EPJ Web of Conferences **38**, 05002 (2012)
DOI: 10.1051/epjconf/20123805002

An *ab initio* theory of double odd-even mass differences in nuclei

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Abstract. Two aspects of the problem of evaluating double odd-even mass differences D_2 in semi-magic nuclei are studied related to existence of two components with different properties, a superfluid nuclear subsystem and a non-superfluid one. For the superfluid subsystem, the difference D_2 is approximately equal to 2Δ, the gap Δ being the solution of the gap equation. For the non-superfluid subsystem, D_2 is found by solving the equation for two-particle Green function for normal systems. Both equations under consideration contain the same effective pairing interaction. For the latter, the semi-microscopic model is used in which the main term calculated from the first principles is supplemented with a small phenomenological addendum containing one phenomenological parameter supposed to be universal for all medium and heavy atomic nuclei.

1 Introduction

Recently, a progress has been made in the *'ab initio'* theory of nuclear pairing, first, by the Milan group [1–3], a bit later by Duguet et al. [4,5], and finally by the Moscow-Catania group [6–10]. By *'ab initio'* we mean a theory of Brueckner type starting from a free NN-potential, not a real *ab initio* approach based on the QCD theory. The quotes indicate the fact that, in any of the cited references, the authors deal with a calculation which is not completely *ab initio* even at such lowest level. Indeed, the self-consistent mean field used in each of them is calculated by means of an Energy Density Functional (EDF) containing phenomenological parameters. Solving the BCS equation for the gap Δ with a realistic NN-potential for the pairing interaction is the main ingredient of all of them. This is exactly the prescription of the simplest version of the Brueckner theory as the ladder diagram summation is already performed in the gap equation itself.

In the first paper of the Milan series, the BCS gap equation for neutrons with the Argonne v_{14} potential was solved for the nucleus ^{120}Sn, the Saxon-Woods Shell-Model basis with the bare neutron mass $m^* = m$. Rather optimistic result was obtained for the gap value, $\Delta_{\rm BCS} = 2.2$ MeV which is bigger, but not dramatically, than the experimental one, $\Delta_{\rm exp} \simeq 1.3$ MeV, leaving some hope of achieving a good agreement by finding corrections to the scheme. In Refs. [2,3], the effective mass $m^* \neq m$ was introduced into the gap equation within the Skyrme–Hartree–Fock (SHF) method with the Sly4 force [11]. In this case the effective mass $m^*(r)$ is space dependent and essentially different from the bare one m. E.g., in nuclear matter the Sly4 effective mass is equal to $m^* = 0.7m$. In the weak coupling limit of the BCS theory, the gap is exponentially dependent, i.e. $\Delta \propto \exp(1/g)$, on the inverse dimensionless pairing strength $g = m^* \mathcal{V}_{\rm eff} k_{\rm F}/\pi^2$, where $\mathcal{V}_{\rm eff}$ is the effective pairing interaction. Therefore, a strong suppression of the gap takes place in the case of $m^* < m$. The value of $\Delta_{\rm BCS} = 0.7$ MeV was obtained in Ref. [2] and $\Delta_{\rm BCS} = 1.04$ MeV, in Ref. [3]. In both cases, the too small value of the gap was explained by invoking various many-body corrections to the BCS approximation. The main correction is due to the exchange of low-lying surface vibrations ("phonons"), contributing to the gap around 0.7 MeV [2], so that the sum turns out to be $\Delta = 1.4$ MeV very close to the experimental value. In Ref. [3], the contribution of the induced interaction caused by exchange of the high-lying in-volume excitations was added either, the sum again is equal to $\Delta \simeq 1.4$ MeV. Thus, the calculations of Refs. [2,3] showed that the effects of $m^* \neq m$ and of many-body corrections to the BCS theory are necessary to explain the difference of $(\Delta_{\rm BCS} - \Delta_{\rm exp})$. In addition, they have different sign and partially compensate each other. Unfortunately, both effects contain large uncertainties. This point was discussed in Refs. [7,9,10].

To avoid such uncertainties, a semi-microscopic model for nuclear pairing was suggested in Refs. [8–10]. It starts from the *ab initio* BCS gap equation with the Argonne force v_{18} treated with the two-step method. The complete Hilbert space S of the problem was split into the model subspace S_0 and the complementary one S'. The gap equation is solved in the model space with the effective interaction $\mathcal{V}_{\rm eff}$ which is found in the complementary subspace in terms of the initial NN-potential $\mathcal{V}$ with $m^* = m$. This *ab-initio* term of $\mathcal{V}_{\rm eff}$ was supplemented by a small addendum proportional to the phenomenological parameter γ that should hopefully embody all corrections to this simplest BCS scheme. This parameter is supposed to be the same for all heavy and medium nuclei, either for neutrons and protons.

The "experimental" gap value $\Delta_{\rm exp}$ for semi-magic nuclei is usually identified with one half of one of that obtained from the following double odd-even mass differ-

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ences:

$$D^{+}_{2n}(N,Z) = M(N+2,Z) + M(N,Z) - 2M(N+1,Z), \quad (1)$$

$$D^{-}_{2n}(N,Z) = -M(N-2,Z) - M(N,Z) + 2M(N-1,Z), \quad (2)$$

$$D^{+}_{2p}(N,Z) = M(N,Z+2) + M(N,Z) - 2M(N,Z+1), \quad (3)$$

$$D^{-}_{2p}(N,Z) = -M(N,Z-2) - M(N,Z) + 2M(N,Z-1), \quad (4)$$

where Z value is magic in Eqs. (1),(2) and N is magic in Eqs. (3),(4). More explicitly, in Refs. [9,10] the experimental gap values were defined as follows:

$$2\Delta^{\rm exp}_{n}(N,Z) = 1/2(D^{+}_{2n}(N,Z) + D^{-}_{2n}(N,Z)), \quad (5)$$

$$2\Delta^{\rm exp}_{p}(N,Z) = 1/2(D^{+}_{2p}(N,Z) + D^{-}_{2p}(N,Z)). \quad (6)$$

The accuracy of such a prescription was estimated in [9] as $\simeq 0.1 \div 0.2$ MeV. Approximately the same accuracy holds for the "developed pairing" approximation in the gap equation, with conservation of the particle number only on average [12], used in [1–10].

