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Microscopic IBM-1 description of collective states in ^{128}Ce

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Abstract. Microscopical quasiparticle theory is developed to calculate the IBM-1 parameters entering the Hamiltonian and $E2$–operator. The theory takes into account the impact of noncollective phonons and predicts alterations of the superconducting properties along the excitation spectrum, whereas the most collective phonon (the image of the IBM d-boson) is practically invariable. Calculations of the energy spectrum and probabilities of $E2$–transitions (without effective nucleon charges) were performed for ^{128}Ce. The results obtained are in satisfactory agreement with experimental data.

1 Introduction

The nucleus ^{128}Ce (Z=58, N=70) can be ascribed to transitional nuclei as $E2$-transitions along its yrast band are enhanced and energies of first excited states lie rather low, though their consequence does not follow strictly the rotation rule (i.e. $E(2_1^+)$ = 0.207 MeV, $E(4_1^+)$ = 0.607 MeV). The phenomenological Interacting Boson Model (IBM) analysis applied to transitional nuclei and, in particular, to ^{128}Ce satisfactory reproduces energy of collective states and probabilities of $E2$–transitions between them. This stimulates constructing a microscopic theory aimed to the calculation of parameters of the IBM Hamiltonian and boson $E2$–operator. In contrast to Iachello, Arima who have calculated parameters for IBM-2 (in this model there are neutron and proton bosons), [1], we deal with IBM-1 (with one type of boson) and exploit a quasiparticle theory and some elements of the Quasiparticle Random Phase Approximation (QRPA).

2 Microscopical theory of IBM-1–parameters

Our approach to the IBM-1 parameter calculations proceeds from the statement that the two-quasiparticle quadrupole phonon (D) of the QRPA type is the microscopical image of the quadrupole d–boson of IBM-1

$$D^+_\mu = \frac{1}{\sqrt{2}} \sum_{1,2;\ \tau=p,n} \{[\psi_{12} a_1^+ a_2^+ + \varphi_{12} a_{\bar{2}} a_{\bar{1}}]^{(2)}_\mu\}_\tau, \qquad (1)$$

where a^+, a are quasiparticle operators, '1' (or '2') is a set of single-particle quantum numbers in a spherically-symmetric mean field.

To construct boson operators we employ arguments by Jolos et al., [2], and Arima, Iachello, [3], and apply the Marumory method for the fermion-boson mapping. In this way we obtain the standard IBM-1 Hamiltonian involving

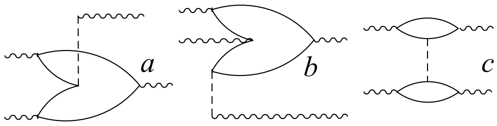

Fig. 1. Diagram illustrations of terms $C_L^{(1)}$ (a), $C_L^{(2)}$ (b), $C_L^{(3)}$ (c) in Eq. (3). Wave and thin lines are phonons and quasiparticles respectively, dotted line is an interaction.

6 parameters: ε_d, k_1, k_2, C_L ($L = 0,\ 2,\ 4$)

$$H_{\mathrm{IBM}} = \varepsilon_d \hat{n}_d + (k_1 d^+ \cdot d^+ ss + k_2 [d^+ d^+]^{(2)} \cdot ds + \mathrm{H.c.}) + \frac{1}{2} \sum_L C_L [d^+ d^+]^{(L)} \cdot [dd]^{(L)}. \qquad (2)$$

In Eq. (2), we make use the representation of $H_{\mathrm{IBM-1}}$ via d- and s-bosons that was given by Arima and Iachello [3]. However, $H_{\mathrm{IBM-1}}$ can be built only from d-bosons [2] that is achieved by replacing $s^+ \rightarrow d^+ \sqrt{\Omega - \hat{n}_d}$. Number Ω is the total amount of d- and s-bosons or the maximum amount of d-bosons in eigenfunctions of $H_{\mathrm{IBM-1}}$. Each parameter is represented via single-particle energies, pairing gaps, Bogoliubov u, v–parameters, phonon amplitudes ψ and φ in Eq. (1) and matrix elements of effective internucleon interactions.

As an example we give in Appendix the explicit form for three components ($C_L^{(1)}$, $C_L^{(2)}$ and $C_L^{(3)}$) of the parameter C_L

$$C_L = C_L^{(1)} + C_L^{(2)} + C_L^{(3)} + \delta C_L. \qquad (3)$$

Diagram illustrations of these three terms are shown in figures 1a, 1b, 1c. Terms $C_L^{(1)}$, $C_L^{(2)}$ and $C_L^{(3)}$ appear if each collective state comprises only D–phonons. The term δC_L and similar terms in other IBM-1 Hamiltonian parameters which are interpreted as their renormalizations are calculated in the lowest perturbation orders. The renormalizations arise by virtue of the connection of collective states (composed by D–phonons) and noncollective ones among which we allow for states with only one noncollective phonon. The latter can have any energy (higher then the D-phonon energy) and any possible multipolarity. Detailed description of our procedure is given in [4].

