Phenomenology of light mesons within a chiral approach

F. Giacosa\textsuperscript{1}, D. Parganlija\textsuperscript{1,2}, P. Kovács\textsuperscript{2,3}, and Gy. Wolf\textsuperscript{3}

\textsuperscript{1}Institute for Theoretical Physics, Johann Wolfgang Goethe University, Max-von-Laue-Str. 1, D-60438 Frankfurt am Main, Germany
\textsuperscript{2}Institute for Theoretical Physics, Vienna University of Technology, Wiedner Hauptstr. 8-10, A–1040 Vienna, Austria
\textsuperscript{3}Institute for Particle and Nuclear Physics, Wigner Research Center for Physics, Hungarian Academy of Sciences, H-1525 Budapest, Hungary

Abstract. The so-called extended linear sigma model is a chiral model with \((\text{pseudo})\)scalar and \((\text{axial-})\)vector mesons. It is based on the requirements of \((\text{global})\) chiral symmetry and dilatation invariance. The purpose of this model is the description of the hadron phenomenology up to 1.7 \(\text{GeV}\). We present the latest theoretical results, which show a good agreement with the experiment.

In this paper we describe a chiral \(\sigma\) model, called \('\text{extended Linear }\sigma\text{ model (El}\sigma\text{m)}', in which scalar, pseudoscalar, vector, axial-vector quark-antiquark mesons and, in addition, a scalar dilaton/glueball field are the basic degrees of freedom. The aim is to develop a model with the basic symmetries of QCD which can describe the vacuum phenomenology up to 1.7 \(\text{GeV}\). The Lagrangian of the model is built by requiring \((\text{i})\) global chiral symmetry and \((\text{ii})\) dilatation invariance. Although chiral models are studied since long time [1], the here presented \(\text{El}\sigma\text{m}\) represents the first attempt to treat the unified chiral model (pseudo)scalar mesons (including the glueball) as well as the (axial-)vector ones. (Previous studies [2] only exist for \(N_\text{f} = 2\) and not all the mentioned d.o.f. were taken into account.) It turns out that the inclusion of (axial-)vector d.o.f. have a very strong influence on the overall phenomenology, influencing also the decays in the \((\text{pseudo})\)scalar sector.

The explicit form of the Lagrangian in the mesonic sector reads (for a generic number of flavors \(N_\text{f}\) [3–6]):

\[
\mathcal{L}_{\text{El}\sigma\text{m}} = \frac{1}{2}(\partial_{\mu}G)^2 - V_{\text{dil}}(G) + \text{Tr}\left[(D^\mu\Phi)^\dagger(D_\mu\Phi) - aG^2\Phi^\dagger\Phi - \lambda_2 \left(\Phi^\dagger\Phi\right)^2 - \lambda_1 \text{Tr}[\Phi^\dagger\Phi]\right] + \frac{c_1}{2}(\text{det} \Phi^\dagger - \text{det} \Phi)^2 + \text{Tr}[H(\Phi^\dagger + \Phi)] - \frac{1}{4}\text{Tr}[(L_{\mu\nu})^2 + (R_{\mu\nu})^2] + g_2^2\text{Tr}[(L_{\mu})^2 + (R_{\mu})^2] + \frac{g_2^2}{2}\text{Tr}[(\Phi^\dagger L_{\mu\nu}L^{\mu\nu}\Phi + \Phi R_{\mu\nu}R^{\mu\nu}\Phi)] + h_2\text{Tr}\left[\Phi^\dagger L_{\mu\nu}L^{\mu\nu}\Phi + \Phi R_{\mu\nu}R^{\mu\nu}\Phi\right] + 2h_3\text{Tr}\left[\Phi R_{\mu\nu}\Phi^\dagger L^{\mu\nu}\Phi\right] + ... ,
\]

where \(D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu)\) and dots represent further terms which are unimportant in the evaluation of decays and (on-shell) scattering lengths. Following comments are in order:

(i) The (pseudo)scalar quark-antiquark mesons are described by the matrix \(\Phi = (S^a + iP^a)\tau^a\) (\(\tau^a\) are the generators of the group \(U(N_\text{f})\)). The pseudoscalar states are the pion, kaon and the \(\eta\) and \(\eta'\) mesons. The assignment of scalar states is controversial [6–8] and represents one of the motivations of our study. It turns out that the best agreement with the experiment is obtained when the quark-antiquark scalar states of the model are assigned to the scalar resonances between 1-2 \(\text{GeV}\) (theory in Refs. [3–5] and experimental results in Ref. [9]).

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(ii) The (axial-)vector mesons are described by the matrices $L^\mu = (V^{\mu\nu} + A^{\mu\nu}) t^\nu$ and $R^\mu = (V^{\mu\nu} - A^{\mu\nu}) t^\nu$. The vector mesons are the usual $\rho$, $\omega$, $K^\ast (892)$, and $\phi$ mesons. The axial-vector mesons are assigned to $a_1(1230)$, $K_1(1270)$, $f_1(1285)$ and $f_1(1510)$.

(iii) The dilaton field is denoted by $G$ and its potential reads $V_{dl}(G) = \frac{1}{4N_c} \left[ G^4 \ln \left( \frac{G}{\Lambda^4} \right) - \frac{G^4}{4} \right]$ [10]. The parameter $\Lambda \sim N_c \Lambda_{QCD}$ sets the energy scale of the theory.

