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Chiral condensate in nuclear matter beyond linear density using chiral Ward identity*

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Abstract. We discuss density corrections of the chiral condensate up to a NLO order using the chiral Ward identity and an in-medium chiral perturbation theory. The in-medium chiral condensate is calculated by a correlation function of the axial current and pseudoscalar density in the nuclear matter as a consequence of the chiral Ward identity. The correlation function is evaluated using the chiral perturbation theory with the hadronic quantities of pion-nucleon dynamics. We assume that the in-vacuum interaction vertices are known, which means that the in-vacuum loop corrections are renormalized to the tree chiral couplings by taking the values of the couplings in chiral Lagrangian as the physical values. We focus on density order in the physical quantities in our perturbative calculation. This study shows that the medium effects to the chiral condensate beyond the linear density come from density corrections to the πN sigma term. It implies that calculating the density dependence of the chiral condensate in nuclear matter is essentially equivalent to describe nuclear matter in chiral effective theory.

1 Introduction

Chiral symmetry breaking (χSSB) $SU(N_f)_L \otimes SU(N_f)_R \rightarrow SU(N_f)_V$ is an important phenomenon, which characterizes low-energy Quantum ChromoDynamics(QCD) and Hadron physics, and the non-vanishing chiral condensate $\langle \bar{q}q \rangle$ as an order parameter of χSSB generates hadron masses.

Recently, partial restoration of chiral symmetry in the nuclear medium has gained considerable attention. It means reduction of the absolute value of the condensate in the nuclear medium and leads to various changes of hadron properties according to the reduction, for example, repulsive enhancement of s-wave π N interaction, the mass difference between the ρ and a_1 mesons and so on. Vast theoretical and experimental effort is devoted in this region[1–3]. In the theoretical side, Ref. [1] discusses a systematic and model-independent derivation of in-medium sum rules and shows some relations between chiral condensate and experimental observables. In experimental side, Ref. [3] has extracted quantitatively the s-wave isovector scattering length of π and nucleus by observing the deeply-bound 1s energy level and decay width of pionic Sn atoms and derived the reduction of the pion decay constant. The reduction of the chiral condensate has been extracted through the Glashow-Weinberg relation and the Gell-Mann-Oakes-Renner relation:

$$\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle_0} \simeq 1 - 0.37 \frac{\rho}{\rho_0} \tag{1}$$

Here, $\langle \bar{q}q \rangle^*$, $\langle \bar{q}q \rangle_0$ are the in-medium and in-vacuum chiral condensates, respectively, and ρ_0 is the normal nuclear density. This estimation suggests that the chiral symmetry breaking is partially restored

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to about 65% in the center of nucleus. Once we know the density dependence of the condensate qualitatively, we can get possibilities to predict other in-medium hadronic quantities through low energy theorems. Therefore, it is important to evaluate the in-medium condensate quantitatively.

2 Chiral Ward identity

In order to calculate the density dependence of the chiral condensate, we make use of the chiral Ward identity in the following correlation function:

$$\Pi_5^{ab}(q) = \int d^4x e^{iq\cdot x} \partial^\mu \langle \Omega | T A_\mu^a(x) P^b(0) | \Omega \rangle, \tag{2}$$

where $A_\mu^a(x)$ denotes the axial vector current associated with the SU(2) chiral transformation, $P^b(0)$ stands for the pseudo-scalar current, and $|\Omega\rangle$ is the nuclear matter ground state normalized as $\langle\Omega|\Omega\rangle = 1$ and characterized by the proton and neutron density,ρ_p and ρ_n. Hereafter we write the expectation value $\langle\Omega|O|\Omega\rangle$ as $\langle O\rangle^*$ for operator O. Differentiating the T-product and the axial current in the soft limit $q_\mu \to 0$ and using the current algebra $[Q_5^a, P^b(x)] = -i\delta^{ab}\bar{q}q$ with the axial transformation generator Q_5^a and the PCAC relation $\partial^\mu A_\mu^a = -m_q P^a$, we find the in-medium chiral condensate given by the Green functions in the soft limit:

$$-i\delta^{ab}\langle \bar{u}u + \bar{d}d\rangle^* = \Pi_5^{ab}(0) + m_q D^{ab}(0), \tag{3}$$

where

$$\Pi_5^{ab}(0) \equiv \lim_{q_\mu \to 0} -iq^\mu \int d^4x e^{iq\cdot x} \langle A_\mu^a(x) P^b(0)\rangle^* \tag{4}$$

$$D^{ab}(0) \equiv \lim_{q_\mu \to 0} \int d^4x e^{iq\cdot x} \langle P^a(x) P^b(0).\rangle^* \tag{5}$$

