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FRpnQRPA approach with the gauge symmetry restored. Application for the $2\nu\beta\beta$ decay

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Abstract. A many body Hamiltonian involving the mean field for a projected spherical single particle basis, the pairing interactions for alike nucleons, a repulsive dipole-dipole proton-neutron interaction in the particle-hole (*ph*) channel and an attractive dipole-pairing interaction is treated by a gauge restored and fully renormalized proton-neutron quasiparticle random phase approximation formalism. Application to the $2\nu\beta\beta$ decay rate show a good agreement with the corresponding data. The Ikeda sum rule is obeyed.

1 Introduction

The $2\nu\beta\beta$ process is interesting by its own but is also very attractive because it constitutes a test for the nuclear matrix elements (m.e.) which are used for the process of $0\nu\beta\beta$ decay. The discovery of this process may provide an answer to the fundamental question, whether neutrino is a Majorana or a Dirac particle. The subject development is described by several review papers [1,2]. The present talk refers to the $2\nu\beta\beta$ process, which is conceived as consisting of two consecutive and virtual single β^- decays. The formalism yielding closest results to the experimental data is the proton-neutron random phase approximation (*pnQRPA*) which includes the particle-hole (*ph*) and particle-particle (*pp*) as independent two body interactions. The second leg of the $2\nu\beta\beta$ process is very sensitive to changing the relative strength of the later interaction, denoted hereafter by g_{pp}. It is worth mentioning that the *ph* interaction is repulsive while the *pp* one is attractive. Consequently, there is a critical value of g_{pp} for which the first root of the *pnQRPA* equation vanishes. Actually, this is the signal that the *pnQRPA* approach is no longer valid. Moreover, the g_{pp} value which corresponds to a transition amplitude which agrees with the corresponding experimental data is close to the mentioned critical value. That means that the result is not stable to adding corrections to the RPA picture. An improvement for the *pnQRPA* was achieved by one of us (AAR), in collaboration, in Ref. [3], by using a boson expansion (BE) procedure. Another procedure, proposed in Ref. [4], renormalizes the dipole two quasiparticle operators by replacing the scalar components of their commutators with their average values. Such a renormalization is, however, inconsistently achieved since the scattering operators do not participate at the renormalization process. This lack of consistency was removed in Ref. [5] where a fully renormalized *pnQRPA* (*FRpnQRPA*) is proposed. Unfortunately, all higher pnQRPA procedures mentioned above have the common drawback of violating the Ikeda sum rule (*ISR*) by an amount of about 20-30% [6]. It is believed that such a violation is caused by the gauge symmetry breaking. Consequently, a method of restoring this symmetry was formulated in Ref. [7].

Recently [8,9], the results of Ref. [7] were improved in two respects: a) aiming at providing a unitary description of the process for the situations when the involved nuclei are spherical or deformed, here we use a projected spherical single particle basis; b) the space of proton-neutron dipole configurations is split in three subspaces, one being associated to the single β^- decay, one to the single β^+ process, and one spanned by the unphysical states. A set of *GRFRpnQRPA* equations is written down in the first two subspaces mentioned above, by linearizing the equations of motion of the basic transition operators corresponding to the two coupled processes.

Results are described according to the following plan. The approach is described in Section 2. Numerical applications and discussions are given in Section 3, while the final conclusions are drawn in Section V.

2 Approximations and the main ingredients

We suppose that the Gamow-Teller transitions dominate the Fermi ones which seems to be a reasonable hypothesis in medium and heavy nuclei. In the exact expression for the transition probability, the leptons energy is replaced by the average value: $\Delta E = mc^2 + \frac{1}{2}Q_{\beta\beta}$, where m denotes the rest mass of the emitted electron while $Q_{\beta\beta}$ the reaction heat of the process. Consequently, the half life is factorized

$$\left[T^{2\nu}_{1/2}(0^+\to 0^+)\right]^{-1} = F|M_{GT}|^2, \qquad (1)$$
$$M_{GT} = \sqrt{3}\sum_m \frac{{}_i\langle 0\|\beta^+\|m\rangle_{ii}\langle m|m'\rangle_{if}\langle m'\|\beta^+\|0\rangle_f}{E_m+\Delta E_1},$$

where $\Delta E_1 = \Delta E + E_{1^+}$ and E_m are the *pnQRPA* energies. E_{1^+} denotes the experimental energy of the first 1^+ state. The GT transition operators for the single beta transitions are denoted by $\beta^{\mp}$.

