

Theoretical description of nuclear masses

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Abstract. Ten different theoretical models are tested for their accuracy in description of nuclear masses. Recent experimental masses of 2011 are used for the tests. The accuracy is studied in three regions of the nuclear chart: global (all nuclei with $Z, N \geq 8$), heavy ($Z \geq 82, N \geq 126$) and the heaviest ones ($Z > 100$). It is found that the accuracy of a given model strongly depends on the region of nuclei in which it is used.

1 Introduction

There is a continuing progress in the measurement and evaluation of nuclear masses. Between the previous evaluation [1] and the recent one [2], masses of more than 140 nuclei have been measured, in particular of more than 30 heavy (heavier than ^{208}Pb) nuclides. Also the accuracy of measured masses has been improved for many nuclei.

In the parallel progress of theoretical description of the masses, a significant reduction of the discrepancy between the calculated values and the measured ones has been achieved. Particularly large reduction has been obtained in the purely microscopic, self-consistent description. For example, a big reduction has been reached in a long series of the (Skyrme)-Hartree-Fock-Bogoliubov (HFB) models: from 0.764 MeV in the first version of the model HFB1 [3], down to 0.574 MeV in the recent one HFB21 [4]. The rms of the HFB1 was obtained with the measured masses of 1888 nuclei, available at that time, while that of the HFB21 was calculated with 2294 masses known presently [2].

The objective of the present paper is to study the quality of the available models in description of the masses. In particular, it is planned to study the dependence of the accuracy of a given model on the region of nuclei in which it is used. Ten models of different kinds: semi-empirical, macroscopic-microscopic (macro-micro), and purely microscopic, are considered.

Much attention is devoted to heaviest nuclei. The reason for this is to get an orientation which of the models can be used for possibly realistic predictions for the experimental studies of superheavy nuclei (SHN) and superheavy elements (SHE). These, presently intensive studies (e.g. [5–12]), are especially difficult and expensive, due to very small cross sections for the synthesis of SHN.

Some discussion of the description of heavy-nuclei masses by macroscopic-microscopic models has been presented in [13].

2 Considered nuclear models

As already stated in the Introduction, ten various models are considered in the study. These are: one semi-empirical

(LMZ) [14], five macroscopic-microscopic, two purely microscopic (self-consistent) and two models of other kind. The macroscopic-microscopic models are: the Finite-Range Droplet Model (FRDM) [15], the Finite-Range Liquid Drop Model (FRLDM) [15], the nuclear Thomas-Fermi (TF) [16], the Warsaw model for Heavy Nuclei (HN) [17] (see also [18]), and the Lublin-Strasbourg (LSD) model [19]. The purely microscopic models are: the HFB21 approach [4], mentioned above, which uses the Skyrme interactions, and the HFB approach exploiting the Gogny forces (GHFB) [20]. Two other models are the following: the model of Duflo and Zuker (DZ) [21] and that of Koura et al. (KTUY) [22]

Two of the models (LMZ and HN) are of a local type, specially adapted to describe heavy nuclei with proton number $Z \geq 82$ and neutron number $N \geq 126$. The others are of a global character describing all nuclei with $Z, N \geq 8$.

3 Results

Table 1 shows the accuracy of the description of masses by the considered models. The results are given for three regions of nuclei: global (all nuclei with $Z, N \geq 8$), heavy ($Z \geq 82, N \geq 126$), and the heaviest ones ($Z > 100$). Experimental masses are taken from [2]. For each model and each region, the number of nuclei taken in the calculation, N_{nucl} , rms of the discrepancies, Rms, average value of the discrepancies, $\bar{\delta}$, and maximum of the absolute values of the discrepancies, $\max |\delta|$, are given.

The dependence of the rms on the model and on the region of considered nuclei is better seen when presented in a graphical form. This is done in figures 1, 2, and 3.

One can see in figure 1 that the models may be separated into three groups. DZ has the lowest rms. LSD, FRDM, TF, and HFB21 give medium and similar to each other values, while FRLDM, GHFB and KTUY have the largest rms.