There is one more physical quantity in semi-magic nuclei which can be evaluated in terms of the same effective interaction as the pairing gap. This is the set of the same double odd-even mass differences (1-4), but for the non-superfluid subsystem, so that now N is magic and Z arbitrary in Eqs. (1), (2) and *vice versa* in Eqs. (3), (4). In non-superfluid nuclei, the mass differences, Eqs. (1), (2), coincide with poles in the total energy E plane of the two-particle Green function $K(1,2,3,4)$ for normal systems [13] in the nn-channel, and Eqs. (3), (4), in the pp-channel. The equation for K in the channel $S = 0, L = 0$ could be expressed in terms of the same effective interaction $\mathcal{V}_{\rm eff}$ as the pairing gap. This point was marked in the old paper [14], where these differences for double-magic nuclei were analyzed within the theory of finite Fermi systems (TFFS) [13]. In this article, the density dependent effective pairing interaction was introduced for the first time and arguments were found in favor of of the surface dominance in this interaction.

It is worth to stress that this calculation of mass difference for the non-superfluid subsystem is a more rigorous operation than the identification with the double gap Δ in the superfluid one. The first results of such calculations with the use of the semi-microscopic model for the effective pairing interaction are presented in [15].

2 The semi-microscopic model for nuclear pairing

The general many-body form of the equation for the pairing gap is [13]

$$\Delta_\tau = \mathcal{U}^\tau G_\tau G^s_\tau \Delta_\tau, \quad (7)$$

where $\tau = (n,p)$ is the isotopic index, $\mathcal{U}^\tau$ is the NN-interaction block irreducible in the two-particle τ-channel, and G_τ (G^s_τ) is the one-particle Green function without (with) pairing. A symbolic multiplication denotes the integration over energy and intermediate coordinates and summation over spin variables as well. The BCS approximation in Eq. (7) means, first, the change of the block $\mathcal{U}$ of irreducible interaction diagrams with the free NN-potential $\mathcal{V}$ and, second, the use of simple quasi-particle Green functions G and G^s, i.e. those without phonon corrections and so on. In this case, Eq. (7) iss greatly simplified and can be reduced to the form typical of the Bogolyubov method,

$$\Delta_\tau = -\mathcal{V}^\tau \varkappa_\tau, \quad (8)$$

where

$$\varkappa_\tau = \int \frac{d\varepsilon}{2\pi i} G_\tau G^s_\tau \Delta_\tau \quad (9)$$

is the anomalous density matrix which can be expressed explicitly in terms of the Bogolyubov functions u and v,

$$\varkappa_\tau(\mathbf{r}_1,\mathbf{r}_2) = \sum_i u^\tau_i(\mathbf{r}_1) v^\tau_i(\mathbf{r}_2). \quad (10)$$

Summation in Eq. (10) scans the complete set of Bogolyubov functions with eigen-energies $E_i > 0$.

Then we split the complete Hilbert space of the pairing problem, $S = S_0 + S'$, where the model subspace S_0 includes the single-particle states with energies less than a separation energy E_0. The gap equation is solved in the model space

$$\Delta_\tau = \mathcal{V}^{\rm BCS}_{\tau,\rm eff} G_\tau G^s_\tau \Delta_\tau|_{S_0}, \quad (11)$$

with the effective pairing interaction $\mathcal{V}^{\rm BCS}_{\tau,\rm eff}$ instead of the block $\mathcal{V}^\tau$ in the BCS version of the original gap equation (7). It obeys the Bethe–Goldstone type equation in the subsidiary space

$$\mathcal{V}^{\rm BCS}_{\tau,\rm eff} = \mathcal{V}^\tau + \mathcal{V}^\tau G_\tau G_\tau \mathcal{V}^{\rm BCS}_{\tau,\rm eff}. \quad (12)$$

In this equation, the pairing effects can be neglected provided the model space is sufficiently large, $E_0 \gg \Delta$. That is why we replaced the Green function G^s_τ for the superfluid system with its counterpart G_τ for the normal system. To solve Eq. (12) in non-homogeneous systems a new form of the local approximation, the Local Potential Approximation (LPA), was developed by the Moscow–Catania group. Originally, it was developed for semi-infinite nuclear matter, then applied to the slab of nuclear matter (see review article [16]) and, finally, to finite nuclei [6,7]. It turned out that, with a very high accuracy, at each value of the average c.m. coordinate $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4)/4$, one can use in Eq. (12) the formulae of the infinite system embedded into the constant potential well $U = U(\mathbf{R})$. This significantly simplifies the equation for $\mathcal{V}_{\rm eff}$, in comparison with the original equation for Δ. As a result, the subspace S' can be chosen as large as necessary to achieve the convergence. The accuracy of LPA depends on the separation energy E_0. For finite nuclei, the value of E_0=40 MeV guarantees an accuracy higher than 0.01 MeV for the gap Δ.

