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Non-yrast quadrupole-octupole spectra

N. Minkov[1,2,a], S. Drenska[1], K. Drumev[1], M. Strecker[2], H. Lenske[2], and W. Scheid[2]

[1] Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Tzarigrad Road 72, BG–1784 Sofia, Bulgaria

[2] Institut für Theoretische Physik, Justus-Liebig-Universität, Heinrich-Buff-Ring 16, D–35392 Giessen, Germany

Abstract. A model of strongly coupled quadrupole and octupole vibrations and rotations is applied to describe non-yrast alternating-parity sequences in even-even nuclei and split parity-doublet spectra in odd-mass nuclei. In even-even nuclei the yrast alternating-parity sequence includes the ground-state band and the lowest negative-parity levels with odd angular momenta, while the non-yrast sequences include excited β-bands and higher negative-parity levels. In odd-mass nuclei the yrast levels are described as low-energy rotation-vibration modes coupled to the ground single-particle (s.p.) state, while the non-yrast parity-doublets are obtained as higher-energy rotation-vibration modes coupled to excited s.p. configurations. We show that the extended model scheme describes the yrast and non-yrast quadrupole-octupole spectra in both even-even and odd-A nuclei. The involvement of the reflection-asymmetric deformed shell model to explain the single-particle motion and the Coriolis interaction in odd nuclei is discussed.

1 Introduction

The appearance of alternating-parity bands in even-even atomic nuclei, and quasi parity-doublet sequences in odd mass nuclei is known to be the result of the presence of quadrupole-octupole deformations [1]. Usually these spectra are attended by enhanced electric $E1$ and $E3$ transitions between levels with opposite parity. While the low-lying (yrast) structure of the octupole spectra was relatively well studied and described within various collective and microscopic model approaches (see [1] and references therein), the interpretation and the model classification of the higher, non-yrast, parts of these spectra is still quite limited. Recently the model of Coherent Quadrupole-Octupole Motion (CQOM) [2,3] has been applied to describe non-yrast alternating-parity bands and the attendant B(E1), B(E2) and B(E3) transition probabilities in even-even nuclei [4]. Thus, in addition to the yrast alternating-parity band composed by the members of the ground-state band and the lowest negative-parity levels the model scheme adopted couples of excited β-bands and higher negative-parity sequences with odd angular momenta.

The purpose of the present work is to show that the model scheme is also capable to describe non-yrast parity-doublet spectra in odd-mass nuclei by extending the original consideration proposed in [3]. It will be shown that the common model mechanism of collective quadrupole-octupole excitations can generate the non-yrast spectra in both even-even and odd-mass nuclei. In this work the coupling of the core to the odd nucleon and the Coriolis interaction in odd-mass nuclei are taken into account phenomenologically, while the model approach remains open for the involvement of a microscopic treatment of the single-particle degrees of freedom.

In Sec. 2 the CQOM model with its extended scheme for the description of non-yrast alternating-parity bands and split-parity doublets is shown. In Sec. 3 numerical results and discussion on the application of the model to the nuclei ^{152}Sm, ^{223}Ra, ^{229}Th and ^{237}U are presented. In Sec. 4 concluding remarks are given.

2 Model of coherent quadrupole–octupole motion

2.1 Model Hamiltonian and analytic solution

The general Hamiltonian of the model is [2,3]

$$H_{qo} = -\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2^2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3^2} + U(\beta_2,\beta_3,I)\,, \qquad (1)$$

where β_2 and β_3 are axial quadrupole and octupole variables, respectively, and

$$U(\beta_2,\beta_3,I) = \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{X(I)}{d_2\beta_2^2 + d_3\beta_3^2}\,. \qquad (2)$$

Here B_2 (B_3), C_2 (C_3) and d_2 (d_3) are quadrupole (octupole) mass, stiffness and inertia parameters, respectively. The quantity X depends on the angular momentum I and has the form

$$X(I) = [d_0 + I(I+1)]/2\,, \qquad (3)$$

for even-even nuclei and

$$X(I^\pi, K) = \frac{1}{2}\Big[d_0 + I(I+1) - K^2 + \pi a \delta_{K,\frac{1}{2}}(-1)^{I+1/2}\left(I+\frac{1}{2}\right)\Big]\,, \qquad (4)$$

[a] e-mail: nminkov@inrne.bas.bg

for odd-even nuclei, where d_0 determines the potential core at $I = 0$ and a is a decoupling factor corresponding to the intrinsic state with angular momentum projection K. In the present work the decoupling factors are considered as model parameters adjusted to the experimental data. In [5] we show that to treat the Coriolis interaction in CQOM microscopically, one can apply an appropriate parity-projection particle-core coupling scheme in which these factors are calculated by using a reflection-asymmetric deformed shell model.