(iv) The $U(1)c$ anomaly term is parametrized by the parameter $c_1$, which has dimension [Energy]$^{4-2N_f}$.

(v) The matrix $H \propto \text{diag}[m_u, m_d, m_s, ...]$ describes explicit symmetry breaking of both chiral and dilatation symmetries due to the bare quark masses $m_i$. Similarly, in the (axial-)vector sector the diagonal matrix $\delta$ has been introduced.

(vi) Chiral symmetry breaking takes place when the parameter $a$ is negative. In fact, upon the condensation of the field $G = G_0$, the ‘wrong’ sign for mesonic masses $aG^2 < 0$ sign is realized.

Once the shifts of the scalar fields $G \rightarrow G_0 + G$ and $\Phi \rightarrow \text{diag} \left[ \sqrt{2}G_N, \sqrt{2}G_N, ... \right] + \Phi$, where the first term is a diagonal matrix with the quark-anti-quark condensates, and necessary redefinitions of the pseudoscalar and axial-vector fields have been performed [3,5], the explicit calculations of physical processes are lengthy but straightforward. (Note, the calculations are performed at tree-level; the inclusion of loops is a task for the future, but only slight changes are expected [11].)

In the following we summarize the results that we have obtained with the model in Eq. (1):

- $N_f = 2$ with frozen glueball ($m_G \rightarrow \infty$) [3,12]: by considering the limit $m_G \rightarrow \infty$ the dilaton/glueball field is not an active d.o.f. One can neglect the gluonic part of the Lagrangian of Eq. (1) and set $G = G_0 = \Lambda_G$. In the works in Refs. [3,12] it has been shown that the inclusion of (axial-)vector mesons has a strong influence on the overall phenomenology. For instance, the width of the scalar meson $\sigma$ (the chiral partner of the pion) decreases substantially w.r.t. the case without (axial-)vector mesons: for this reason, the identification of this field with the resonance $f_0(500)$ is not favoured, because the theoretically evaluated width is smaller than 200 MeV; this is at odd with the experiment, according to which it is larger than 400 MeV. On the other hand, the identification of the $\sigma$ field with the resonance $f_0(1370)$ turns out to be in agreement with the experimental results. The description of the (axial-)vector resonances is also in agreement with the experiments reported in Ref. [9].

- $N_f = 2$ with active glueball ($m_G \sim 1.5$ GeV) [4]: the glueball with a bare mass $m_G \sim 1.5$ GeV, in agreement with the lattice results [13], has been investigated for the first time in a chiral model with (axial-)vector mesons. (For other approaches see Ref. [8] and refs. therein). The state $f_0(1500)$ results as the predominantly (75\%) glueball state, and the rest of the phenomenology is only slightly affected w.r.t. the previous case, in which $m_G \rightarrow \infty$. Moreover, also the gluon condensate has been evaluated and is in agreement with lattice results.

- $N_f = 3$ with frozen glueball ($m_G \rightarrow \infty$) [5,14]: the results for this scenario are interesting because, for the first time, it is possible to study in a chiral framework the overall vacuum’s phenomenology up to 1.7 GeV. The results for the best-fit scenario are listed in Table I. Eleven free parameters enter in the fit and the total $\chi^2 \approx 1$ signals a good agreement of the theory with the experiment, see Ref. [5] for details. It is visible that the resonances $a_0(1450)$ and $K_0^\ast (1430)$ are well described as quark-anti-quark fields. The scalar-isoscalar mesons are not included in the fit, but can be studied as a consequence of it: the decay pattern and the masses suggest that $f_0(1370)$ and $f_0(1710)$ are (predominantly) the non-strange and strange scalar-isoscalar fields.

- Other works related to the model: in Ref. [15] the baryonic part of the model has been presented for $N_f = 2$ and in Ref. [16] the non-zero density and nuclear matter saturation have been described. In Ref. [17] part of the model has been investigated at nonzero temperature.

In the future we plan to perform the following investigations with the chiral model in Eq. (1): (i) The case $N_f = 3$ with $m_G \sim 1.5$ GeV: the inclusion of an active glueball represents the most straightforward extension of our approach; the aim is a full description of scalar-isoscalar states between 1-2 GeV, including also the resonance $f_0(1500)$ [8]. (ii) The case $N_f = 4$ (that is, including charmonia states) can be studied. In this way we can test the implications of chiral symmetry also beyond the low-energy sector. Note, only two additional parameters (both connected to the charm mass) w.r.t. the
case $N_f = 3$ are needed. The Lagrangian is still the one in Eq. (1). (iii) $\tau$ decays involving (axial-)vector mesons: after including the weak-gauge bosons $W^\pm, Z^0$ in the model the spectral functions of vector and axial-vector states can be investigated. (iv) Scalar states below 1 GeV: these states are not part of the model, but could be added as tetraquarks along the line of Ref. [18] (see also Ref. [19] and refs. therein). (v) Inclusion of the pseudoscalar glueball and the evaluation of its decays: this project can be relevant for future experiments, such as the PANDA experiment at FAIR. (vi) In-medium properties and the phase diagram: the study of chiral phase transition represents an important outlook of this work (for preliminary works see Refs. [16,17]). In fact, a great advantage of the linear realization of chiral symmetry is the straightforward extension to nonzero temperature and density. Moreover, also inhomogeneous condensates can be investigated.

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