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The model proposed by our group has two main ingredients: **i)** The single particle basis is obtained from a deformed basis by projecting out the good angular momentum

$$\Phi^{IM}_{nlj}(d) = \mathcal{N}^I_{nlj} P^I_{MI}[|nljI\rangle \Psi_g] \equiv \mathcal{N}^I_{nlj} \Psi^{IM}_{nlj}(d),$$
$$\Psi_g = exp[d(b^+_{20} - b_{20})]|0\rangle_b. \qquad (2)$$

The single particle energies are described by the average values of a particle-core Hamiltonian on the projected basis. b)To describe the states involved in Eq. (2) we used the following many body Hamiltonian

$$H = \sum_{\tau,\alpha,I,M} \frac{2}{2I+1}(\epsilon_{\tau\alpha I} - \lambda_{\tau\alpha}) c^\dagger_{\tau\alpha IM} c_{\tau\alpha IM} \qquad (3)$$
$$- \sum_{\tau,\alpha,I,I'} \frac{G_\tau}{4} P^\dagger_{\tau\alpha I} P_{\tau\alpha I'} + 2\chi \sum_{pn;p'n';\mu} \beta^-_\mu(pn)\beta^+_{-\mu}(p'n')(-)^\mu$$
$$- 2X_{dp} \sum_{pn;p'n';\mu} \left(\beta^-_\mu(pn)\beta^-_{-\mu}(p'n') + \beta^+_\mu(pn)\beta^+_{-\mu}(p'n')\right)(-)^\mu.$$

In the qp representation the Hamiltonian is expressed in terms of the dipole 2qp and dipole density operators

$$A^\dagger_{1\mu}(pn) = \sum C^{I_p\ I_n\ 1}_{m_p\ m_n\ \mu} a^\dagger_{pI_pm_p} a^\dagger_{nI_nm_n}, \qquad (4)$$
$$B^\dagger_{1\mu}(pn) = \sum C^{I_p\ I_n\ 1}_{m_p\ -m_n\ \mu} a^\dagger_{pj_pm_p} a_{nI_nm_n}(-)^{I_n-m_n},$$
$$A_{1\mu}(pn) = \left(A^\dagger_{1\mu}(pn)\right)^\dagger,\ B_{1\mu}(pn) = \left(B^\dagger_{1\mu}(pn)\right)^\dagger.$$

Linearized equations of motion of the above operators determine the dipole excitations of the many body system. Such equations are obtained by the mutual commutators

$$\left[A_{1\mu}(k), A^\dagger_{1\mu'}(k')\right] \approx \delta_{k,k'}\delta_{\mu,\mu'}\left[1 - \frac{\hat{N}_n}{\hat{I}^2_n} - \frac{\hat{N}_p}{\hat{I}^2_p}\right],$$
$$\left[B^\dagger_{1\mu}(k), A^\dagger_{1\mu'}(k')\right] \approx \left[B^\dagger_{1\mu}(k), A_{1\mu'}(k')\right] \approx 0,$$
$$\left[B_{1\mu}(k), B^\dagger_{1\mu'}(k')\right] \approx \delta_{k,k'}\delta_{\mu,\mu'}\left[\frac{\hat{N}_n}{\hat{I}^2_n} - \frac{\hat{N}_p}{\hat{I}^2_p}\right],\ k = (I_p, I_n),$$

with $\hat{N}_\tau$ denoting the quasiparticle number operator of type τ(=p,n). There are three distinct approximations for these equations: 1) *pnQRPA*, 2) Standard renormalized pnQRPA [4] 3) Fully renormalized pnQRPA [5].

Denoting by $C^{(1)}_{I_p,I_n}$ and $C^{(2)}_{I_p,I_n}$ the averages of the right hand sides, with the renormalized pnQRPA vacuum state, the renormalized operators defined as

$$\bar{A}_{1\mu}(k) = \frac{1}{\sqrt{C^{(1)}_k}} A_{1\mu},\ \bar{B}_{1\mu}(k) = \frac{1}{\sqrt{|C^{(2)}_k|}} B_{1\mu}, \qquad (5)$$

obey boson-like commutation relations

$$\left[\bar{A}_{1\mu}(k), \bar{A}^\dagger_{1\mu'}(k')\right] = \delta_{k,k'}\delta_{\mu,\mu'}, \qquad (6)$$
$$\left[\bar{B}_{1\mu}(k), \bar{B}^\dagger_{1\mu'}(k')\right] = \delta_{k,k'}\delta_{\mu,\mu'} f_k,\quad f_k = sign(C^{(2)}_k).$$

Further, these operators are used to define the phonon operator

$$C^\dagger_{1\mu} = \sum_k \left[X(k)\bar{A}^\dagger_{1\mu}(k) + Z(k)\bar{D}^\dagger_{1\mu}(k)\right. \qquad (7)$$
$$\left. - Y(k)\bar{A}_{1-\mu}(k)(-)^{1-\mu} - W(k)\bar{D}_{1-\mu}(k)(-)^{1-\mu}\right],$$

where $\bar{D}^\dagger_{1\mu}(k)$ is equal to $\bar{B}^\dagger_{1\mu'}(k')$ or $\bar{B}_{1\mu}(k)$ depending on whether f_k is + or -. The phonon amplitudes are determined by the equations

$$\left[H, C^\dagger_{1\mu}\right] = \omega C^\dagger_{1\mu},\ \left[C_{1\mu}, C^\dagger_{1\mu'}\right] = \delta_{\mu\mu'}. \qquad (8)$$

Unfortunately both the renormalized and fully renormalized pnQRPA violates the ISR by an amount of about 20-30%. The boson expansion procedure overestimate ISR, while the standard renormalized pnQRPA underestimate it. In Ref. [6] we have used a boson expansion formalism on the top of a renormalized pnQRPA. The result was that the departure of our predictions from ISR was diminished up to about 10%.