To avoid uncertainties that affect the corrections to the BCS scheme discussed above, a semi-microscopic model was suggested in Refs. [8–10]. In this model, a small phenomenological addendum to the effective pairing interaction is introduced which embodies approximately all these corrections. The simplest ansatz for it is

$$\mathcal{V}^\tau_{\rm eff}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = \mathcal{V}^{\rm BCS}_{\tau,\rm eff}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) + \gamma^\tau C_0 \frac{\rho(r_1)}{\bar\rho(0)} \prod_{i=2}^{4} \delta(\mathbf{r}_1 - \mathbf{r}_i). \quad (13)$$

Here $\rho(r)$ is the density of nucleons of the kind under consideration, and γ^τ are dimensionless phenomenological

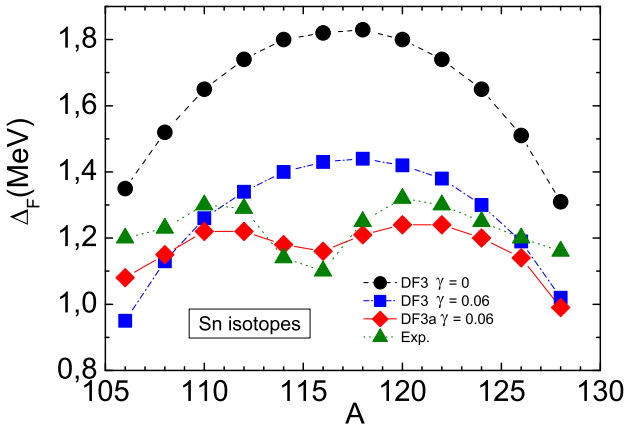

Fig. 1. Neutron gap in Sn isotopes

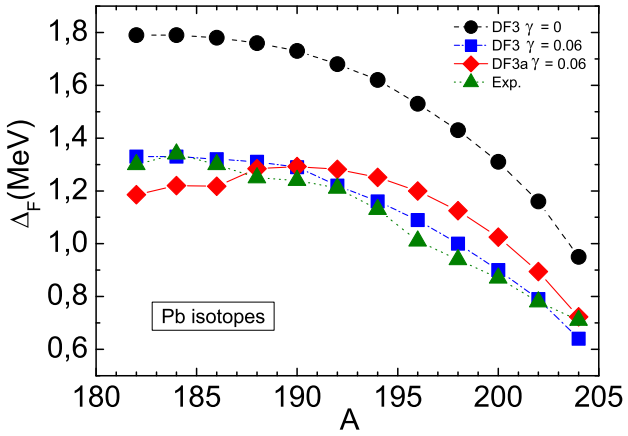

Fig. 2. Neutron gap in Pb isotopes

parameters. To avoid any influence of the shell fluctuations in the value of $\rho(0)$, the average central density $\bar{\rho}(0)$ is used in the denominator of the additional term. It is averaged over the interval of r<2 fm. The first, *ab initio*, term in the r.h.s. of Eq. (13) is the solution of Eq. (12) in the framework of the LPA method described above, with m^*=m in the subspace S'.

In Ref. [9], the above equations were solved in the self-consistent λ-basis of the EDF by Fayans et al. [17,18]. Two sets of the functional were used, the original one DF3 [18] and its modification DF3-a [19]. In the latter, the spin-orbit and effective tensor terms of the original functional were modified. The results for the pairing gap in three chains of semi-magic nuclei are displayed in figures 1–3.

In accordance with the recipe of Ref. [3], we represent the theoretical gap with the "Fermi average" combination

$$\Delta_{\rm F} = \sum_{\lambda} (2j{+}1)\Delta_{\lambda\lambda} / \sum_{\lambda} (2j{+}1), \qquad (14)$$

where the summation is carried out over the states λ in the interval of $|\varepsilon_\lambda - \mu|$<3 MeV. The "experimental" gap is determined by the symmetric 5-term odd-even mass differences (5), (6).

Let us begin from neutron pairing and consider first the tin isotopes, figure 1. We see that the BCS gap (γ=0) is approximately 30% greater than the experimental one. Switching on the phenomenological addendum with γ=0.06 makes the theoretical gap values closer to experiment. However, the predictions of the two versions of the functional

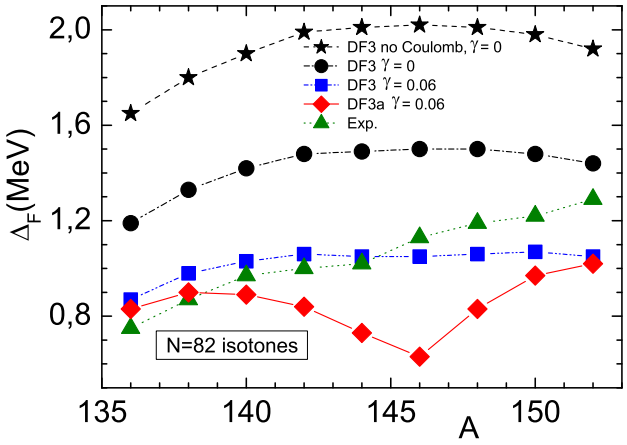

Fig. 3. Proton gap for N = 82 isotones

used are significantly different, being much better for the DF3-a functional. In particular, the A-dependence of the experimental gap is reproduced with a pronounced minimum in the center of the chain. As the analysis in Ref.[9] has shown, this strong difference between results for two functionals is due to the strong influence on the gap of the high j intruder state $1h_{11/2}$. Its position depends essentially on the spin-orbit parameters and is noticeably different for DF3 and DF3-a functionals.

In the lead chain, see figure 2, the overall pattern is quite similar. Again the BCS gap is approximately 30% bigger of the experimental one and again the inclusion of the phenomenological term with γ=0.06 gives a qualitative agreement. Now, the difference between the two functionals is much smaller. In this case, the agreement is rather perfect for the DF3 functional and a little worse for the DF3-a one, but also within limits for the accuracy discussed above.

