

Chiral dynamics with vector fields: an application to $\pi\pi$ and πK scattering

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Abstract. A theoretical study of Goldstone boson scattering based on the chiral Lagrangian with vector meson fields is presented. In application of a recently developed novel approach we extrapolate subthreshold partial-wave amplitudes into the physical region. The constraints set by micro-causality and coupled-channel unitarity are kept rigorously. It is shown that already the leading order subthreshold amplitudes lead to s- and p-wave $\pi\pi$ and πK phase shifts are in agreement with the experimental data up to about 1.2 GeV.

1 Introduction

Chiral perturbation theory (χPT) is a powerful tool for a systematic study of the interaction of Goldstone bosons [1,2]. However the convergence of the chiral expansion is limited to the threshold region and a generalization to higher energy is desirable. It was shown that coupled-channel unitarity techniques may lead to a systematic description of scattering phases in the resonance region [3].

The purpose of this contribution is to apply a recently proposed unitarization scheme [4] to Goldstone boson scattering based on the chiral Lagrangian supplemented with light vector mesons. There are several reasons for considering the light vector mesons as explicit degrees of freedom. First of all we recall a resonance saturation mechanism [5]. It was shown that the size of the low energy constants (LECs) of Q^4 counter terms are basically saturated by the light vector mesons. Second, the light vector mesons play a particular role in the hadrogenesis conjecture [6–8]. Together with the Goldstone bosons, they are identified to be the relevant degrees of freedom that are expected to generate the meson spectrum. For instance it was shown that the leading chiral interaction of Goldstone bosons with the light vector mesons generates an axial-vector meson spectrum that is quite close to the empirical one [7].

The Chiral Lagrangian at order Q^2 includes two relevant and known parameters only, the chiral limit value of the pion decay constant and a coupling constant that characterize the decay of the rho meson into a pair of pions. As a result we recover the empirical pion-pion and pion-kaon scattering up to about $\sqrt{s} \approx 1.2$ GeV [9].

2 Description of the method

The relevant leading-order Lagrangian [6,8] is given by

$$\mathcal{L} = \frac{1}{48f^2} \text{tr} \left\{ [\Phi, \partial^\mu \Phi]_- [\Phi, \partial_\mu \Phi]_- + \Phi^4 \chi_0 \right\} - i \frac{f_V h_P}{8f^2} \text{tr} \left\{ \partial_\mu \Phi \Phi^{\mu\nu} \partial_\nu \Phi \right\},$$

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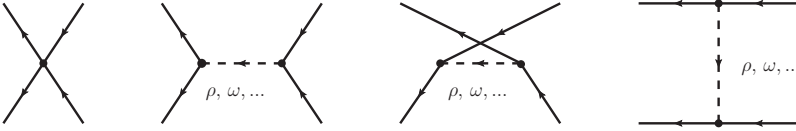


Fig. 1. Tree level diagrams for Goldstone boson scattering with the exchange of light vector mesons (dashed line) in the s -, t - and u -channels.

where the pseudo-scalar and vector mesons are collected in

$$\Phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}} \eta & \sqrt{2} \pi^+ & \sqrt{2} K^+ \\ \sqrt{2} \pi^- & -\pi^0 + \frac{1}{\sqrt{3}} \eta & \sqrt{2} K^0 \\ \sqrt{2} K^- & \sqrt{2} \bar{K}^0 & -\frac{2}{\sqrt{3}} \eta \end{pmatrix} \quad \text{and} \quad \Phi_{\mu\nu} = \begin{pmatrix} \rho_{\mu\nu}^0 + \omega_{\mu\nu} & \sqrt{2} \rho_{\mu\nu}^+ & \sqrt{2} K_{\mu\nu}^+ \\ \sqrt{2} \rho_{\mu\nu}^- & -\rho_{\mu\nu}^0 + \omega_{\mu\nu} & \sqrt{2} K_{\mu\nu}^0 \\ \sqrt{2} K_{\mu\nu}^- & \sqrt{2} \bar{K}_{\mu\nu}^0 & \sqrt{2} \phi_{\mu\nu} \end{pmatrix}, \quad (1)$$

respectively. In order to generate a faithful resonance saturation mechanism, the vector mesons are represented in terms of anti-symmetric fields $\Phi_{\mu\nu} = -\Phi_{\nu\mu}$. In the Lagrangian (1), $f = 90$ MeV may be identified with the pion-decay constant at leading order. The value of the parameter $f_V h_P$ was determined in [6],

$$f_V h_P \approx 0.23 \text{ GeV}. \quad (2)$$

The tree level p.w. scattering amplitudes are given by the Weinberg-Tomozawa and the s -, t - and u -channel vector meson exchange terms. In Fig.1 the set of diagrams that we take into account is depicted.