We believe that such a deviation from the ISR is caused by the fact that the renormalized ground state is not eigenstate of the nucleon total number operator.

The state $C^\dagger_{1\mu}|0\rangle$, where $|0\rangle$ is the vacuum state for the phonon operator defined by the FRpnQRPA approach, with both the *ph* and *pp* interactions included, is a superposition of components describing the neighboring nuclei $(N-1, Z+1)$, $(N+1, Z-1)$, $(N+1, Z+1)$, $(N-1, Z-1)$. The first two components conserve the total number of nucleons (N+Z) but violate the third component of isospin, T_3. By contrast, the last two components violate the total number of nucleons but preserve T_3. Actually, the last two components are those which contribute to the ISR violation. However, one can construct linear combinations of the basic operators $A^\dagger, A, B^\dagger, B$ which excite the nucleus (N, Z) to the nuclei $(N-1, Z+1)$, $(N+1, Z-1)$, $(N+1, Z+1)$, $(N-1, Z-1)$, respectively. These operators are

$$\mathcal{A}^\dagger_{1\mu}(pn) = -\left[c^\dagger_p c_{\tilde{n}}\right]_{1\mu},\ \mathcal{A}_{1\mu}(pn) = -\left[c^\dagger_p c_{\tilde{n}}\right]^\dagger_{1\mu},$$
$$\mathbf{A}^\dagger_{1\mu}(pn) = \left[c^\dagger_p c^\dagger_n\right]_{1\mu},\ \mathbf{A}_{1\mu}(pn) = \left[c^\dagger_p c^\dagger_n\right]^\dagger_{1\mu}. \qquad (9)$$

In terms of the new operators, the many body model Hamiltonian is

$$H = \sum_{\tau jm} E_{\tau j} a^\dagger_{\tau jm} a_{\tau jm}$$
$$+ 2\chi \sum_{pn,p'n';\ \mu} \sigma_{pn;p'n'} \mathcal{A}^\dagger_{1\mu}(pn)\mathcal{A}_{1\mu}(p'n')$$
$$- X_{dp} \sum_{pn;p'n';\ \mu} \sigma_{pn;p'n'}(-)^{1-\mu}$$
$$\times \left(\mathcal{A}^\dagger_{1\mu}(pn)\mathcal{A}^\dagger_{1,-\mu}(p'n') + \mathcal{A}_{1,-\mu}(p'n')\mathcal{A}_{1\mu}(pn)\right),$$
$$\sigma_{pn;p'n'} = \frac{2}{3\hat{I}_n\hat{I}_{n'}}\langle I_p||\sigma||I_n\rangle\langle I_{p'}||\sigma||I_{n'}\rangle. \qquad (10)$$

The equations of motion of the operators involved in the phonon operator are determined by the commutation relations

$$\left[\mathcal{A}_{1\mu}(pn), \mathcal{A}^\dagger_{1\mu'}(p'n')\right] \approx \delta_{\mu,\mu'}\delta_{j_p,j_{p'}}\delta_{j_n,j_{n'}} \qquad (11)$$
$$\times \left[U^2_p - U^2_n + \frac{U^2_n - V^2_n}{\hat{I}^2_n}\hat{N}_n - \frac{U^2_p - V^2_p}{\hat{I}^2_p}\hat{N}_p\right].$$

The quasi-boson approximation replaces the r.h. side of the above equation by its average with the *GRFRpnQRPA* vacuum state denoted by

$$D_1(pn) = U_p^2 - U_n^2 + \frac{1}{2I_n+1}(U_n^2 - V_n^2)\langle \hat{N}_n \rangle \quad (12)$$
$$- \frac{1}{2I_p+1}(U_p^2 - V_p^2)\langle \hat{N}_p \rangle.$$

Equations of motion show that the two qp energies are also renormalized

$$E^{ren}(pn) = E_p(U_p^2 - V_p^2) + E_n(V_n^2 - U_n^2). \quad (13)$$

The space of the pn dipole states, $\mathcal{S}$, is written as a sum of three subspaces defined as

$$\begin{aligned} \mathcal{S}_+ &= \{(p,n)|D_1(pn) > 0, \quad E^{ren}(pn) > 0, \}, \\ \mathcal{S}_- &= \{(p,n)|D_1(pn) < 0, \quad E^{ren}(pn) < 0, \}, \\ \mathcal{S}_{sp} &= \mathcal{S} - (\mathcal{S}_+ + \mathcal{S}_-), \\ \mathcal{N}_\pm &= \dim(\mathcal{S}_\pm), \quad \mathcal{N}_{sp} = \dim(\mathcal{S}_{sp}), \\ \mathcal{N} &= \mathcal{N}_+ + \mathcal{N}_- + \mathcal{N}_{sp}. \end{aligned} \quad (14)$$