Let us go to proton pairing, N=82 chain, see figure 3. In this case, the Coulomb interaction should be included into the pairing effective interaction,

$$\mathcal{V}^p_{\rm eff} = \mathcal{V}^n_{\rm eff} + \mathcal{V}_{\rm C}. \qquad (15)$$

As it is argued in Ref.[9], the bare Coulomb potential could be used in this equation with high accuracy. The strong Coulomb effect in the gap is demonstrated in figure 3. It is also explained with the exponential dependence of the gap on the pairing interaction. It should be mentioned that Duguet and co-authors [5] were the first who inserted explicitly the Coulomb interaction into the pairing force for protons. Only after inclusion of the Coulomb interaction into $\mathcal{V}_{\rm eff}$, we can use the same value of γ = 0.06 for protons and neutrons. As for tin isotopes, the difference between DF3 and DF3-a results is rather strong, now in favor of the DF3 functional. This effect is again due to different positions of the $1h_{11/2}$ level, but now for protons. Overall agreement with experiment is for protons worse, maybe, because of closeness of some nuclei to the region of the phase transition to deformed state.

As it is argued in Introduction, we may expect to reach accuracy for the gap of the order of $\simeq$ (0.1 ÷ 0.2) MeV. In practice, the overall disagreement of the theory with the data is $\sqrt{\overline{(\Delta_{\rm th} - \Delta_{\rm exp})^2}} \simeq 0.13$ MeV for the DF3 functional and $\simeq$0.14 MeV, for the DF3-a one. For the mass differ-

ences D_2 these numbers should be multiplied by a factor of two.

3 Double mass differences in non-superfluid subsystems

The Lehmann expansion for the two-particle Green function K in a non-superfluid system reads, in the single-particle wave functions $|1\rangle=|n_1, l_1, j_1, m_1\rangle$ representation, [13]:

$$K_{12}^{34}(E) = \sum_s \frac{\mathcal{X}_{12}^s \mathcal{X}_{34}^{s*}}{E - E_s^{+,-} \pm i\gamma}, \qquad (16)$$

where E is the total energy in the two-particle channel and $E_s^{+,-}$ denote the eigen-energies of nuclei with two particles and two holes, respectively, added to the original nucleus. They are often interpreted as the "pair vibrations" [20]. Instead of the Green function K, it is convenient to use the two-particle interaction amplitude Γ:

$$K = K_0 + K_0 \Gamma K_0, \qquad (17)$$

where $K_0 = GG$. The amplitude Γ obeys the following equation [13]:

$$\Gamma = \mathcal{U} + \mathcal{U}GG\Gamma, \qquad (18)$$

where $\mathcal{U}$ is the same irreducible interaction block as in Eq. (7). Again, within the Brueckner theory, the block $\mathcal{U}$ should be replaced with the realistic potential $\mathcal{V}$ which does not depend on the energy. Then the integration over the relative energy can be readily carried out in Eq. (18)

$$A_{12} = \int \frac{d\varepsilon}{2\pi i} G_1\left(\frac{E}{2}+\varepsilon\right) G_2\left(\frac{E}{2}-\varepsilon\right) = \frac{1-n_1-n_2}{E-\varepsilon_1-\varepsilon_2}, \qquad (19)$$

where $\varepsilon_{1,2}$ are the single-particle energies and $n_{1,2}$=(0; 1), the corresponding occupation numbers. As the result, we obtain

$$\Gamma = \mathcal{V} + \mathcal{V}A\Gamma. \qquad (20)$$

Simple transformations in Eq. (17) or (20), in vicinity of the pole E=$E_s^{+,-}$, lead to the following equation for the eigenfunctions $\mathcal{X}^s$:

$$(E_s - \varepsilon_1 - \varepsilon_2)\mathcal{X}_{12}^s = (1 - n_1 - n_2) \sum_{34} \mathcal{V}_{12}^{34} \mathcal{X}_{34}^s. \qquad (21)$$

It is different from the Shrödinger equation for two interacting particles in an external field only for the factor $(1 - n_1 - n_2)$ which reflects the many-body character of the problem, in particular, the Pauli principle. The direct solution of this equation is complicated by the same reasons as of the *ab initio* BCS gap equation described in Sec. 2. The same two-step method is used. The usual renormalization of Eq. (21) transforms it into the analogous equation in the model space

$$(E_s - \varepsilon_1 - \varepsilon_2)\mathcal{X}_{12}^s = (1 - n_1 - n_2) \sum_{34}^{0} (\mathcal{V}_{\rm eff})_{12}^{34} \mathcal{X}_{34}^s, \qquad (22)$$

where the effective interaction $\mathcal{V}_{\rm eff}$ coincides with that of pairing problem, Eq. (12), provided the same value of the separation energy E_0 is used. The next step consists in the use of ansatz (13) to take into account corrections to the

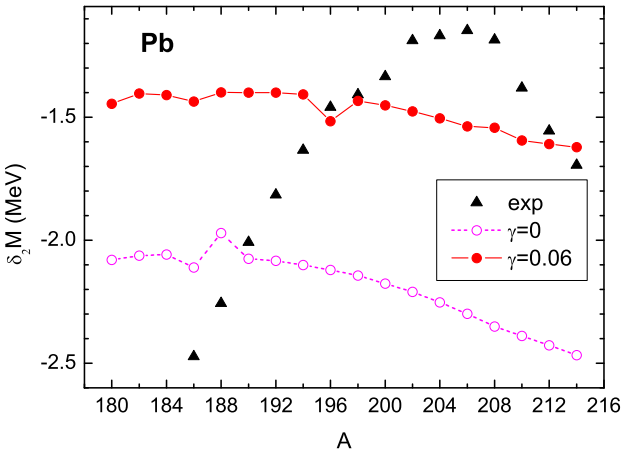

Fig. 4. Double mass differences D_{2p}^{+} for adding to Pb isotopes of two protons