When one goes to the heavy nuclei (figure 2), the picture changes quite much. The rms of LSD goes down, close to that of DZ, while the rms of GHFB and KTUY go up to about 1 MeV or even more. The accuracy of LMZ and HN is rather high, as could be expected for these local models, specially adapted for heavy nuclei. Finally, for the heaviest nuclei (figure 3), the picture is very different from the

Table 1. The Rms, average discrepancy, $\bar{\delta}$, and maximum of the absolute values of the discrepancies, $\text{Max} |\delta|$ (all given in MeV), calculated for all ($Z, N \geq 8$), heavy ($Z \geq 82, N \geq 126$), and heaviest ($Z > 100$) nuclei, with the use of the specified models. The year of publication of each model and the number of nuclei with measured mass in each of the considered regions, N_{nucl} , are also indicated.

Model	LMZ (2000)	HN (2001)	LSD (2003)	FRDM (1995)	TF (1996)	FRLDM (1995)	HFB21 (2010)	GHFB (2009)	DZ (1995)	KTUY (2005)
$N_{\text{nucl}}(Z \geq 8)$	-	-	2267	2294	2293	2294	2294	2294	2294	2294
Rms	-	-	0.600	0.645	0.629	0.768	0.574	0.784	0.374	0.690
$\bar{\delta}$	-	-	-0.028	-0.062	0.027	0.057	0.031	-0.108	-0.029	-0.048
$\text{Max} \delta $	-	-	4.34	3.64	4.61	4.17	3.20	3.23	3.01	2.63
$N_{\text{nucl}}(Z \geq 82)$	297	297	289	297	296	297	297	297	297	297
Rms	0.202	0.358	0.352	0.455	0.476	0.731	0.484	1.057	0.333	0.986
$\bar{\delta}$	0.028	-0.133	0.163	0.131	0.340	0.562	0.132	-0.118	-0.011	-0.307
$\text{Max} \delta $	1.12	1.13	1.43	1.95	1.75	1.92	1.33	3.23	3.01	2.38
$N_{\text{nucl}}(Z > 100)$	24	24	23	24	23	24	24	24	24	24
Rms	0.215	0.147	0.295	0.676	0.484	1.251	0.314	1.172	0.761	1.326
$\bar{\delta}$	-0.140	-0.063	-0.157	-0.525	0.457	1.218	-0.095	1.046	-0.151	1.303
$\text{Max} \delta $	0.69	0.39	1.09	1.95	0.82	1.92	0.92	1.70	3.01	1.93

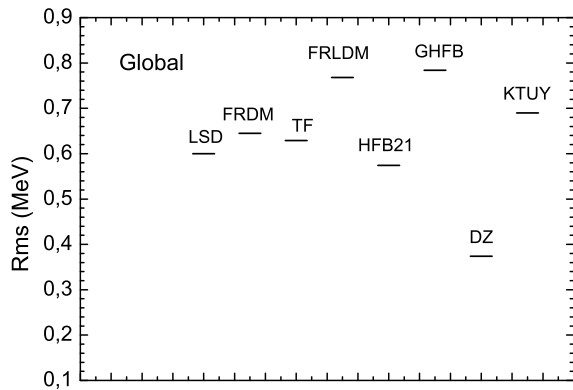


Fig. 1. Rms values of the discrepancies between calculated within 8 global models (see text for the notation of the models) for the whole (global: $Z, N \geq 8$) region of nuclei and the experimental values.

previous ones. The rms of HN is especially low, about 0.15 MeV. Generally good DZ model does not appear to be very successful for nuclei in this region. The rms of FRLDM, GHFB and KTUY are especially large, much above 1 MeV.