In $\mathcal{S}_+$ one defines the renormalized operators

$$\bar{\mathcal{A}}^\dagger_{1\mu}(pn) = \frac{1}{\sqrt{D_1(pn)}}\mathcal{A}^\dagger_{1\mu}(pn), \quad (15)$$
$$\bar{\mathcal{A}}_{1\mu}(pn) = \frac{1}{\sqrt{D_1(pn)}}\mathcal{A}_{1\mu}(pn),$$

while in $\mathcal{S}_-$ the renormalized operators are

$$\bar{\mathcal{F}}^\dagger_{1\mu}(pn) = \frac{1}{\sqrt{|D_1(pn)|}}\mathcal{A}_{1\mu}(pn), \quad (16)$$
$$\bar{\mathcal{F}}_{1\mu}(pn) = \frac{1}{\sqrt{|D_1(pn)|}}\mathcal{A}^\dagger_{1\mu}(pn).$$

An pnQRPA treatment within $\mathcal{S}_{sp}$ would yield either vanishing or negative energies. The corresponding states are therefore spurious.

FRpnQRPA with the gauge symmetry projected defines the phonon operator as

$$\Gamma^\dagger_{1\mu} = \sum_k \left[X(k)\bar{\mathcal{A}}^\dagger_{1\mu}(k) + Z(k)\bar{\mathcal{F}}^\dagger_{1\mu}(k)\right.$$
$$\left. - Y(k)\bar{\mathcal{A}}_{1-\mu}(k)(-)^{1-\mu} - W(k)\bar{\mathcal{F}}_{1-\mu}(k)(-)^{1-\mu}\right], \quad (17)$$

with the amplitudes determined by the GRFRpnQRPA equations

$$[H, \Gamma^\dagger_{1\mu}] = \omega\Gamma^\dagger_{1\mu}, \ [\Gamma_{1\mu}, \Gamma^\dagger_{1\mu'}] = \delta_{\mu,\mu'}. \quad (18)$$

In order to solve the GRFRpnQRPA equations we need to know $D_1(pn)$ and, therefore, the averages of the qp's number operators, $\hat{N}_p$ and $\hat{N}_n$. These are written first in particle representation and then the particle number conserving term is expressed as a linear combination of $\mathcal{A}^\dagger\mathcal{A}$ and $\mathcal{F}^\dagger\mathcal{F}$ chosen such that their commutators with $\mathcal{A}^\dagger, \mathcal{A}$ and $\mathcal{F}^\dagger, \mathcal{F}$ are preserved. The final result is

$$\langle \hat{N}_p \rangle = V_p^2(2I_p+1) + 3(U_p^2 - V_p^2) \quad (19)$$
$$\times \Big(\sum_{\substack{n',k \\ (p,n')\in\mathcal{S}_+}} D_1(p,n')(Y_k(p,n'))^2 - \sum_{\substack{n',k \\ (p,n')\in\mathcal{S}_-}} D_1(p,n')(W_k^2)\Big),$$
$$\langle \hat{N}_n \rangle = V_n^2(2I_n+1) + 3(U_n^2 - V_n^2) \quad (20)$$
$$\times \Big(\sum_{\substack{p',k \\ (p',n)\in\mathcal{S}_+}} D_1(p',n)(Y_k(p',n))^2 - \sum_{\substack{p',k \\ (p',n)\in\mathcal{S}_-}} D_1(p',n)(W_k^2)\Big).$$

GRFRpnQRPA equations, the average qp numbers and the normalization factor equations are to be simultaneously considered and solved iteratively.

2.1 The gauge symmetry and the pp interaction

At this stage we have to explain why the pp interaction is not effective, i.e. does not contribute at all within our approach. Indeed, within the gauge preserved picture the operators $\mathcal{A}_{1\mu}$ and $\mathbf{A}^\dagger_{1\mu}$ commute with each other. Consequently, the gauge projected phonon operator cannot comprise terms like $\mathbf{A}^\dagger_{1\mu}$ since they violate the total number of nucleons. If the mentioned commutator would be different from zero, but equal to the average with the new vacuum state of its scalar part, then the equations of motion for the operators $\mathcal{A}_{1\mu}$ and $\mathcal{A}^\dagger_{1\mu}$ would be linear not only in the nucleon number conserving operators, but also in those which do not conserve the total number operator. In order that the equations of motion constitute a closed algebra, we have to add the equations corresponding to the number non-conserving operators. Consequently, the phonon operator is a linear combination of both nucleon number conserving and non-conserving terms. It is conspicuous now that in order to conserve the nucleon total number it is necessary to accept that the operators $\mathcal{A}_{1\mu}$ and $\mathbf{A}^\dagger_{1\mu}$ commute with each other. In this context the pp interaction is becoming inefficient for properties described by gauge preserving wave functions and therefore we have to ignore it. In this respect our formalism contrasts the picture of Ref. [10] where the phonon operator is commuting with the nucleon total number operator and at a time the pp interaction contributes to the renormalized *pnQRPA* equations.