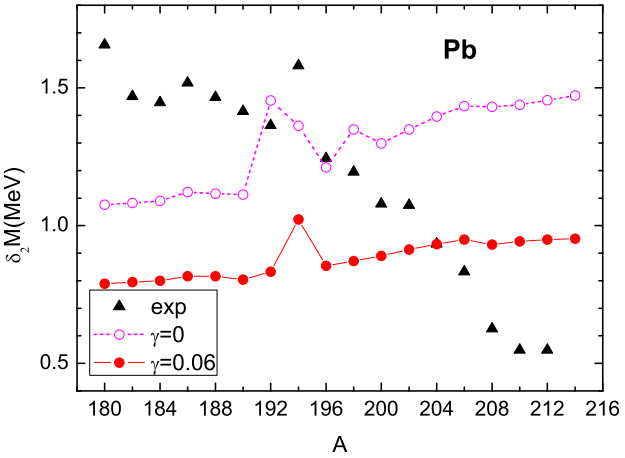

Fig. 5. Double mass differences D_{2p}^{-} for adding to Pb isotopes of two proton holes

Brueckner theory with a phenomenological addendum (~ γ).

The double mass differences (1– 4) are identified with the two first solutions $E_s^{+,-}$ of Eq. (22), corresponding to the addition of two particles (holes) to the magic core into the state $\varepsilon_1 = \varepsilon_2 = \mu^{+,-}$, where the chemical potentials $\mu^{+,-}$ are defined in a usual way as mass differences, e.g., $\mu_p^+ = E_{\rm B}(N, Z+1) - E_{\rm B}(N, Z)$. Then, the energy difference in the left-hand side of Eq. (22) is $E_s^{+,-} - 2\mu^{+,-} = D^{+,-}$. If we are interested only in these solutions, we may rewrite Eq. (22) as follows:

$$E_s - 2\mu = (1-2n_1)\,(\Gamma'(E_s))_{11}^{11}, \qquad (23)$$

where

$$(\Gamma'(E_s))_{12}^{34} = (\mathcal{V}_{\rm eff})_{12}^{34} + \sum_{56}{}' (\mathcal{V}_{\rm eff})_{12}^{56} \times \frac{1-n_5-n_6}{E_s-\varepsilon_5-\varepsilon_6} (\Gamma'(E_s))_{56}^{34}. \qquad (24)$$

The accent on the sum denotes that the two-particle state 5=6=1 is excluded. In all formulas (22 – 24) the angular momenta of two-particle states $|12\rangle$, $|34\rangle$ are coupled to the total angular momentum I=0.

The set of Eqs. (23), (24) was solved with the functional DF3 only. For lead isotopes, the proton differences

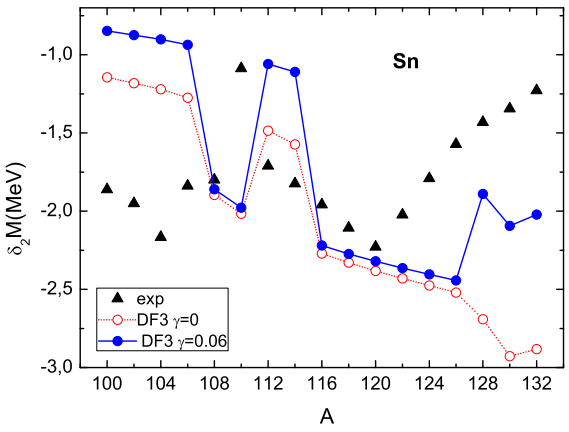

Fig. 6. Double mass differences D^{+}_{2p} for adding to Sn isotopes of two protons

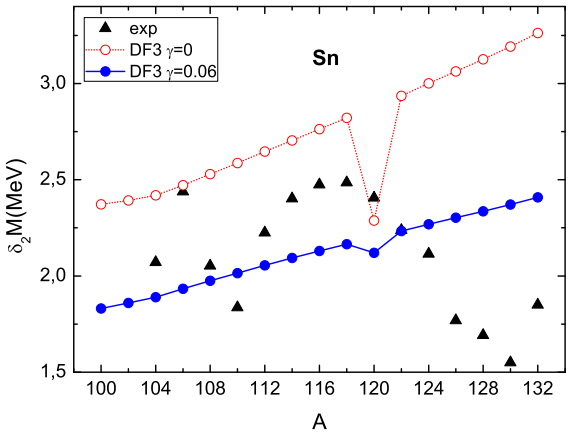

Fig. 7. Double mass differences D^{-}_{2p} for adding to Sn isotopes of two proton holes

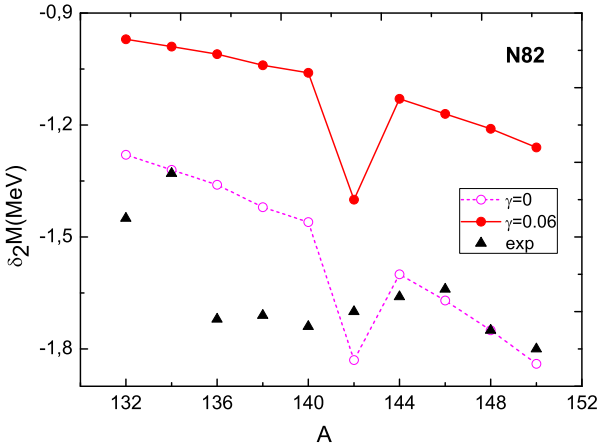

Fig. 8. Double mass differences D^{+}_{2n} for adding to $N = 82$ isotones of two neutrons

