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# Check of phenomenological amplitudes for the $\pi\pi$ scattering using the dispersion relations

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**Abstract.** The multichannel S- and P-wave amplitudes for the $\pi\pi$ scattering, constructed requiring analyticity and unitarity of the S-matrix and using the uniformization procedure, are elaborated using the dispersion relations which, in addition, impose the crossing symmetry condition. The amplitudes are modified in the low-energy region to improve their consistency with experimental data and the dispersion relations. Agreement with data is achieved for both amplitudes from the threshold up to 1.8 GeV and with dispersion relations up to 1.1 GeV. Consequences of the applied modifications, e.g. changes of the S-wave lowest-pole positions, are presented.

## 1 Introduction

A model independent analysis of the $\pi\pi$ scattering is important tool in getting information about the spectrum of light mesons. A reliable description of the process is therefore desirable to allow us to learn more on nature and parameters of the mesons.

The phenomenological multichannel amplitudes for the S and P waves in the $\pi\pi$ scattering were constructed without any specific assumptions about dynamics of the process, only requiring analyticity and unitarity of the S-matrix and applying the uniformization procedure [1]. This procedure can be applied exactly in the two-channel case. In the three-channel case, simplifying approximations had to be made which has resulted in a poor description of experimental data in the threshold region [1].

The crossing symmetry condition, which relates the S and P waves and which is important below the inelastic threshold, was not taken into account in the construction of these amplitudes [1]. Since the crossing symmetry is properly included in the Roy-like dispersion relations [2], it is possible and desired to improve the low-energy behavior of the three-channel amplitudes [1] and to check their consistency with the dispersion relations (DR).

In the multichannel uniformizing (MI) approach a heavy and broad $\sigma$ meson is predicted, $m = 829 \pm 10$ MeV and $\Gamma = 1108 \pm 22$ MeV [1], in disagreement with results from DR [3]. It is therefore also interesting to show how much the modifications of the three-channel amplitudes affect the position of the pole connected with the $\sigma$ meson, which can contribute to disclosing reasons of differences between some results from the MI and DR approaches.

In this note we present an example of using the dispersion relations to improve the low-energy behavior of a phenomenological three-channel partial-wave $\pi\pi$ amplitudes and to impose the crossing symmetry condition on the amplitudes.

## 2 Multichannel amplitudes and dispersion relations

Two channels coupled to the $\pi\pi$ channel, $K\bar{K}$ and $\eta\eta'$ for S wave and $\rho 2\pi$ and $\rho\sigma$ for P wave, were explicitly considered in construction of the three-channel amplitudes [1]. The eight-sheeted Riemann

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surface was transformed into a uniformization plane using a uniformizing variable $w$ in which the left-hand branch point connected with the crossed channels was not taken into account. The crossing symmetry condition was not therefore considered in the construction. A contribution of the left-hand cut was included in the background part of the amplitude. Note, that in Ref. [4] the left-hand branch point in $w$ was already included in the S-wave analysis.

In the uniformization plane an influence of the $\pi\pi$-threshold branching point was neglected, keeping however the unitarity on the $\pi\pi$ cut, which was a necessary approximation in the three-channel case [1]. We got, therefore, a four-sheeted model of the initial Riemann surface in which the near-threshold data are not described properly. Note that in the two-channel case this approximation is not needed and the threshold data are described correctly [5].

The resonance part of the matrix element $S_{11}$ of the S-matrix is generated by clusters of complex-conjugate poles and zeros on the Riemann surface, which represent resonances [1]. For example, the $f_0(600)$ resonance is represented by a cluster which possesses zero only in the $S_{11}$ matrix element on the physical sheet. Location of poles on the unphysical sheets is given by the analytic continuation of the matrix elements [4]. The background and resonant parts of the S-matrix are separated and expressed via the Le Couteur–Newton relations with the Jost matrix determinant $d(w)$ [1]

$$S_{11} = S_{11}^{bgr} S_{11}^{res} = \frac{d_{bgr}(-k_1, k_2, k_3)}{d_{bgr}(k_1, k_2, k_3)} \frac{d^*_{res}(-w^*)}{d_{res}(w)}\,, \tag{1}$$

where $k_j$ are the channel momenta. The resonant part is $d_{res}(w) = w^{-\frac{M}{2}} \prod(w + w_r^*)$, where the product includes all zeros $w_r$ of the chosen resonances and $M$ is a number of resonance zeros. The background part is modeled via complex energy-dependent phases $\alpha_j(s)$, $j$=1,2,3 representing mainly an influence of other channels and the neglected left-hand cut: $d_{bgr}(k_j) = \exp\left[-\mathrm{i} \sum \alpha_j(s)\right]$. The resonance zeros $w_r$ and background parameters were obtained from fitting the phase shifts and inelasticity parameters in the assumed channels to experimental data [1].

The dispersion relations with one subtraction for S ($l$=0, $I$=0) and P ($l$=1, $I$=1) waves read as

$$\mathrm{Re} f_l^I(s)^{out} = ST_l^I + \sum_{I'=0}^{2} \sum_{l'=0}^{3} \mathrm{vp} \int_{4m_\pi^2}^{s'_{max}} ds'\, K_{ll'}^{II'}(s, s')\, \mathrm{Im} f_{l'}^{I'}(s')^{in} \; + \; d_l^I(s)\,, \tag{2}$$

where $ST_l^I$, $K_{ll'}^{II'}(s, s')$ and $d_l^I(s)$ are the subtracting, kernel and driving terms, respectively [2]. $f_l^I(s)^{out}$ and $f_{l'}^{I'}(s')^{in}$ are the output and input amplitudes. The difference between $\mathrm{Re} f_l^I(s)^{out}$ and $\mathrm{Re} f_l^I(s)^{in}$ demonstrates a consistency of the amplitudes with the dispersion relations. The smaller the difference the better consistency, see the last term in eq. (4) below. The summation includes also D and F waves.