However, aiming at a quantitative description of the double beta process, the presence of an attractive proton-neutron interaction is necessary. Due to this reason we replace the pp interaction, which is ineffective anyway, with a dipole-pairing interaction:

$$\Delta H = -X_{dp} \sum_{\substack{pn;p' \\ n';\mu}} \Big(\beta^-_\mu(pn)\beta^-_{-\mu}(p'n')$$
$$+ \beta^+_{-\mu}(p'n')\beta^+_\mu(pn)\Big)(-1)^{1-\mu}. \quad (21)$$

We remark that the two terms of ΔH are changing the charge by +2 and -2 units respectively, and therefore one may think that it is not justified within the meson-dynamic theory of nuclear forces. That is not true, having in mind the isospin charge independence property of the nuclear forces. Also, we note that ΔH is Hermitian and invariant to rotation. This Hamiltonian should be looked at as an effective Hamiltonian in the same manner as the standard pairing Hamiltonian is. Indeed, within the BCS approximation the initial pairing Hamiltonian is replaced by an effective one $\Delta(c^\dagger c^\dagger)_0 + \Delta^*(cc)_0$, with $c^\dagger$ (c) denoting the single particle creation (annihilation) operator. This Hamiltonian does not preserve the charge too, but this is consistent with the trial variational state $|BCS\rangle$ which is a mixture of components with different even number of particles. In the present case the *pnQRPA* state is built on the top of the *BCS* ground state which is a product of the *BCS* states for protons and neutrons respectively, which results in obtaining a linear superposition of components with different isospin third component, T_3. Of course, at the *BCS* level T_3 is preserved in the average. Therefore, in the quasiparticle picture the condition that the Hamiltonian commutes separately with the proton and neutron number operators is anyway not fulfilled by any of the composing terms from the model Hamiltonian. Note that ΔH commutes with the total number of nucleons and preserves this feature after the linearization procedure is performed, contributing to the equations of motion of the basic operators with the gauge restored. Concerning the T_3 symmetry let us denote by $\mathcal{N}_\tau$ the τ $(=p, n)$ particle number operators respectively, and calculate the commutator

$$\left[\Delta H, \mathcal{N}_p - \mathcal{N}_n\right] = 4X_{dp} \qquad (22)$$
$$\times \sum_{\substack{pn;p' \\ n';\mu}} \left(\beta^-_\mu(pn)\beta^-_{-\mu}(p'n') - \beta^+_{-\mu}(p'n')\beta^+_\mu(pn)\right)(-1)^{1-\mu}.$$

Note that the right hand side of the above equation is an anti-Hermitian operator. Consequently, its average value with any state is vanishing. In particular it is vanishing if the chosen state is the *BCS* ground state or the vacuum state of the *GPFRpnQRPA* phonon operator. Concluding, in the present formalism the third isospin component is conserved in the average. Clearly, this happens since while one term of ΔH increasing the charge by two units the other term is decreasing it by the same amount. Note that this isospin non-conserving term shows up even at the level of the standard *pnQRPA*. Indeed, within this formalism the two-body interaction is approximated by a linear combination of the operators

$$A^\dagger_{1\mu}(pn)A_{1\mu}(pn), \qquad (23)$$
$$(-1)^{1-\mu}\left(A^\dagger_{1\mu}(pn)A^\dagger_{1-\mu}(pn) + A_{1,-\mu}(pn)A_{1\mu}(pn)\right). \qquad (24)$$

Writing these terms in the particle representation one finds that the effective two-body interaction comprises, among other terms, a term which is proportional to ΔH. Therefore in a formalism using approximations which violates the T_3 symmetry, the use of a Hamiltonian ΔH which is not preserving the T_3 component does not produce a special inconsistency.

3 Numerical application and discussions

The approach presented in the previous sections was applied for the transitions of fourteen double beta emitters. The parameters defining the single particle energies are those of the spherical shell model, the deformation parameter d and the parameter k are fixed as described in Ref. [11]. The proton and neutron pairing strengths are different slightly from those from the quoted reference since the dimension of the single particle basis used in the present paper is different from that from Ref. [11]. The strength χ was taken to be

$$\chi = \frac{5.2}{A^{0.7}} MeV. \qquad (25)$$

This expression was obtained by fitting the positions of the GT resonances in ^{40}Ca, ^{90}Zr and ^{208}Pb [12]. The strength for the attractive pn two-body interaction was chosen such that the result for the log ft value associated to one of the