3 In-medium chiral perturbation theory

In order to calculate the in-medium chiral condensate, we calculate the correlation functions appearing in the right hand side of Eq. (3) in an in-medium chiral perturbation theory developed in Ref. [4–6]. In Ref. [4], the generating functional of the in-medium SU(2) CHPT with external sources is derived. Starting with the SU(2) chiral Lagrangian having nucleon bilinear terms, we consider the generating functional Z defined by a transition from the in-state to the out-state:

$$Z[J, \eta, \eta^\dagger] = e^{iW[J,\eta,\eta^\dagger]} = \langle \Omega_{\rm out} | \Omega_{\rm in}\rangle_{J,\eta,\eta^\dagger}, \tag{6}$$

where $J = (s, p, v, a)$ represents the scalar, pseudo-scalar, vector and axial-vector sources, respectively and $\eta, \eta^\dagger$ are external sources for the nucleon field. The states $\langle\Omega_{\rm out}|, |\Omega_{\rm in}\rangle$ are the out and in states defined by the nuclear Fermi gas in which nucleons are occupied up to the Fermi momentum k_F:

$$|\Omega_{\rm in,out}\rangle \equiv \prod_n^N a^\dagger(\mathbf{p}_n)|0\rangle \tag{7}$$

with the nucleon creation operator $a^\dagger(\mathbf{p}_n)$. In the path integral representation the nucleon bilinear interactions are integrated out by the Gauss integral formula. Setting the external sources $\eta, \eta^\dagger$ to be zero at the final stage, we obtain the in-medium CHPT generating functional with pion-nucleon dynamics. It is shown that the obtained generating functional is characterized by double expansion of Fermi sea insertion and chiral counting. Similar to the in-vacuum CHPT, in which one counts the pion momentum p_π and mass m_π as $O(p)$, we have a counting rule of the chiral power order [5].

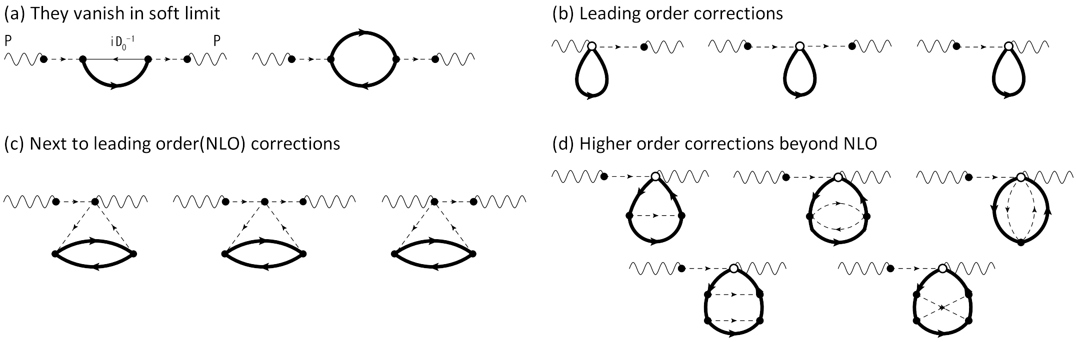

Fig. 1. Feynman diagrams for the evaluation of the in-medium chiral condensate. The solid, thick and dashed lines represent the free nucleon, the nucleon propagation in the Fermi sea, and pion, respectively. The filled and open circles are the leading and subleading order πN vertices.

This counting scheme allows us to count Fermi momentum k_F as $O(p)$ because Fermi momentum $k_F \approx 270\text{MeV} \approx 2m_\pi$ at the normal nuclear density ρ_0. Using the in-medium CHPT, we can perform the order counting for density corrections systematically. We take πN chiral Lagrangian as interactions between pions and nucleons in the Fermi gas, and assume that the in-vacuum interactions are fixed by in-vacuum pion-nucleon dynamics. In other words, new parameters characterizing nuclear matter is not necessary up to a certain order. Describing the Green functions as follows:

$$\langle \Omega_{\text{out}} | \hat{O}_1 \cdots \hat{O}_i | \Omega_{\text{in}} \rangle = (-i)^n \frac{\delta}{\delta J_1} \cdots \frac{\delta}{\delta J_i} Z[J] = \sum_n C_n k_F^n$$

with the QCD current operator $\hat{O}_i$ corresponding to the external source J_i, we can evaluate the low energy QCD Green functions in the nuclear matter by perturbative calculations in terms of the Fermi momentum k_F. In our study, we use the renormalization idea and focus on density counting by using the observed values for the couplings in the chiral Lagrangian.