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Our method of IBM-1 parameter calculation involves three groups of independent variables. The first one is a set of u–, v–parameters, the second includes ψ– and φ–amplitudes and the third group consists of amplitudes of many boson components (we use SU_5 boson basis) which define the eigenfunction of the IBM-1 Hamiltonian for each collective state. The last group of variables is essential for transitional nuclei where as the phenomenological IBM-1 shows, each state is formed by several components with different boson numbers. All these variables are found by minimization of the expectation value of the fermion Hamiltonian mapped onto bosons. Thus, we permit variables u, v and ψ, φ to be dependent on excitation energy. The expectation value of the Hamiltonian, $E(I)$ (I is the state spin), consists of the quasiparticle-phonon vacuum $E_0(I)$ and the energy of the boson part of the Hamiltonian $\langle H_{\rm IBM}(I)\rangle$ that is calculated with the boson eigenfunction

$$E(I) = E_0(I) + \langle H_{\rm IBM}(I)\rangle. \tag{4}$$

The minimization is performed with several additional constraints. A part of them is traditional. This is the normalizations of the whole boson function, u–, v– parameters ($u_j^2 + v_j^2 = 1$ for each single-particle state with angular momentum j) and D– phonon operator, Eq. (1), $(\sum_{1,2,\tau}(\psi_{12}^2 - \varphi_{12}^2)_\tau = 1,\ \ \tau = p,\ n)$. The conservation on the average of neutron and proton numbers is attained by the choice of the respective chemical potentials which alter with excitation energy in our approach. Such specific feature of IBM-1 as the existence of the maximum d–boson number Ω is reflected in the constraint on the expectation value of the commutator $1/5\sum_\mu[D_\mu,\ D_\mu^+]$ mapped onto bosons and calculated for each state. Since we employ the simplest way for mapping D–phonons onto d–boson

$$D_\mu^+ \longrightarrow d_\mu^+\sqrt{1-\frac{\hat{n}_d}{\Omega}} \tag{5}$$

the value of the commutator has to be equal to $1-0.8\Omega^{-1}\langle\hat{n}_d\rangle$, where $\langle\hat{n}_d\rangle$ is the averaged d–boson number in the same state. We adopt that Ω is identical in all states. The value Ω calculated through the commutator can be more than the canonical $\Omega_{\rm can}$ (the half of the valence nucleon (or hole) number) as D–phonons are determined on a wide single-particles basis. In the case of ^{128}Ce $\Omega = 13$ while $\Omega_{\rm can} = 10$.

Equation (5) limits the dimension of the d–boson space. However, to achieve more complete fulfillment of the Pauli principle we implement minimizing the expectation value of H provided that mentioned above independent variables alter in such a way that the averaged quasiparticle number on each j–level does not exceed $j + 1/2$. This requirement gives a series of additional constraints.

Our approach presupposes that the main share of the correlations in the phonon vacuum is caused by the D–mode. However, we extract these correlations from the vacuum and take them into account by the direct diagonalization in boson space. The rest of correlations has to be small. Therefore we impose the condition

$$\sum_{12\tau}(\varphi_{12}^2)_\tau \ll \sum_{12\tau}(\psi_{12}^2)_\tau$$

and adopt in calculations that

$$\sum_{12\tau}(\varphi_{12}^2)_\tau\Big/\sum_{12\tau}(\psi_{12}^2)_\tau = 0.05.$$

The minimization with allowing for all enumerated above additional constraints gives rise to noticeable alteration of the quasiparticle-phonon vacuum energy, $E_0(I)$ in Eq. (4), and IBM Hamiltonian parameters that does not agree with the phenomenological IBM. However, the alteration of $E_0(I)$ is correlated with alterations of the expectation values of the boson number $\hat{n}_d$ and pairing $(d^+\cdot d^+ss + s^+s^+d\cdot d)$ operators which are constituents of $H_{\rm IBM}$, Eq. (2). We can therefore redistribute terms in $E(I)$ and separate a part $\widetilde{E}_0$ that does not vary with excitation energy. The rest of terms, $\langle\widetilde{H}(I)\rangle$, can be arranged so that all alterations will be mainly generated only by expectation values of operators entering $H_{\rm IBM}$ whereas the Hamiltonian parameters remain practically invariable.