D^{+}_{2p} are displayed in figure 4 and the differences D^{-}_{2p}, in figure 5 and for the tin isotopes, analogously, in figure 6 and 7. In similar way, the neutron double differences for isotones N=82, are displayed in figures 8 and 9. Note that the opposite signs of the quantities D^{+}_{2} and D^{-}_{2} occur owing to the factor of $(1 - 2n_1)$ in Eq. (23): particles attract each other whereas holes, repel. The results of the *ab initio* calculation (γ=0) and with the phenomenological addendum (γ=0.06) are shown. To quantify the accuracy of calculations, just as for the pairing gap in Sec. 2, we calculated

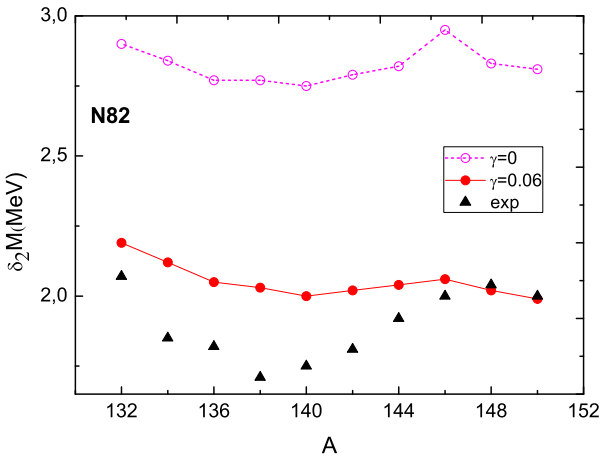

Fig. 9. Double mass differences D^{-}_{2n} for adding to $N = 82$ isotones of two neutron holes

the rms deviations of the theory from experiment

$$\overline{\delta M_{2p,2n}} = \sqrt{\frac{1}{N_{p,n}} \sum_i \left(D^{(i)}_{\text{theor}} - D^{(i)}_{\text{exp}}\right)^2}, \qquad (25)$$

with obvious notation. For the lead isotopes, we find $\overline{\delta M_{2p}}(\gamma{=}0)$=0.64 MeV and $\overline{\delta M_{2p}}(\gamma{=}0.06)$=0.42 MeV, $N_p = 26$. For the tin isotopes, we get $\overline{\delta M_{2p}}(\gamma{=}0)$=0.68 MeV and $\overline{\delta M_{2p}}(\gamma{=}0.06)$=0.45 MeV, N_p=21. For isotones with N=82 we obtain $\overline{\delta M_{2n}}(\gamma{=}0)$=0.66 MeV and $\overline{\delta M_{2n}}(\gamma{=}0.06)$ =0.42 MeV, N_n=19. We included into analysis only nuclei with the experimental error of the double difference less than 0.1 MeV. The total disagreement for neutrons and protons is $\overline{\delta M_{\text{tot}}}(\gamma{=}0)$=0.66 MeV and $\overline{\delta M_{\text{tot}}}(\gamma{=}0.06)$=0.43 MeV. Thus, introducing the phenomenological addendum makes the agreement with data better but not so much as for the gap.

One can see that the agreement is worse for lighter isotopes and isotones which are close to the drip line, the chemical potential μ in Eq. (23) being small. The reason is that in vicinity of drip line the concept of energy independent mean field becomes erroneous [21,22]. If we limit the analysis to nuclei with μ<−4 MeV, we obtain $\overline{\delta M_{2p}}(\gamma{=}0)$=0.53 MeV and $\overline{\delta M_{2p}}(\gamma{=}0.06)$=0.27 MeV, N_p= 10 for lead isotopes and $\overline{\delta M_{2p}}(\gamma{=}0)$=0.64 MeV and $\overline{\delta M_{2p}}(\gamma{=}0.06)$ =0.40 MeV, N_p=16 for the tin chain. For isotones N=82 we have $\overline{\delta M_{2n}}(\gamma{=}0)$ = 0.71 MeV and $\overline{\delta M_{2n}}(\gamma{=}0.06)$ = 0.39 MeV, N_n=16. It is worth mentioning that for the D^{+}_{2n} quantity, figure 8, inclusion of the phenomenological addendum makes agreement significantly worse. The total disagreement for neutrons and protons is now $\overline{\delta M_{\text{tot}}}(\gamma{=}0)$=0.64 MeV and $\overline{\delta M_{\text{tot}}}(\gamma{=}0.06)$=0.36 MeV. Thus, after the inclusion of the phenomenological correction, we obtain the agreement only a little worse than for the superfluid subsystem.

We observe that our calculations, which reproduce the value of D_2 on average, do not catch the trend of the A-dependence of this quantity. We can suppose that it could be due to the fact that phonon corrections, included into the parameter γ, are not considered explicitly. Estimations show that for this problem they are not so universal as in the gap problem as far as the diagonal element $(\mathcal{V}_{\text{eff}})^{11}_{11}$ in Eq. (24) for $(\Gamma'(E_s))^{11}_{11}$ often dominates. In such a situation,

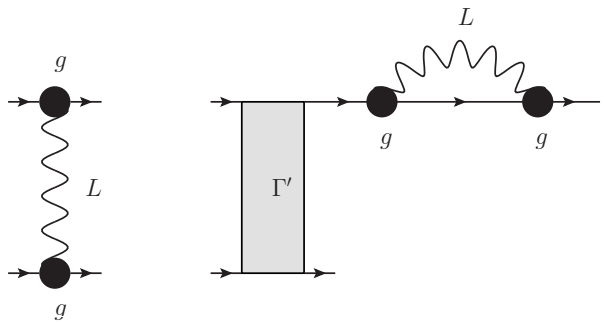

Fig. 10. Phonon corrections to the interaction amplitude Γ'

the phonon corrections could strongly depend on the state $|1\rangle$.