Under the assumption of coherent quadrupole-octupole oscillations with a frequency

$$\omega = \sqrt{\frac{C_2}{B_2}} = \sqrt{\frac{C_3}{B_3}} \equiv \sqrt{\frac{C}{B}}, \tag{5}$$

and after introducing ellipsoidal coordinates

$$\beta_2 = p\eta\cos\phi, \qquad \beta_3 = q\eta\sin\phi, \tag{6}$$

with

$$p = \sqrt{\frac{d}{d_2}}, \quad q = \sqrt{\frac{d}{d_3}}, \quad d = \frac{1}{2}(d_2 + d_3), \tag{7}$$

the collective energy of the system is obtained in the form [2,3]

$$E_{n,k}(I) = \hbar\omega\left[2n + 1 + \sqrt{k^2 + bX}\right], \tag{8}$$
$$n = 0, 1, 2, \ldots, \quad k = 1, 2, 3, \ldots,$$

where $b = 2B/(\hbar^2 d)$. The quadrupole-octupole vibration wave function is

$$\Phi^{\pi}_{nkI}(\eta,\phi) = \psi^{I}_{nk}(\eta)\varphi^{\pi}_{k}(\phi), \tag{9}$$

where the "radial" part

$$\psi^{I}_{n,k}(\eta) = \sqrt{\frac{2c\Gamma(n+1)}{\Gamma(n+2s+1)}}e^{-c\eta^2/2}(c\eta^2)^s L^{2s}_{n}(c\eta^2) \tag{10}$$

involves generalized Laguerre polynomials in the variable η with

$$c = \frac{1}{\hbar}\sqrt{BC}, \quad s = \frac{1}{2}\sqrt{k^2 + bX}. \tag{11}$$

The "angular" part in the variable ϕ appears with a positive or negative parity as follows

$$\varphi^{+}_{k}(\phi) = \sqrt{\frac{2}{\pi}}\cos(k\phi), \qquad k = 1, 3, 5, \ldots, \tag{12}$$

$$\varphi^{-}_{k}(\phi) = \sqrt{\frac{2}{\pi}}\sin(k\phi), \qquad k = 2, 4, 6, \ldots. \tag{13}$$

2.2 Wave functions, yrast and non-yrast model spectra

The wave function for a state with angular momentum I^{π} belonging to an alternating-parity sequence in even-even nuclei has the form [2]

$$\Psi^{\pi}_{nIM0}(\eta,\phi) = \sqrt{\frac{2I+1}{8\pi^2}}D^{I}_{M\,0}(\theta)\psi^{I}_{n}(\eta)\varphi^{\pi}(\phi), \tag{14}$$

where $D^{I}_{M\,0}(\theta)$ is the Wigner function defined according to the phase convention in [6]. The total core+particle wave function for a state with angular momentum I^{π} belonging to a parity-doublet sequence in odd-even nuclei is given by [3]

$$\Psi^{\pi}_{nkIMK}(\eta,\phi) = \sqrt{\frac{2I+1}{16\pi^2}}\Phi^{\pi}_{n,k,I}(\eta,\phi)\left[D^{I}_{M\,K}(\theta)\mathcal{F}_K + \pi(-1)^{I+K}D^{I}_{M\,-K}(\theta)\mathcal{F}_{-K}\right], \tag{15}$$

where $\mathcal{F}_K$ is the wave function of the unpaired nucleon.

The energy spectrum is determined in (8) by the quantum numbers n and k. In even-even nuclei the alternating-parity band is formed under the condition $\pi(-1)^I = 1$. Then the band is characterized by a given n and a pair of odd and even k-values, $k^{(+)}_n$ and $k^{(-)}_n$ with $k^{(+)}_n < k^{(-)}_n$, corresponding to the positive- and negative-parity sequences, respectively. The difference between $k^{(+)}_n$ and $k^{(-)}_n$ gives the parity shift between the states with even and odd angular momentum. The sets of levels labeled by $n = 0, 1, 2$ and $k^{(+)}_n$ involve the ground-state band, first and second β-bands, respectively, while those with $k^{(-)}_n$ involve the respective yrast and non-yrast negative-parity bands.