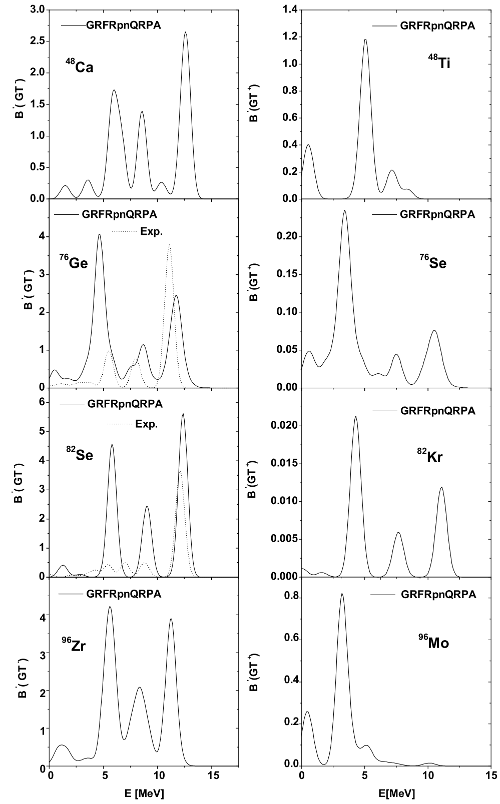

Fig. 1. One third of the single β^- (left column) and one third of the β^+ (right column) strengths, denoted by $B'(GT^-)$ and $B'(GT^+)$, for the mother , ^{48}Ca, ^{76}Ge, ^{82}Se and ^{96}Zr, and daughter, ^{48}Ti, ^{76}Se, ^{82}Kr and ^{96}Mo, nuclei respectively, folded by a Gaussian function with a width of 1 MeV, are plotted as functions of the corresponding energies yielded by the present formalism. The experimental data for the β^- strengths of ^{76}Ge and ^{82}Se are also presented [13].

single beta decay of the intermediate odd-odd nucleus, be close to the corresponding experimental data. If the experimental data are missing, the restriction refers to the existent data in the neighboring region. Since for ^{100}Mo and ^{116}Cd, experimental data for the log ft values associated to the $\beta^\pm$ decays of the intermediate odd-odd nuclei ^{100}Tc and ^{116}In respectively, are available, the parameters χ and χ_1 were fixed such that the mentioned data are reproduced. For these cases, the results are compared with the data from [14] in Table 1.

Let us just enumerate the results obtained with the formalism described above: **a**) The ISR is satisfied. **b**) We calculated the single $\beta^\pm$ strength distributions. For some of them experimental data are available. For example, β^- strength for the transitions ^{76}Ge→ ^{76}Se and ^{82}Se→ ^{82}Kr was extracted from the reactions ^{76}Ge(p,n)^{76}As, and ^{82}Se (p,n) ^{82}Br, respectively. The agreement of the calculated strength distribution and the corresponding experimental data is quite good. For illustration, four cases are presented in figure 1.

c Also, the summed single β^- and β^+ strengths, denoted conventionally by $\sum B(GT^-)$ and $\sum B(GT^+)$ respectively, were calculated and compared with the available experimental data. These single β decay total strengths quenched with a factor of 0.6 [15], accounting for the polarization effects on the single-β transition operator, ignored in the

Table 1. The calculated summed strengths for the β^- strength associated to the mother nuclei and the summed β^+ strengths for the daughter nuclei, quenched by a factor 0.6, are are compared with the corresponding available data. Experimental data for total $B(GT^-)$ are taken from Refs. [16] ($^{a)}$), [13] ($^{b)}$),[17] ($^{c)}$), [18] ($^{d)}$).

Nucleus	$0.6\sum B(GT)^-$	$\sum [B(GT)^-]_{exp}$
^{48}Ca	14.54	14.4±2.5 $^{a)}$
^{76}Ge	23.037	23.3 $^{b)}$
^{82}Se	25.372	24.6 $^{b)}$
^{96}Zr	29.163	-
^{104} Ru	32.921	-
^{110}Pd	32.932	-
^{128}Te	43.485	40.08 $^{b)}$
^{130}Te	47.432	45.90 $^{b)}$
^{148}Nd	51.74	-
^{150}Nd	54.11	-
^{154}Sm	54.68	-
^{160}Gd	57.93	-
Nucleus	$0.6\sum B(GT)^+$	$\sum [B(GT)^+]_{exp}$
^{48}Ti	3.666	1.9±0.5 $^{a)}$
^{76}Se	1.125	1.45±0.07 $^{c)}$
^{82}Kr	0.079	-
^{96}Mo	2.537	0.29±0.08 $^{d)}$
^{104}Pd	3.990	-
^{110}Cd	7.239	-
^{128}Xe	2.917	-
^{130}Xe	13.040	-
^{148}Sm	1.29	-
^{150}Sm	0.02	-
^{154}Gd	0.54	-
^{160}Dy	0.21	-

Table 2. The strengths $B(GT)$ of the single β^- transitions from the mother nuclei to the intermediate odd-odd nuclei excited in the states of the two components, GTR1 and GTR2, of the GT giant resonance are listed. The experimental [14] (Exp.) and theoretical (Th.) values for the centroid energies are also specified.

Exc. st.	^{100}Tc			
	Ex [MeV]		B	
	Exp.	Th.	Exp.	Th.
G1	13.3	11.16	23.1± 3.8	15.63
G2	8.0	8.05	2.9±0.5	5.87
	^{116}In			
	Ex [MeV]		B	
	Exp.	Th.	Exp.	Th.
G1	14.5	12.37	25.8± 4.1	18.9
G2	8.9	7.87	6.6±1.1	7.2

present paper, are listed in Table 1. Actually, the quenched values are to be compared with the experimental data, since the measured B(GT) strength represents about 60%-70% of the strength corresponding to the ISR.