The main corrections to Eq. (23) caused by the L-phonon are shown in diagrams of figure 10. The first one is the so-called induced interaction, the second one is the "ends correction". In magic and semi-magic nuclei, the vertex g as a rule can be considered as a small parameter, and the so-called g^2-approximation can be used. The g^2-corrections are displayed in figure 10. The wavy line corresponds to the phonon $\mathcal{D}$-function, $\mathcal{D}(\omega)=2\omega_L/(\omega^2-\omega_L^2)$, where ω_L is the excitation energy of the L-phonon, and the transferred energy $\omega=0$ for the diagonal matrix element. The vertex $g(r)$ of creating L-phonon and the frequency ω_L were found [23] by within the self-consistent TFFS [24]. Note that all low-lying phonons possess the normal parity $\pi=(-1)^L$, therefore we do not show it explicitly. The first diagram can be easily evaluated

$$\delta^{(1)}_{\rm ph}D_2 = -\frac{2(\langle j_1 l_1\|Y_L\|j_1 l_1\rangle g_{11})^2}{\omega_L(2j_1+1)}, \qquad (26)$$

where $\langle\ \|Y_L\|\ \rangle$ denotes the reduced matrix element [25], and g_{11} is the radial matrix element of $g(r)$.

As to the second diagram, one has the same corrections for each of the ends. They can be readily summed [13]. As the result, each of the ends should be multiplied by $\sqrt{Z_1}$, where Z_1 is the residue of the Green function $G(\varepsilon)$ at the point $\varepsilon=\varepsilon_1$. The corresponding correction to the double mass difference is equal to

$$\delta^{(2)}_{\rm ph}D_2 = (Z_1^2-1)D_2. \qquad (27)$$

This expression contains terms of higher order in g^2 but such partial summation has clear physical meaning. In addition, sometimes the difference Z_1-1 is not small. The matter is that the ratio $\bar{g}=g_{12}/\delta\varepsilon_{12}$, where $\delta\varepsilon_{12}=\varepsilon_1-\varepsilon_2\pm\omega_L$, is the actual dimensionless "small" parameter. For collective L-phonons the matrix elements $g_{12}\simeq 1$ MeV, whereas the typical value of the denominator is $\delta\varepsilon_{12}\simeq\varepsilon_{\rm F}/A^{1/3}$, $\varepsilon_{\rm F}$ being the Fermi energy and A the mass number of the nucleus under consideration. Thus the condition $\bar{g}\ll 1$ is valid. However, sometimes the "resonance" cases occur when the quantity $\delta\varepsilon_{12}$ is anomalously small. Then the g^2-approximation does not work and higher order terms should be summed up. In such cases, it is natural to multiply by $\sqrt{Z_1}$ also the ends of the first diagram in figure 10. Then the expression for the double mass difference with phonon correction is as follows:

$$\tilde{D}_2 = \left(D_2+\delta^{(1)}_{\rm ph}D_2\right)Z_1^2. \qquad (28)$$

Let us calculate from this formula the corrections to the proton double mass difference D_{2p} first for the double-magic ^{208}Pb nucleus. In this nucleus the collective 3^--phonon with the energy ω_3=2.684 MeV plays the main role. Adding two proton particles, we have $D^+_{2p}(\gamma=0)=-2.35$ MeV, $|1\rangle=1h_{9/2}$, Z_1=0.98, $\delta^{(1)}_{\rm ph}D_{2p}$=−0.14 MeV and $\tilde{D}_{2p}=-2.38$ MeV. Thus, in this case the phonon correction, $\delta_{\rm ph}D_{2p}=\tilde{D}_{2p}-D_{2p}$=−0.03 MeV is negligible. Adding two proton holes, we have $D^-_{2p}(\gamma=0)=+1.43$ MeV, $|1\rangle=3s_{1/2}$, obviously, we have $\delta^{(1)}_{\rm ph}D_{2p}=0$ due to the angular momentum conservation. Then, we have Z_1=0.96 and $\tilde{D}_{2p}$=1.32 MeV, thus the phonon effect $\delta_{\rm ph}D_{2p}$=−0.11 MeV is more than for the $1h_{9/2}$ state but also less than 10%. Another situation takes place for the semi-magic nucleus ^{204}Pb. Here the phonon corrections appear mainly due to the low-lying 2^+-phonon, ω_2=0.882 MeV. In this case, for the state $1h_{9/2}$ we have Z_1=0.74, $\delta^{(1)}_{\rm ph}D_{2p}$=−0.55 MeV, $\tilde{D}_{2p}$=−1.53 MeV and $\delta_{\rm ph}D^+_{2p}$=0.72 MeV. For the state $3s_{1/2}$ again we have $\delta^{(1)}_{\rm ph}D_{2p}$=0, but Z_1=0.82, $\tilde{D}_{2p}$=0.93 MeV and $\delta_{\rm ph}D^-_{2p}$=−0.47 MeV. Thus, for both states the phonon correction is approximately of the same magnitude as the effect of the phenomenological addendum in Eq. (13) at γ=0.06. Hence now we must take smaller value of γ. In particular, for the state $3s_{1/2}$ the value of γ=0 looks preferable. Evidently, more consistent approach is necessary with systematic account for the phonon corrections with a new readjustment of the parameter γ.

