explained in the PDG, these changes are heavily influence by the results of recent dispersive approaches, including the one we are reviewing here. For completeness, let us also provide the position of the pole found in this analysis, which is $M_{f_0(980)} = 996 \pm 7 \text{ MeV}$ and $\Gamma_{f_0(980)} = 50^{+20}_{-12} \text{ MeV}$.

3 Conclusions

The presented method of combined analysis of experimental data together with theoretical constraints expressed by a set of dispersion relations turns out to be easy to use, very efficient and precise. The results obtained in this analysis, together with other dispersive works like [6], with which we find a good agreement, although we make no use of Chiral Perturbation Theory predictions, have provided additional and convincing evidence for the significant change of parameters of the two lightest scalar-isoscalar mesons $f_0(500)$ and $f_0(980)$ in latest edition of the particle data tables [1]. We hope that our method will be widely used in other studies to determine or to correct $\pi\pi$ amplitudes in many partial waves. In this way, it will ensure that causality and crossing symmetry constraints are satisfied.

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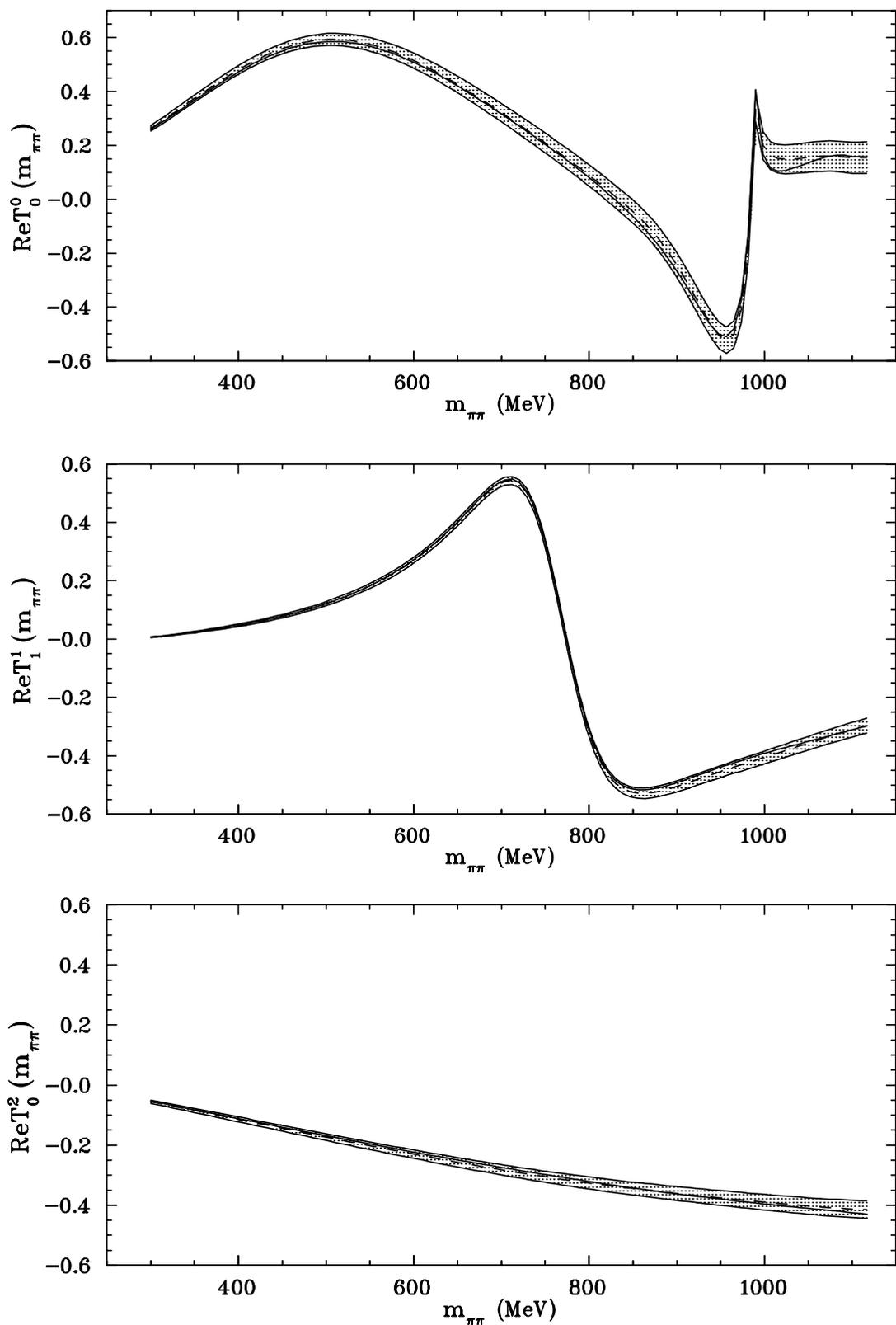