In odd-mass nuclei the parity-doublet structure is imposed by the condition $\pi = \pi_{\varphi}\cdot\pi_{\mathrm{sp}}$, where π_{φ} is the parity of the quadrupole-octupole vibration wave function determined by (12) and (13) and π_{sp} is the parity of the single-particle (s.p.) state. Similarly to the even-even nuclei the parity doublet is determined by a given n and a pair of odd and even k-values, $k^{(+)}_n$ and $k^{(-)}_n$ ($k^{(+)}_n < k^{(-)}_n$). However, now one has two doublet counterparts with the same angular momentum $I^{\pm}$ (half-integer) and opposite parities. Their k-values are determined so that $k = k^{(+)}_n$ for $I^{\pi=\pi_{\mathrm{sp}}}$ and $k = k^{(-)}_n$ for $I^{\pi=-\pi_{\mathrm{sp}}}$. Again, similarly to the even-even nuclei, the difference between $k^{(+)}_n$ and $k^{(-)}_n$ generates the splitting of the parity-doublet. The yrast doublet is formed above the ground state whose parity is $\pi^{(0)}_{\mathrm{sp}}$. The non-yrast doublets are formed on the top of excited s.p., or quasi-particle (q.p.) states (if the pairing correlations are taken into account) whose parity $\pi^{(n)}_{\mathrm{sp}}$ determines the doublet structure, with the $k^{(+)}_n$ and $k^{(-)}_n$ accordance, as in the case of the ground-state (yrast) doublet. The index n also labels the different intrinsic configurations on which the non-yrast doublets are built.

The ground state, the excited q.p. states, the respective decoupling factors (in case of $K = 1/2$), as well as the Coriolis mixing contributions can be obtained through a reflection-asymmetric deformed shell model (DSM). This possibility provides a natural way to connect the collective CQOM model with the microscopic approach. Although the work in this direction is in an essential progress, here we consider the s.p. degree of freedom phenomenologically. Thus for a given doublet we take the s.p. parity and the K value as suggested by the experimental analysis and by microscopic calculations reported in the literature. On the other hand the doublet band-heads are obtained as different rotation-vibration modes characterized by the CQOM oscillator quantum number $n = 0, 1, 2, \ldots$, and the respective decoupling factors a_n entering the expression (4) (in case of $K = 1/2$) are adjusted according to the experimental data. It should be noted that the knowledge of these phenomenological decoupling factors