4 Conclusions

Two aspects of the problem of evaluating double odd-even mass differences D_2 in semi-magic nuclei are analyzed. A semi-magic nucleus contains two components with different properties, a superfluid subsystem and a non-superfluid one. For the superfluid subsystem, D_2 is supposed to be equal to 2Δ, the gap Δ being solution of the gap equation. Such equalization contains an inherent inaccuracy for Δ of the order of $\simeq 0.1\div 0.2$ MeV [9] and an additional one due to particle number non-conservation effects $\simeq 0.1$ MeV [12]. For D_2 value they should be multiplied by a factor of 2. In Ref. [9] the gap equation was solved with the effective pairing interaction of the semi-microscopic model in which the main term found from first principles is supplemented with a small phenomenological addendum containing one phenomenological parameter γ supposed to be the same for all medium and heavy atomic nuclei. The data for 34 nuclei were analyzed for isotopes of the lead and tin chains and isotones $N=82$. Neighbors of the double magic nuclei were excluded, as for them particle number non-conservation effects are especially large [12]. Overall disagreement of the theory with the data is $\sqrt{\overline{(\Delta_{\rm th}-\Delta_{\rm exp})^2}}\simeq$ 0.13 MeV for the DF3 functional [18] and ≃0.14 MeV, for the DF3-a one [19]. For the mass differences D_2 it corresponds to accuracy a little better then 0.3 MeV.

For the non-superfluid subsystem, D_2 is found solving the equation for two-particle Green function of normal systems. This equation contains exactly the same effective pairing interaction as the gap equation. Results of calculations for the lead and tin isotopes and $N=82$ isotones with the DF3 functional and with the same value γ=0.06

of the phenomenological parameters are presented here in figures 4–9. It is worth to note that the problem for non-superfluid nuclei, in principle, does not contain so serious inaccuracies as the pairing problem. The main approximation which is made is that the two-particle Green function K which concerns the array of $A, A \pm 1, A \pm 2$ nuclei is described in the basis of the A-nucleus. Estimation of the accuracy of this approximation is $\sim 2/A$. However, the agreement turned out to be a little worse than for superfluid case, $\overline{\delta M_{\rm tot}}$=0.36 MeV. The reason is, evidently, in phonon corrections which are in this case not so regular as in the gap equation. Estimation of these corrections shows that, indeed, a more consistent approach is necessary with systematic account for the phonon corrections, the parameter γ being readjusted anew.

Acknowledgments

We thank S. V. Tolokonnikov for valuable discussions. The work was partly supported by the DFG and RFBR Grants Nos.436RUS113/994/0-1 and 09-02-91352NNIO-a, by the Grants NSh-7235.2010.2 and 2.1.1/4540 of the Russian Ministry for Science and Education, and by the RFBR Grants 11-02-00467-a and 12-02-00955-a. Two of us (E. S. and S. P.) thank the INFN, Seczione di Catania, for hospitality.

References

1. F. Barranco, R. A. Broglia, H. Esbensen, and E. Vigezzi, Phys. Lett. B **390**, 13 (1997)
2. F. Barranco, R. A. Broglia, G. Colo, *et al*., Eur. Phys. J. A **21**, 57 (2004)
3. A. Pastore, F. Barranco, R. A. Broglia, and E. Vigezzi, Phys. Rev. C **78**, 024315 (2008)
4. T. Duguet and T. Lesinski, Eur. Phys. J., Special Topics **156**, 207 (2008)
5. K. Hebeler, T. Duguet, T. Lesinski, and A. Schwenk, Phys. Rev. C **80**, 044321 (2009)
6. S. S. Pankratov, M. Baldo, M. V. Zverev, U. Lombardo, E. E. Saperstein, S. V. Tolokonnikov, JETP Lett. **90**, 612 (2009)
7. M. Baldo, U. Lombardo, S. S. Pankratov, E. E. Saperstein, J. Phys. G: Nucl. Part. Phys. **37**, 064016 (2010)
8. S. S. Pankratov, M. Baldo, M. V. Zverev, U. Lombardo, E. E. Saperstein, JETP Lett. **92**, 92 (2010)
9. S. S. Pankratov, M. V. Zverev, M. Baldo, U. Lombardo, E. E. Saperstein, Phys. Rev. C **84**, 014321 (2011)
10. E. E. Saperstein, M. Baldo, U. Lombardo, S. S. Pankratov, M. V. Zverev, Phys. At. Nucl. **74**, 1644 (2011)
11. E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A **627**, 710 (1997)
12. Abhishek Mukherjee, Y. Alhassid, and G. F. Bertsch, Phys. Rev. C **83**, 014319 (2011)
13. A. B. Migdal *Theory of finite Fermi systems and applications to atomic nuclei* (Wiley, New York, 1967)
14. E. E. Saperstein, M. A. Troitsky, Yad. Fiz. **1**, 400 (1965)
15. N. V. Gnezdilov, E. E. Saperstein, JETP Lett., to be published.
16. M. Baldo, U. Lombardo, E. E. Saperstein, and M. V. Zverev, Phys. Rep. **391**, 261 (2004)
17. A. V. Smirnov, S. V. Tolokonnikov, S. A. Fayans, Sov. J. Nucl. Phys. **48**, 995 (1988)
18. S. A. Fayans, S. V. Tolokonnikov, E. L. Trykov, and D. Zawischa, Nucl. Phys. A **676**, 49 (2000)
19. S. V. Tolokonnikov and E. E. Saperstein, Phys. At. Nucl. **73**, 1684 (2010)
20. A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, Amsterdam, 1974.), Vol. 2.
21. M. Baldo, U. Lombardo, E. E. Saperstein, and M. V. Zverev, Phys. Lett. B **533**, 17 (2002)
22. E. E. Saperstein, S. V. Tolokonnikov, JETP Lett. **78**, 343 (2003)
23. S. V. Tolokonnikov, S. P. Kamerdzhiev, D. Voytenkov, S. Krewald, and E.E. Saperstein, arXiv: 1107.2432v2[nucl-th]; Phys. Rev. C **84**, 064324 (2011)
24. V. A. Khodel and E. E. Saperstein, Phys. Rep. **92**, 183 (1982)
25. A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, Amsterdam, 1969), Vol. 1.