Our approach is based on partial-wave dispersion relations. Following [4] a generalized potential $U_{ab}^J(s)$ is constructed from the chiral Lagrangian in the subthreshold region and analytically extrapo-

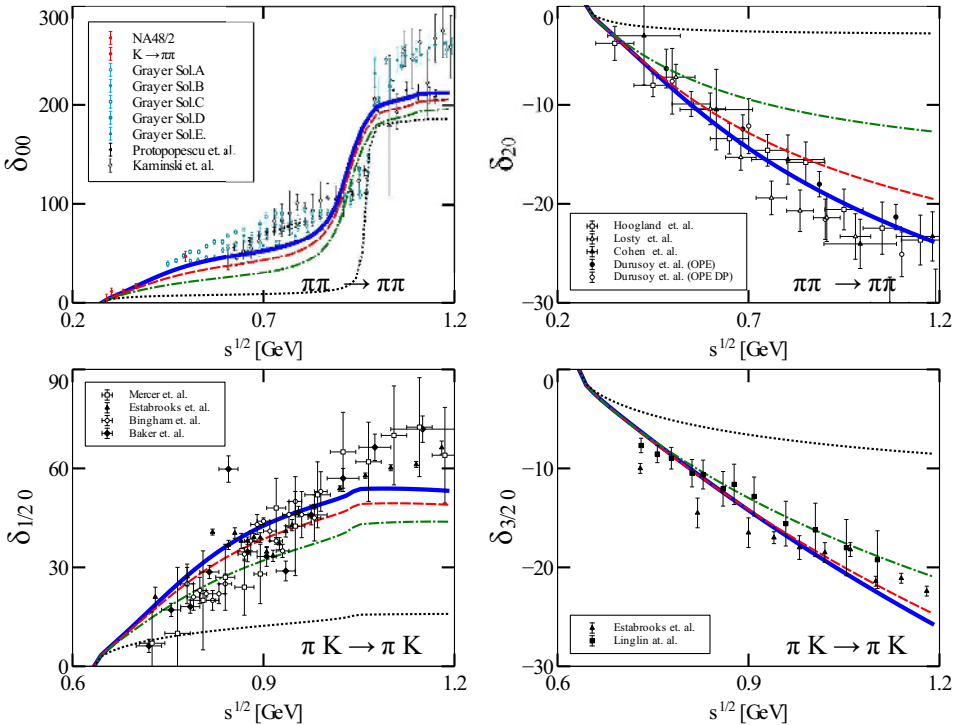


Fig. 2. Results for the s -wave $\pi\pi$ and πK phase shifts δ_{IJ} . The dotted, dash-dotted, dashed and solid lines correspond to a truncation in the expansion (4) at order 0, 1, 2, 3 respectively. For data references see [9].