## 3 Method of improvement of the amplitudes at low energies

Behavior of the S- and P-wave amplitudes near thresholds is given with a generalized expansion [2]

$$\mathrm{Re} f_l^I(s) = \frac{\sqrt{s}}{4k} \sin 2\delta_l^{(I)} = m_\pi k^{2l} [a_l^I + b_l^I k^2 + c_l^I k^4 + d_l^I k^6 + O(k^8)]\,, \tag{3}$$

where $k = \sqrt{s/4 - m_\pi^2}$ is the pion momentum. $a_l^I$ is the scattering length and $b_l^I$ is the slope parameter fixed at values: $a_0^0 = 0.22\, m_\pi^{-1}$, $b_0^0 = 0.278\, m_\pi^{-3}$, $a_1^1 = 0.0381\, m_\pi^{-3}$, and $b_1^1 = 0.00523\, m_\pi^{-5}$ [2]. $c_l^I$ and $d_l^I$ are calculated from the continuity conditions for the phase shift and its first derivative at matching energies $s_{0l}$ fitted to data. The low-energy corrected original amplitudes are denoted *extended* amplitudes. Above the matching energies the *original* and *extended* amplitudes are equivalent.

Parameters of the extended amplitudes, which strongly influence the low-energy behavior of the amplitudes, were optimized (re-fitted) to fit the experimental data and to achieve a better consistency with the dispersion relations, minimizing

$$\chi^2 = \sum_i \left( \frac{\delta_i^{exp} - \delta_i^{th}}{\Delta\delta_i^{exp}} \right)^2 + \sum_i \left( \frac{\eta_i^{exp} - \eta_i^{th}}{\Delta\eta_i^{exp}} \right)^2 + \sum_i \left( \frac{\mathrm{Re} f_i^{out} - \mathrm{Re} f_i^{in}}{\Delta_{DR}} \right)^2\,, \tag{4}$$

where $\delta_i$ and $\eta_i$ are experimental and calculated values of the phase-shift and inelasticity parameter in the assumed channels of the $S$ and $P$ waves. The summation therefore runs also over the channels and partial waves. Re$f^{out}$ is calculated using the dispersion relations, eq. (2), and Re$f^{in}$ is the real part of the input amplitude on the right-hand side in (2). The scale parameter $\Delta_{DR}$ makes a reasonable weight of the DR contribution to $\chi^2$. Note, that the last term in (4) provides a coupling between the S and P waves which would be otherwise independent in the analysis.

The parameters varied in the fitting are zeros of the lowest poles, $f_0(600)$ and $\rho(770)$, the matching points $s_{00}$ and $s_{01}$, and the background parameters in the $\pi\pi$ channel. Experimental data used in this analysis are from Ref. [1] supplemented near the threshold with phases from the dispersive analysis [2] and data from the NA48 Collaboration.

## 4 Results

Applying the modifications we got the S- and P- wave amplitudes which describe very well the experimental data on the $\pi\pi$ scattering from the threshold up to 1.8 GeV. The threshold expansion (3) provided a reasonable agreement with data, $\chi^2/n.d.f.$ = 2.36 for the extended amplitudes, but the refitting parameters still significantly improved the result, $\chi^2/n.d.f.$ = 1.39 for the re-fitted amplitudes. The biggest improvement was for the DR contribution, the last term in eq. (4) changed from 571 to 67, which suggests a significant improvement of consistency of the amplitudes with the Roy-like dispersion relations. The re-fitted amplitudes provide also proper values of the phase shifts and inelasticity parameters in the assumed coupled channels as the original amplitudes.

Positions of poles changed strongly for the $f_0(600)$ resonance, e.g. on the II sheet the pole shifted from (616.5, i554.0) MeV for the original amplitude to (479.5, i302.1) MeV for the re-fitted one. This results in a reduction of the $\sigma$ meson mass, 829 MeV $\rightarrow$ 567 MeV and width, 1108 MeV $\rightarrow$ 604 MeV. The pole of $\rho(770)$ moved up by less than 1%. Note that the pole of $f_0(980)$ not changed in our analysis is located at (1013, i31) MeV which rather differ from the original value (1009, i31) MeV [1] as here we use slightly different values of the pion and kaon masses to be consistent with DR [2].

Most of the background parameters have now very small values which means that some part of the contribution from the left-hand cut was included in the new values of parameters, the resonance zeros. However, both background S- and P-wave phase shifts in the $\pi\pi$ channel are forced to be negative.

To summarize, agreement of the phase-shifts with low-energy data was improved for the new re-fitted S- and P-wave $\pi\pi-$scattering amplitudes. The amplitudes are calculated with the scattering lengths and slope (effective-range) parameters consistent with results of calculations based on ChPT and DR. Consistency of the three-channel amplitudes with the dispersion relations was improved significantly from the threshold up to 1.1 GeV which means that the amplitudes better fulfill the crossing symmetry condition. The lowest pole in S wave is shifted to lower energy and nearer to the real axis which results in smaller values of the mass and width for the $\sigma$ meson. The negative S-wave phase-shift in the background, $a_{11} = -0.099$, starting at the $\pi\pi$ threshold seems to be important for a good description of data.

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