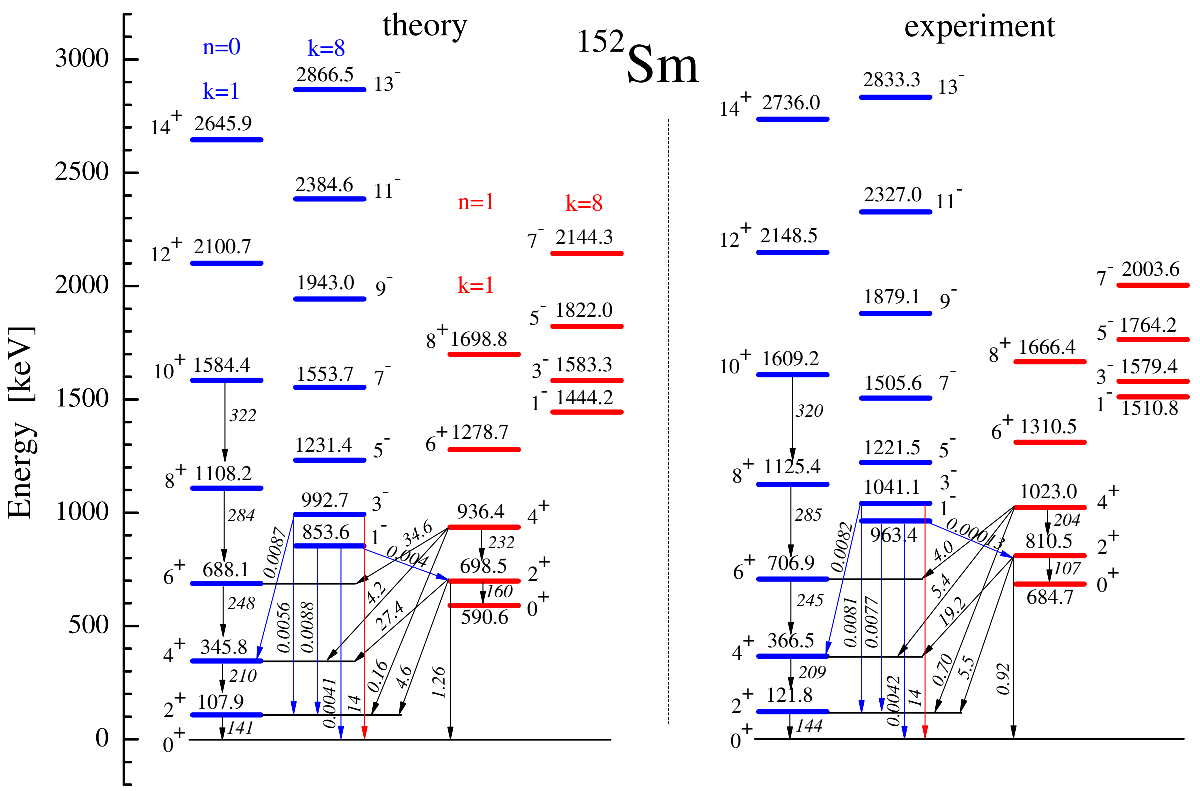

Fig. 1. Theoretical and experimental alternating-parity levels and transition probabilities for ^{152}Sm. Data for the energy levels from [7]. B($E1$) and B($E2$) data from [8] and B($E3$) data from [9]. Parameter values: ω = 0.295 MeV/$\hbar$, b = 2.450 $\hbar^{-2}$, d_0 = 78.8 $\hbar^2$, c = 113.2, p = 0.854, e^1_{eff} = 1.01 e.

is of great importance to determine the physically reasonable deformation regions [5] where the DSM calculations have to be performed after involving the microscopic part in CQOM.

By having the wave functions (14) and (15) one can calculate B($E1$), B($E2$) and B($E3$) transition probabilities in the yrast and non-yrast octupole spectra of both even-even and odd-mass nuclei. For the even-even nuclei the relevant formalism was originally developed in the case of yrast alternating-parity bands [2] and further extended to the case of the non-yrast states [4]. Results of its application describing transition data in ^{152}Sm are illustrated in the next section. For odd-mass nuclei the formalism is so far utilized only for the yrast parity-doublets [3], while its application to the non-yrast part of the spectrum is currently in progress.

3 Numerical results and discussion

In this section results of the CQOM model application to the even-even nucleus ^{152}Sm and the odd-mass nuclei ^{223}Ra, ^{229}Th and ^{237}U are presented. The model energy levels are determined by Eq. (8) as

$$\tilde{E}_{n,k}(I) = E_{n,k}(I) - E_{0,k_{gs}}(I_{gs}), \qquad (16)$$

where I_{gs} and k_{gs} are the angular momentum and k-values of the ground state respectively. In ^{152}Sm the model parameters ω, b, d_0, c, p and e^1_{eff} are adjusted by simultaneously taking into account experimental data on the energy bands [7] and the available B($E1$)-B($E3$) transition probabilities [8,9]. The parameter e^1_{eff} is an effective charge involved in [4] to tune the scale for the $E1$ transitions. In ^{223}Ra, ^{229}Th and ^{237}U presently only the energy levels are fitted through the parameters ω, b, d_0 and the considered decoupling factors in the cases of $K = 1/2$ bandheads. For each nucleus the calculations are performed in a net over the values of $k_n^{(+)}$ and $k_n^{(-)}$ providing the sets of k-values for the best description. The theoretical and experimental energy levels of the considered nuclei are compared in figures 1-4. The obtained parameter values and k-numbers are also given there.