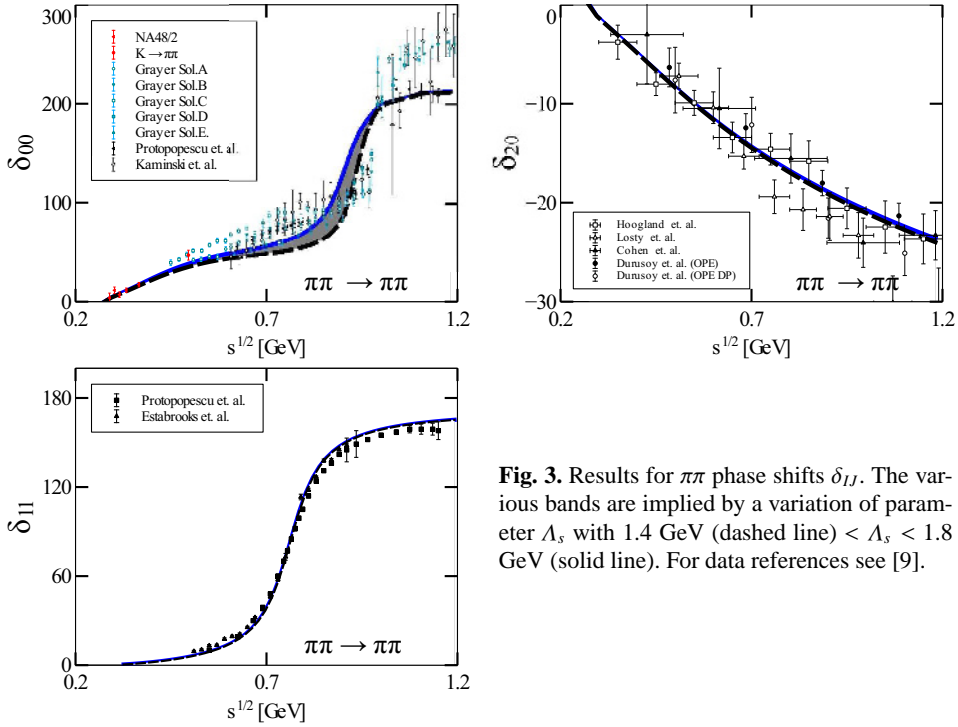


Fig. 3. Results for $\pi\pi$ phase shifts δ_{IJ} . The various bands are implied by a variation of parameter Λ_s with $1.4 \text{ GeV} < \Lambda_s < 1.8 \text{ GeV}$ (solid line). For data references see [9].

lated to higher energies. The partial-wave scattering amplitudes $T_{ab}^J(s)$ are obtained as solutions of the non-linear integral equation

$$T_{ab}^J(s) = U_{ab}^J(s) + \sum_{c,d} \int_{\mu_{thr}^2}^{\infty} \frac{d\bar{s}}{\pi} \frac{s - \mu_M^2}{\bar{s} - \mu_M^2} \frac{T_{ac}^J(\bar{s}) \rho_{cd}^J(\bar{s}) T_{db}^{J*}(\bar{s})}{\bar{s} - s - i\epsilon}, \quad (3)$$

which leads to a controlled realization of the causality and coupled-channel unitarity condition. Here $\rho_{cd}^J(s)$ is the phase-space matrix and μ_M is a matching scale which is identified with the smallest two-body threshold accessible in a sector with isospin and strangeness (I, S).

As was pointed out in [4], the generalized potential $U_{ab}^J(s)$ can be reconstructed unambiguously in terms of its derivatives at an expansion point that lies within its analyticity domain and where the results of χ PT are reliable. It holds

$$U(s) = \sum_{k=0}^n c_k \xi^k(s) \quad \text{for } s < \Lambda_s^2, \quad (4)$$

where the coefficients c_k are determined by the first k derivatives of $U(s)$ at the expansion point. The function $\xi(s)$ is a suitable conformal map constructed as to analytically continue the potential to larger energies. Explicitly, it is given by

$$\xi(s) = \frac{a(\Lambda_s^2 - s)^2 - 1}{(a - 2b)(\Lambda_s^2 - s)^2 + 1}, \quad a = \frac{1}{(\Lambda_s^2 - \mu_E^2)^2}, \quad b = \frac{1}{(\Lambda_s^2 - \Lambda_0^2)^2}, \quad (5)$$

where the parameter Λ_0 is identified unambiguously such that the mapping domain of the conformal map touches the closest left-hand branch point. The value of Λ_s sets the scale from where on s-channel physics is integrated out. For $s > \Lambda_s^2$ the generalized potential is set to a constant.

In the p-wave amplitude, vector mesons show up as poles above threshold. In that case CDD poles have to be included explicitly in order to solve Eq.(3).