Fig. 1. Real parts of the input (dashed lines) and output (solid lines) amplitudes for the GKPY equations together with the uncertainties in the difference between both of them (gray bands). From top to bottom: for the S_0 wave, P wave and S_2 wave.

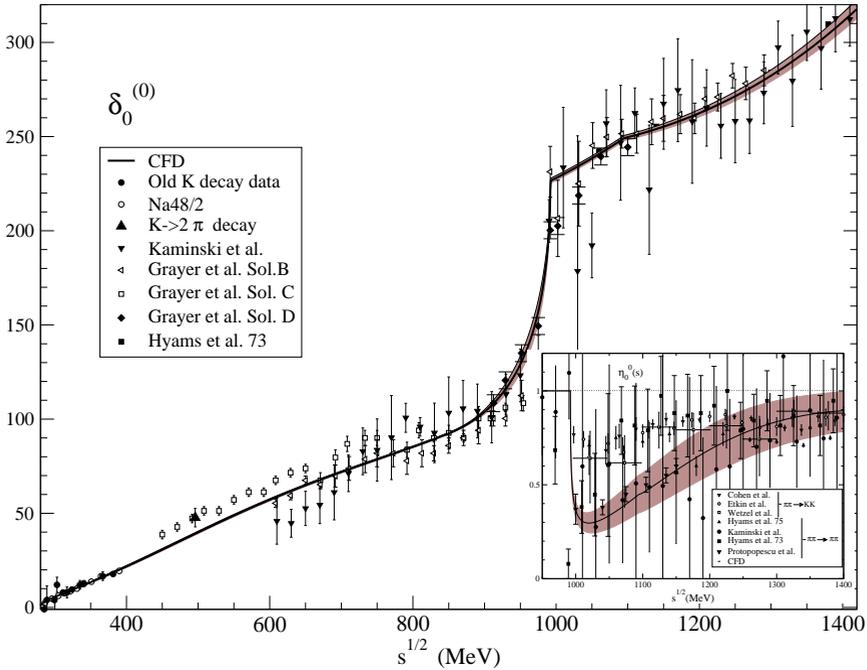


Fig. 2. Phase shifts and inelasticities together with their uncertainties (gray bands) for the S_0 wave calculated in the data fit constrained with dispersion relations (called CFD, see [2] for details).

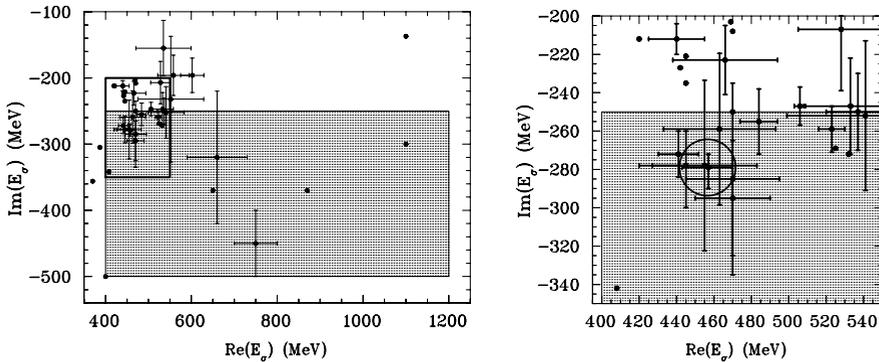


Fig. 3. Pole positions of the $f_0(600)$ (or σ) resonance cited in PDG'2010 [5] - black dots. Gray bands represent uncertainties in PDG'2010. The rectangle on the left panel indicates the magnified area shown on the right drawing and corresponds to the errors of mass and minus half of the width of the $f_0(500)$ in PDG'2012 [1]. The pole calculated in our work, which we have reviewed here, lies well in the center of the circle. Here $E_\sigma = \sqrt{s_\sigma}$.

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