In figure 1 it is seen that the model correctly reproduces the structure of the yrast and non-yrast alternating-parity bands in ^{152}Sm. Also there, it is seen that the B($E1$) transition probabilities between the ground-state band (gsb) and the first negative-parity band of this nucleus are described quite well. The value of the interband B($E1$) probability B($E1$; $1^-_1 \to 2^+_2$) in ^{152}Sm is overestimated, while the B($E2$) intraband probabilities within the gsb are well described. The B($E3$; $3^-_1 \to 0^+_1$) value is exactly reproduced due to the adjustment of the parameter p.

In figure 2 the model description of the yrast and two non-yrast split parity-doublet level sequences in ^{223}Ra are shown. The yrast band is obtained by imposing $K = 1/2^+$ instead of the experimentally suggested $K = 3/2^+$. This is motivated by the previously indicated staggering behaviour of the doublet splitting which is considered as the manifestation of a Coriolis decoupling effect in that band [3]. The first non-yrast doublet is based on $K = 5/2^+$, while the second one is again with $K = 1/2^+$. Thus, two de-

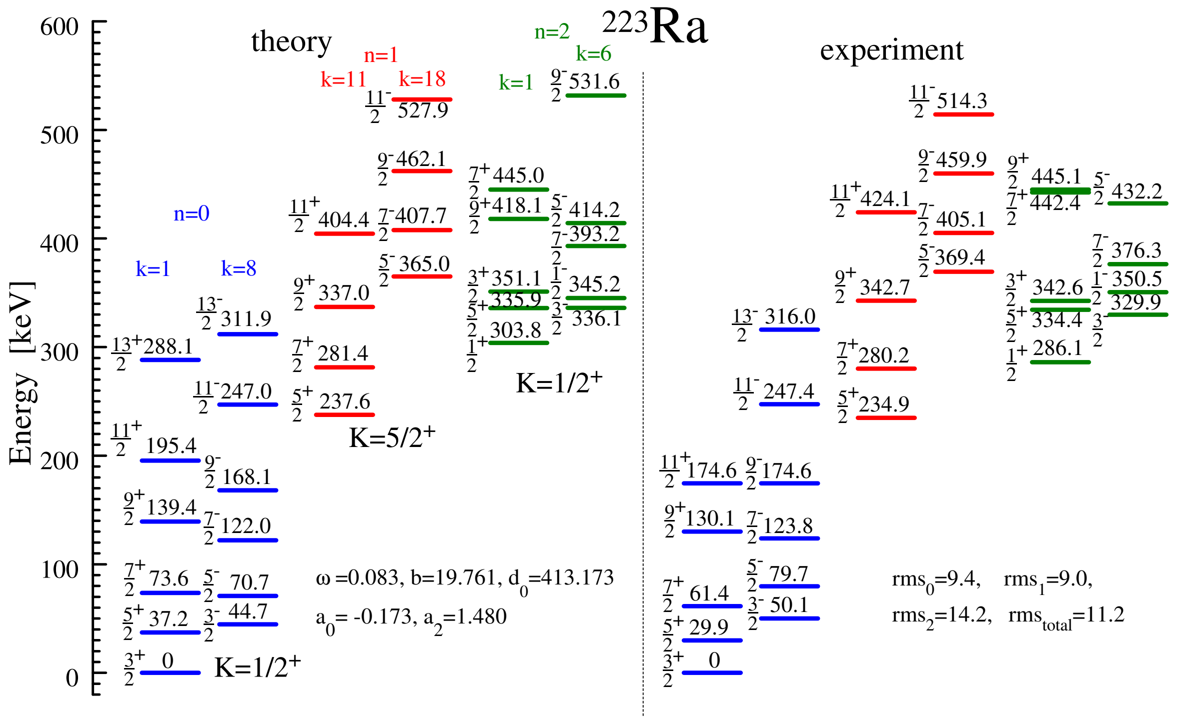

Fig. 2. Theoretical and experimental parity-doublet levels for ^{223}Ra. The root mean square (rms) factors are given in keV. Data from [7].