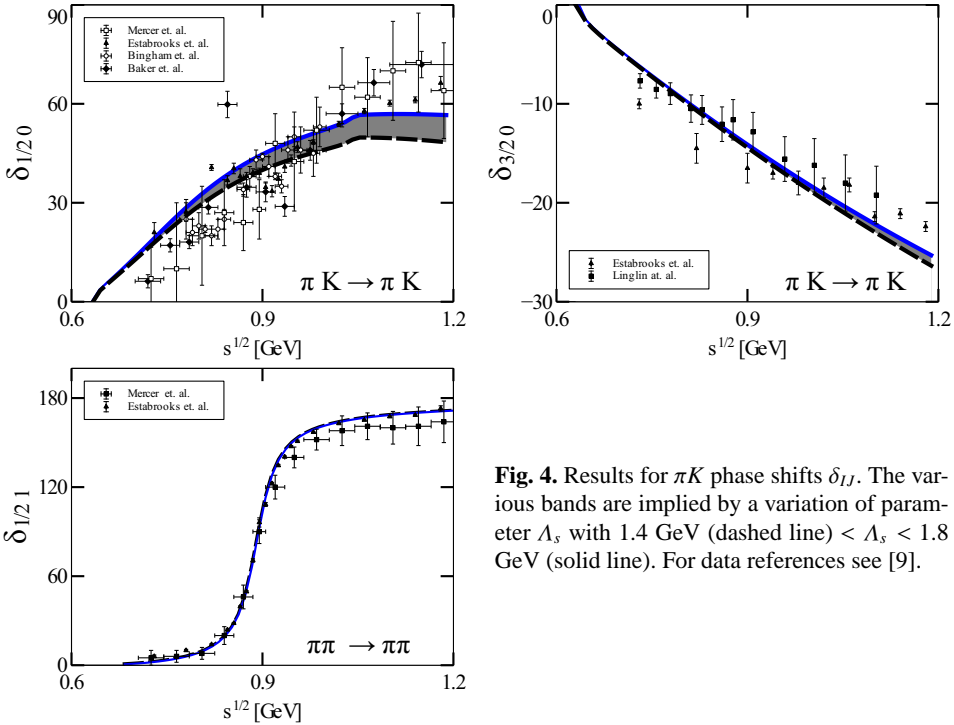


Fig. 4. Results for πK phase shifts $\delta_{l,j}$. The various bands are implied by a variation of parameter Λ_s with $1.4 \text{ GeV} < \Lambda_s < 1.8 \text{ GeV}$ (solid line). For data references see [9].

3 Results

We first study the dependence of our results on the truncation index n in (4) at given $\Lambda_s = 1.6 \text{ GeV}$. In Fig.2 the s -wave phase shifts for $\pi\pi$ and πK are shown for the cases $n = 0, 1, 2, 3$. The strong dependence on the truncation index we interpret as a clear signal of vector meson dynamics in the s -wave phase shifts. The convergence pattern towards the empirical phase shifts is quite reassuring pointing at the reliability of our approach. We note that in the p -wave scattering phases there is a very minor dependence on the truncation index n only.

In Fig. 3 and 4 we show the dependence of the scattering phases for fixed $n = 3$ but a variation of Λ_s from 1.4 GeV to 1.8 GeV . In all cases this causes a rather small error band only, where for $n < 3$ the error bands get even smaller. We conclude that within our scheme the s - and p -wave $\pi\pi$ and πK phase shifts are in a good agreement with the data set up to 1.2 GeV .

References

1. S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264.
2. J. Gasser and H. Leutwyler, Annals Phys. **158** (1984) 142.
3. A. Gomez Nicola and J. R. Pelaez, Phys. Rev. D **65** (2002) 054009.
4. A. Gasparyan and M. F. M. Lutz, Nucl. Phys. A **848** (2010) 126; I. V. Danilkin, A. M. Gasparyan and M. F. M. Lutz, Phys. Lett. B **697** (2011) 147; A. M. Gasparyan, M. F. M. Lutz and B. Pasquini, Nucl. Phys. A **866** (2011) 79.
5. G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321** (1989) 311.
6. M. F. M. Lutz and S. Leupold, Nucl. Phys. A **813** (2008) 96.
7. M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. A **730** (2004) 392.
8. C. Terschlusen, S. Leupold and M. F. M. Lutz, arXiv:1204.4125 [hep-ph].
9. I. V. Danilkin, L. I. R. Gil and M. F. M. Lutz, Phys. Lett. B **703** (2011) 504.