The experimental value for the summed $B(GT^-)$ of ^{48}Ca is taken from Ref. [16], where from the total strength, which amounts about 15.3±2.2, the contribution of isovector spin monopole states was extracted. The result was obtained with the reaction ^{48}Ca(p,n)^{48}Sc, and corresponds to a large energy excitation interval, from 0 to 30 MeV.

In Ref.[13] the total GT strength, for ^{76}Ge and ^{82}Se, consists of the sum of the strength observed in the peaks plus the estimated contribution from the background. The experimental results correspond to 65 and 59% of the $3(N-Z)$ sum rule. According to Ref. [19], by adding to the GT cross section in discrete states the contribution from the background and that of continuum, the total strength magnitude is much improved to a better obey of the sum rule. We note a good agreement between the results of our calculations for the summed β^- strength and the corresponding experimental data.

The experimental data for the summed $B(GT^+)$ transition of ^{48}Ti, was taken from Ref. [16]. This result was obtained after extracting the contribution of the isovector spin monopole states from the total strength of 2.8±0.3. The reaction ^{48}Ti(n,p)^{48}Sc was used to study the $B(GT^+)$ strength for excitation energies up to 30 MeV. This value for the total strength is larger than that reported by Alford *et al.*, in Ref. [20]

$$\sum B(GT^+) = 1.42 \pm 0.2. \tag{26}$$

where only contribution of states with excitation energies up to 15 MeV are taken into account. This comparison shows that, indeed, the $B(GT)$ strength is sensitive to the magnitude of the considered energy interval. In this context we mention the results obtained through the charge exchange reactions (^{3}He,t) and (d,^{2}He) on ^{48}Ca and ^{48}Ni respectively [21], for $B(GT^-)$ and $B(GT^+)$ with an excitation energy interval $E_x \leq 5$ MeV: 1.43(38), 0.45.

The GT strength from the ^{76}Se(n,p)^{76}As reaction [17] is 1.45 ± 0.07 and corresponds to and excitation energy $E_x \leq 10MeV$. The authors used the multipole decomposition method. In Ref. [22] the $B(GT^+)$ strength was measured in a different reaction, ^{76}Se(d,^{2}He)^{76}As, and different excitation energy interval, $E_x \leq 4MeV$. The result reported is $\sum_{0-4MeV} B(GT^+) = 0.54 \pm 0.1$, which is smaller than that from Ref.[17]. The length of the energy intervals justifies the mentioned differences. We remark that the results for the summed β^+ strength in ^{48}Ti and ^{76}Se are in reasonable good agreement with the corresponding experimental data.

The last strength mentioned in Table 2 refers to the daughter nucleus ^{96}Mo. Through the ^{96}Mo(d,^{2}He)^{96}Nb reaction the strength taken mainly by a single state, placed at 0.69 MeV, was measured. However, from figure 1 we note that, indeed, there is a state at 0.69 Mev which catch a certain β^+ strength, but that strength is smaller than that distributed among the states lying in the energy interval of 1.8 to 7.5 MeV. More complete measurement through a (p,n) reaction on ^{96}Mo and an energy range of 0-10 MeV is necessary in order to make a fair comparison with the results presented here.

The quenched values of the total β^- strength of ^{128,130}Te are compared with the experimental data since the measured $B(GT^-)$ strength, as we already mentioned before, represents about 56% and 59% respectively, of the strength corresponding to the ISR. There are some claims [19] saying that adding the strength carried by the states from the continuum, the total B(GT) strength are corrected up to 90% of the simple sum rule. We remark the good agreement between the calculated and experimental total strength. Note that if we replace the quenching factor by 0.56 for ^{128}Te and by 0.59 for ^{130}Te the results for the total strength would be 40.586 and 46.56 respectively which are closer to the experimental data. Unfortunately for the last four mother and for the last four daughter nuclei, there are no

Table 3. The Gamow-Teller amplitude for the $2\nu\beta\beta$ decay, in units of MeV^{-1}, and the corresponding half life ($T_{1/2}$), in units of yr, are listed. The references list for experimental data is given in Ref. [23, 24].