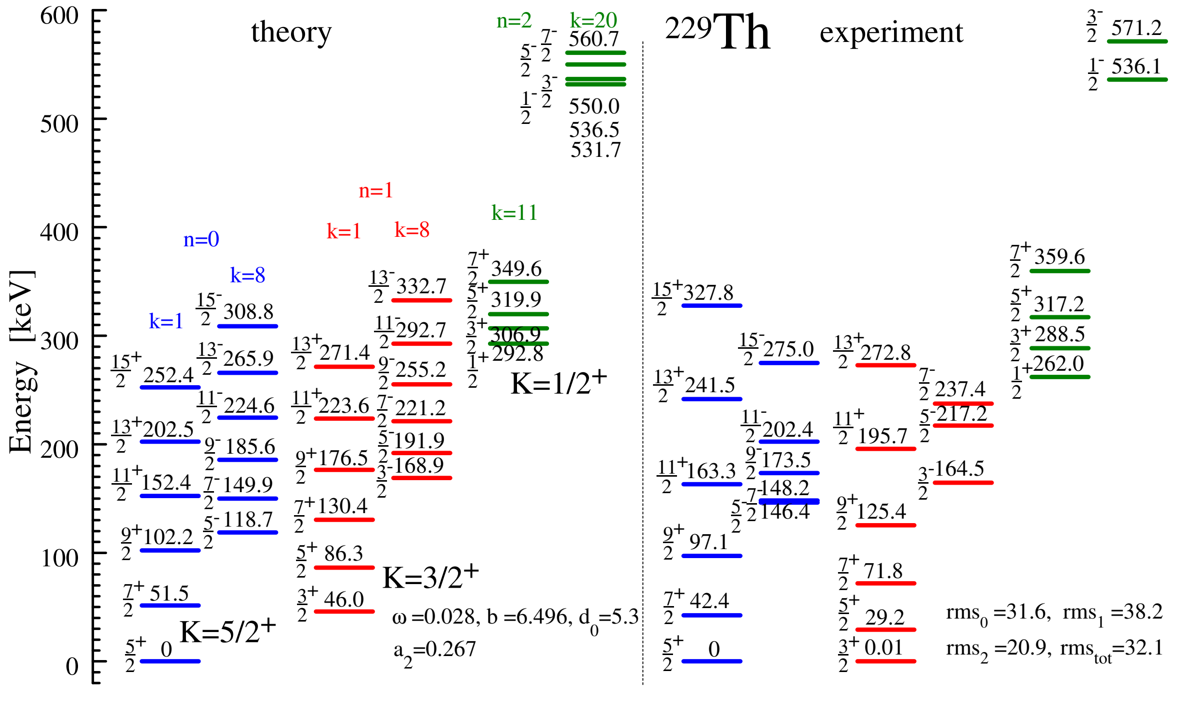

Fig. 3. The same as in figure 2, but for ^{229}Th.

coupling parameter values, $a_0 = -0.173$ and $a_2 = 1.480$, corresponding to the yrast band and the second non-yrast sequence, are obtained in this energy fit. It is seen that the Coriolis decoupling in the second doublet is considerably larger, which explains the observed rearrangement of the experimental levels in dependence on the angular momentum. The observed interchanges of the neighbouring angular momenta are reproduced except for the levels $7/2^+$ and $9/2^+$ where the theoretical value of the decoupling factor is a bit insufficient to provide it. The values of the quantum number k obtained for the different energy sequences, and shown in the figure, carry information about their mutual disposition. The root mean square (rms) deviation between the theory and experiment obtained for the different sequences as well as the total rms factor show the good quality of the model description.

In figure 3 the description of parity-doublet sequences in ^{229}Th is shown. Here the K projection values are K =