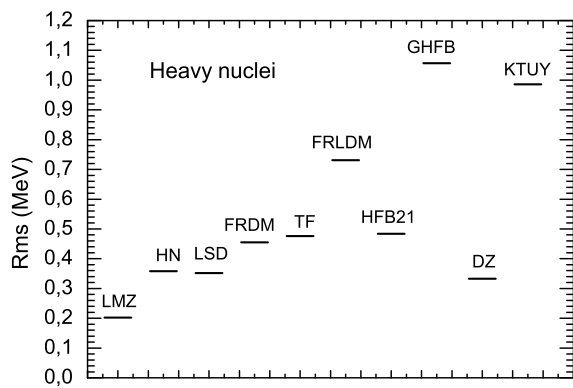


Fig. 2. Same as in figure 1, but for the heavy-nuclei region ($Z \geq 82, N \geq 126$). Results for the two local models (LMZ and HN) are also shown.

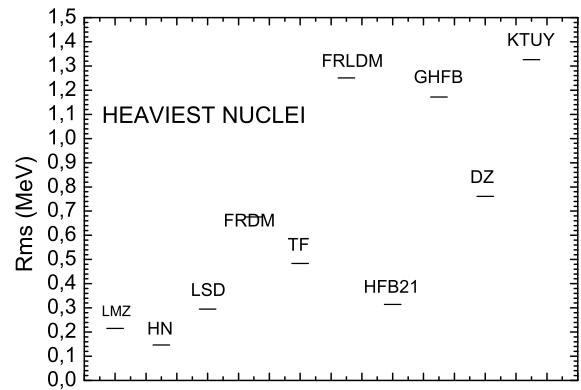


Fig. 3. Same as in figure 2, but for the heaviest region of nuclei ($Z > 100$).

It is worth noting that the self-consistent model HFB21 gives relatively low rms for all three considered regions of nuclei. For the heaviest nuclei, it appears among the models with the smallest rms.

Figure 4 shows a detailed map of the discrepancies obtained for this model in the region of heavy nuclei. The experimental masses of Ref. [2] are taken. The nuclei with

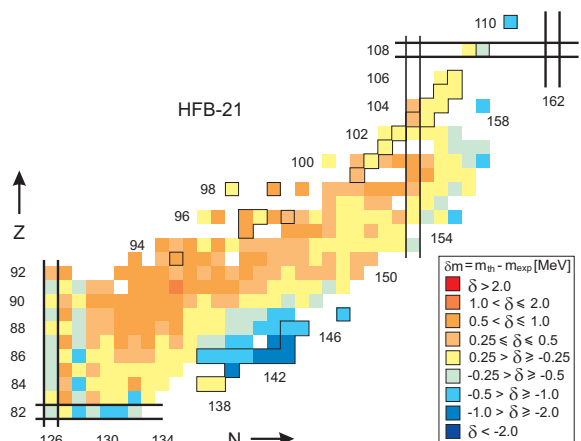


Fig. 4. (Color-online) Detailed map of the discrepancies obtained for the HFB21 model in the heavy-nuclei region.

masses measured recently (i.e., with masses given in Ref. [2], but absent in Ref. [1]) are put in the thin black frames. Positions of the magic numbers: $Z=82, 108$ and $N=126, 152, 162$ are indicated by thick black lines. One can see that the discrepancies are below 0.5 MeV (in their absolute values) for most of the nuclei. Only for relatively small number of them, it is larger (up to 1.33 MeV).

4 Summary

Ten different theoretical models are tested for their accuracy in description of nuclear masses. Recent experimental masses of 2011 are used for the tests. The accuracy is studied in three regions of the nuclear chart: global (all nuclei with $Z, N \geq 8$), heavy ($Z \geq 82, N \geq 126$) and heaviest ones ($Z > 100$).

The following conclusions may be drawn from the study:

(1) The accuracy of a given model usually strongly depends on the region of nuclei in which it is used. For example, the Duflo-Zuker model (DZ), which has very good average accuracy in both the global and heavy-nuclei regions, is rather poor for description of heaviest nuclei.

(2) The self-consistent model HFB21 gives relatively low rms, lower than that of a number of widely used macroscopic-microscopic models, for all three considered regions of nuclei. This probably happens for the first time, for a purely microscopic model, in the history of description of nuclear masses.

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