4 Density dependence of chiral condensate

We calculate the Green functions $\Pi_5^{ab}(0), D^{ab}(0)$ to evaluate the in-medium chiral condensate based on the chiral power counting scheme. $\Pi_5^{ab}(0)$ vanishes in soft limit because axial current is coupled with pion with derivative interaction but pion is not zero-mode. Therefore we consider $D^{ab}(0)$ in the following. In fig. 1, we list up the Feynman diagrams of $D^{ab}(0)$ in the symmetric nuclear matter. The Feynman diagrams in (a) are the leading corrections in the chiral counting scheme but vanishes in the soft limit, and diagram (b) is the leading order density corrections $O(\rho)$ and reproduces the result in the linear density approximation obtained in Ref. [7], which is known as the linear density correction proportional to the πN sigma term. Diagram (c) is the next to leading order(NLO) corrections with $O(\rho^{4/3})$. They are the density corrections to the πP vertex. In other words, these are density corrections to the πN sigma term. We calculate the in-medium chiral condensate within NLO density corrections and obtain the analytic expression

$$\frac{\langle \bar{u}u + \bar{d}d \rangle^*}{\langle \bar{u}u + \bar{d}d \rangle_0} \approx 1 + \frac{4c_1}{f^2}\rho + \frac{g_A^2 k_F^4}{4 f^4 \pi^4}\left(\frac{3}{8} - \frac{3a^2}{4} - \frac{3a}{2}\arctan\frac{1}{a} + \frac{3a^2(a^2+2)}{4}\ln\frac{a^2+1}{a^2}\right)$$

Here, $a = m_\pi / 2k_F$.

In fig. 2, the density dependence of the condensate is plotted as a function of the nuclear density normalized by ρ_0 . The green, blue and red lines are the chiral condensates obtained by the linear density approximation in chiral limit, by the linear density approximation off the chiral limit and by the NLO corrections off the chiral limit. We find that the NLO density correction amounts to 5% at the

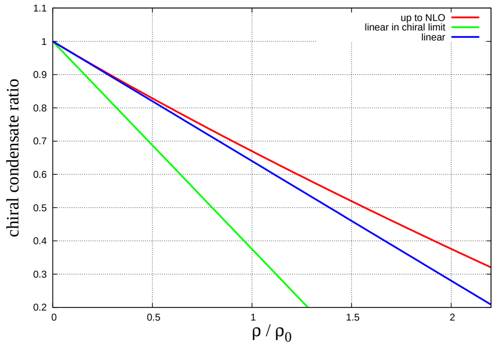

Fig. 2. The density dependence of chiral condensate in symmetric nuclear matter. The green, blue and red lines are the chiral condensates obtained by the linear density approximation in chiral limit, by the linear density approximation off the chiral limit and by the NLO corrections off the chiral limit.

normal nuclear density. Therefore the linear density approximation will be good in low density region up to around ρ_0. When we go to further higher density, the NLO correction becomes 10% at $2\rho_0$.

Diagram (d) shows fig. 1 is possible higher order corrections beyond NLO. We find that these are 1 and 2 pion-exchange effects in the Fermi gas. Higher order calculation has UV divergences from pion-loops, so that one needs renormalization of these vertices. This means that one needs not only in-vacuum πN couplings but also the NN contact terms obtained the NN dynamics. Recently, as a step in this direction, a non-perturbative chiral effective theory has been developed to improve the NN correlation by including NN contact terms using a resummation method [8]. Moreover, in Ref. [9] $\Delta(1232)$ resonance contributions have been evaluated and it has been found that Δ resonance effects are not small.

5 Summary

We calculate the density dependence of the chiral condensate up to NLO by using the chiral Ward identity and an in-medium chiral perturbation theory. We make use of a renormalization idea in which we take physical values of the couplings and focus on the density corrections. We find that the NLO correction amounts to about 5% at the normal nuclear density and to about 10% at twice of the normal density. The linear density approximation seems to be good in the normal density. In higher order corrections beyond NLO, we need information of the NN dynamics, and Δ resonance contributions could be large. Therefore, we need to include the NN contact term interactions and delta resonance effects, in other words, we need dynamical information beyond πN dynamics information such as nuclear force dynamics and Δ resonance dynamics.

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