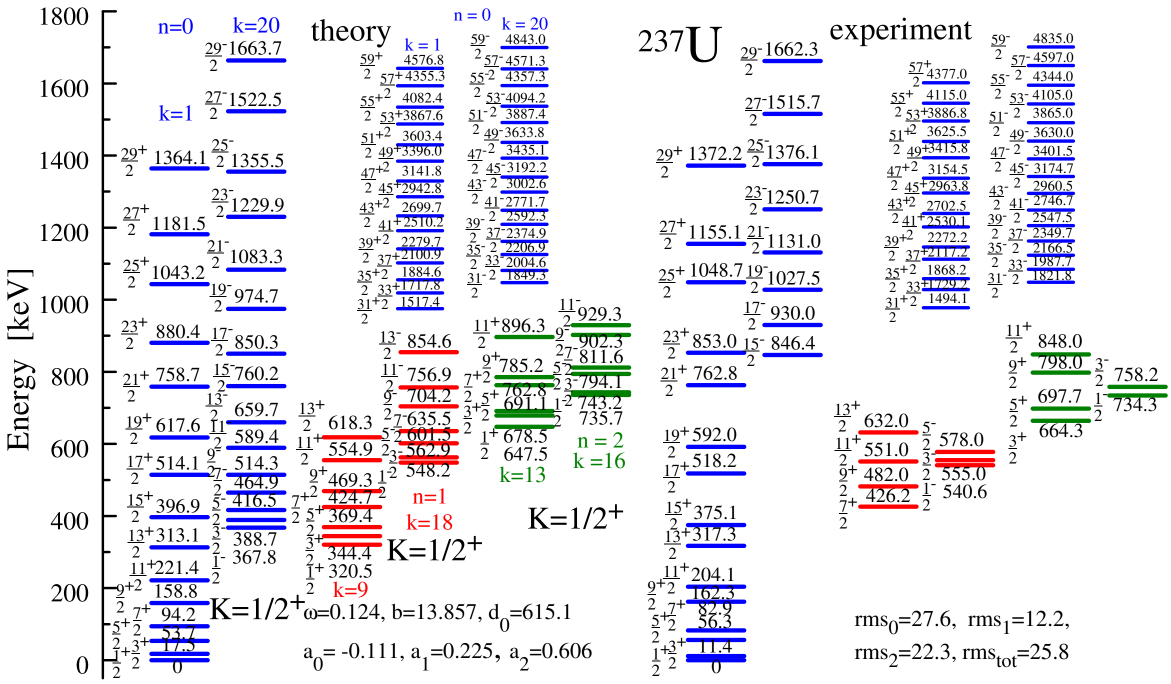

Fig. 4. Theoretical and experimental parity-doublet levels for ^{237}U. The rms factors are given in keV. Data from [7] and [10].

$5/2^+$ for the yrast band and $K = 3/2^+$ and $1/2^+$ for the non-yrast sequences. There is only one decoupling factor with a value $a_2 = 0.267$ obtained for the third quasi doublet. Also, we remark that the parity splitting in this doublet is quite large. This is taken into account by the obtained values of the quantum number k shown in figure 3.

In figure 4 three split parity-doublet sequences in ^{237}U are described. All of them are built on $K = 1/2^+$ intrinsic configurations and, therefore, three decoupling parameter values $a_0 = -0.111$, $a_1 = 0.225$ and $a_2 = 0.606$ are obtained. The two non-yrast sequences are well reproduced with some higher-lying energy levels being predicted. It is noticeable that at the same time the yrast band is described up to a quite high angular momentum 59/2.

The CQOM model descriptions of yrast and non-yrast octupole spectra of even-even and odd-mass nuclei illustrated in figures 1-4 show that the model is capable to reproduce a wide range of spectroscopic properties related to the simultaneous manifestation of quadrupole and octupole degrees of freedom in these nuclei. It should be recognized that the successful descriptions are particularly due to the allowed jumps of the angular-oscillator quantum number k over several low-lying model states within the considered alternating-parity and quasi-doublet structures. So far, this is only justified by the meaning of the differences in the k-values as characteristics of the mutual displacement of the opposite-parity sequences. The search for a deeper meaning and more sophisticated correlation between the quadrupole and octupole modes capable to compensate or explain these jumps is still an open issue. On the other hand even on the present level of phenomenology, the CQOM model descriptions provide a useful information which can serve in the initiation of further developments. As it was mentioned in the end of Sec. 2 the knowledge of the decoupling factors as well as the model mechanism for the forming of parity-doublet spectra in odd-mass nuclei can guide the inclusion of microscopic calculations in the study.

4 Conclusion

In conclusion, the present work provides a unified approach based on the collective model of Coherent Quadrupole and Octupole Motion (CQOM) for the description of yrast and non-yrast alternating-parity spectra in even-even nuclei and quasi-doublet spectra in odd-mass nuclei. The capability of the model to obtain the attendant B(E1), B(E2) and B(E3) transition probabilities was illustrated in ^{152}Sm. In the odd-mass nuclei ^{223}Ra, ^{229}Th and ^{237}U the split parity-doublet structures were described together with the observed Coriolis decoupling effects. On this basis the present CQOM model descriptions can serve as a starting point for the application of a deeper collective+microscopic approach in the exploration of nuclear quadrupole-octupole collectivity. Work in this direction is in progress.

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