	M_{GT}	$T_{1/2}$[yr] present	Exp.	Klapdor *et al*
^{48}Ca	0.045	4.72×10^{19}	$4.2\pm1.2) \times 10^{19}$	3.2×10^{19}
^{76}Ge	0.177	0.938×10^{21}	$1.5\pm0.1 \times 10^{21}$	2.61×10^{20}
^{82}Se	0.083	1.293×10^{20}	$1.1^{+0.8}_{-0.3} \times 10^{20}$	0.85×10^{20}
^{96}Zr	0.115	1.59×10^{19}	$(1.4^{+3.5}_{-0.5}) \times 10^{19}$	5.2×10^{17}
^{100}Mo	0.221	8.79×10^{18}	$(8.0\pm0.16) \times 10^{18}$	2.9×10^{18}
^{104}Ru	0.453	2.26×10^{21}	-	1.8×10^{21}
^{110}Pd	0.188	3.11×10^{20}	-	1.2×10^{21}
^{116}Cd	0.160	2.02×10^{19}	$(3.2\pm0.3) \times 10^{19}$	5.1×10^{19}
^{128}Te	0.056	1.43×10^{24}	$(7.2\pm0.3) \times 10^{24}$	1.2×10^{23}
^{130}Te	0.023	1.56×10^{21}	$(1.5\text{-}2.8)\times10^{21}$	1.9×10^{19}
^{148}Nd	0.422	2.00×10^{19}	-	1.19×10^{21}
^{150}Nd	0.042	2.50×10^{19}	$\geq 1.8 \times 10^{19}$	1.66×10^{19}
^{154}Sm	0.303	2.02×10^{21}	-	1.49×10^{22}
^{150}Gd	0.111	1.02×10^{21}	-	2.81×10^{21}

Table 4. The $\log ft$ values characterizing the β^+/EC and β^- processes associated to the intermediate odd-odd nuclei are listed.

Mother			odd-odd		Daughter
^{48}Ca	Th.	8.44	^{48}Sc	4.63	^{48}Ti
^{76}Ge	Th.	4.57	^{76}As	6.13	^{76}Se
^{82}Se	Th.	8.11	^{82}Br	7.18	^{82}Kr
^{96}Zr	Th.	5.67	^{96}Nb	7.00	^{96}Mo
^{100}Mo	Exp.	$4.45^{+0.18}_{-0.30}$	^{100}Tc	4.66	^{100}Ru
	Th.	4.65		4.1	
^{104}Ru	Exp.	4.32	^{104}Rh	4.55	^{104}Pd
	Th.	4.71		6.47	
^{110}Pd	Exp.	4.08	^{110}Ag	4.66	^{110}Cd
	Th.	4.14		6.32	
^{116}Cd	Exp.	$4.45^{+0.18}_{-0.30}$	^{116}In	4.66	^{116}Sn
	Th.	4.65		4.1	
^{128}Te	Exp.	5.049	^{128}	6.061	^{128}Xe
	Th.	5.87		6.06	
^{130}Te	Th.	6.08	^{130}I	5.80	^{130}Xe
^{148}Nd	Th.	6.8	^{148}Pm	7.33	^{148}Sm
^{150}Nd	Th.	5.55	^{150}Pm	8.46	^{150}Sm
^{154}Sm	Th.	5.52	^{154}Eu	5.13	^{154}Gd
^{160}Gd	Th.	5.25	^{160}Tb	4.20	^{160}Dy

data available for the single β^- and single β^+ strengths, respectively.

d) The experimental value [14] of the transition $0^+_i \rightarrow 1^+$ m.e. describing the β^- strength of ^{100}Mo and ^{116}Cd was derived from the reactions output ^{100}Mo(^{3}He,t)^{100}Tc, and ^{116}Cd(^{3}He,t)^{116}In at $\theta_t \approx 0^0$, while the m.e. $1^+ \rightarrow 0^+_f$ was derived from the corresponding experimental log ft value. These quantities were compared with the results of our calculations in Table 2.

e) Transition amplitudes and half lives were calculated for 14 double beta emitters and the results are shown in Table 3.

f) We calculated the $\log ft$ value associated with the single beta transitions of the intermediate odd-odd nucleus towards the daughter and mother nuclei respectively. Results are given in Table 4.

4 Conclusions

Summarizing the results of this paper, one may say that restoring the gauge symmetry from the fully renormalized *pnQRPA* provides a consistent and realistic description of the transition rate and, moreover, the ISR is obeyed.

As shown in this paper, it seems that there is no need to include the pp interaction in the many body treatment of the process. Indeed, in the framework of a *pnQRPA* approach this interaction violates the total number of particles and consequently the gauge projection process makes it ineffective. The proton-neutron correlations in the ground state are however determined by an attractive dipole pairing interaction. The results of our calculations are compared with those obtained by different methods as well as with the available experimental data. Here the strength of the *ph* interaction was taken as given by Eq. (25), while the one for the dipole-pairing interaction was approximately fixed such that one decay branch of the intermediate odd-odd nucleus has the $\log ft$ value close to those known for the given nuclei or for the nuclei belonging to the neighboring region. Small deviations of the predicted and experimental GT resonance centroids suggest that the parameter χ should be fixed by fitting the centroids within the *GRFRpnQRPA*. By contrast to the standard *pnQRPA* models where the strength of the pp interaction is not affecting the position of the GT resonance centroids, here the attractive interaction contributes to the distribution of the β^- strength. Therefore, the two strengths should be fixed at a time by fitting two data, either the GT resonance centroid and the log ft value of one decay of the intermediate odd-odd nuclei or by fixing the log ft values corresponding to the single beta decays of the odd-odd intermediate nucleus.

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