$$E(I) = E_0(I) + \langle H_{\rm IBM}(I)\rangle = \widetilde{E}_0 + \langle\widetilde{H}_{\rm IBM}(I)\rangle. \tag{6}$$

In spite of achieved weak dependence of the Hamiltonian parameters on excitation energy, microscopical structure of the quasiparticle-phonon vacuum and D–phonon undergoes variations from state to state. First of all this concerns u–, v–superconducting parameters. Since the phonon number on the average increases with excitation energy in transitional nuclei, this leads to increasing quasiparticle number on valence and adjacent shells. Thus, in many phonon states the Fermi surface turns out to be washed away stronger as compared with the quasiparticle vacuum in which the distribution of particles over levels arises only due to the pairing force. At the end, parameters v decrease for levels occupied in the ground state and increase for upper levels. Nevertheless, owing to the high collectivity of the D–phonon (it is formed by a great number of two-quasiparticle states) variations of u and v do not practically affect amplitudes ψ and φ. Therefore, the D–phonon structure weakly changes with excitation energy.

The interaction with noncollective phonons leading to Hamiltonian parameter renormalizations turns out to be essential in calculations of $E2$–probabilities. The impact of such phonons reveals itself both in the effective charge e^* of the standard IBM $E2$–operator (T_0) and in the appearance of additional terms (δT) in the boson $E2$–operator (T)

$$\widehat{T} = \widehat{T}_0 + \delta\widehat{T};\ \widehat{T}_0 = e^*(s^+d + d^+s + \chi d^+d)_\mu^{(2)}. \tag{7}$$

One of these additional terms contributing noticeably to the $E2$–probability can be written as

$$\delta\widehat{T} = \delta e^*(d_\mu^+\hat{n}_d s + s^+\hat{n}_d d_{\tilde{\mu}}). \tag{8}$$

3 Results of calculations

The numerical values of energies and $E2$–probabilities have been obtained with the Saxon-Woods mean field, factorized particle-hole forces (the strengths are chosen to be close to the Bohr-Mottelson estimations, [5]) and monopole and separable quadrupole-quadrupole particle-particle forces. The single-particle basis envelops practically all bound states. Since we have three systems of nonlinear equations (for u, v, for ψ, φ and for boson amplitudes in the whole boson function of each state) their solutions were found by successive iterations, i.e. the solutions were the result of the triple selfconsistency on each group of the independent variables.

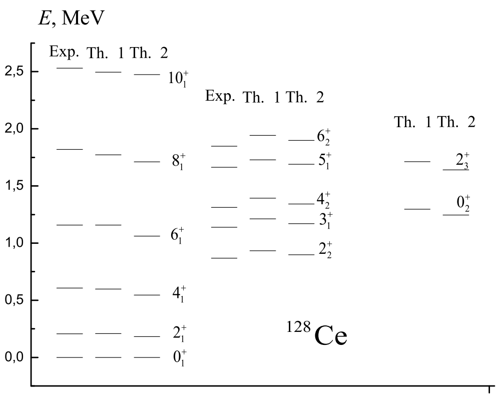

Fig. 2. Experimental and theoretical values of collective state energies in ^{128}Ce.

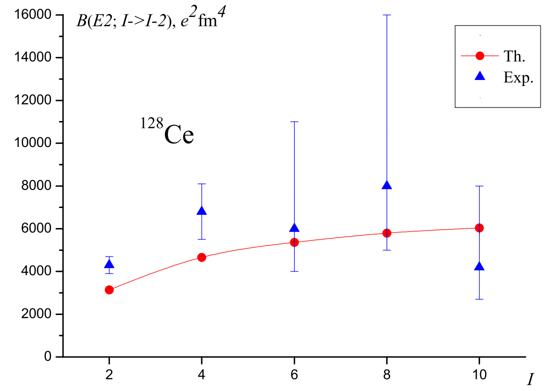

Fig. 3. Experimental [6] and theoretical $B(E2)$-values inside yrast band for ^{128}Ce.

Figure 2 shows that our theoretical energies of collective states (variants Th.1 and Th.2) are in satisfactory agreement with experimental data in ^{128}Ce. The values of Th.1 are calculated with the variable Hamiltonian parameters obtained after the redistribution of terms in $E(I)$, Eq. (6), as it was described above. The values of Th.2 are found with parameters averaged over all states under consideration. The insignificant difference between Th.1 and Th.2 confirms the weak variation of the parameters with state spin and energy.

Calculated reduced probabilities, $B(E2)$, are compared with experimental data in figure 3. As mentioned above, the D–phonon structure practically does not undergo changes along the energy spectrum. Therefore, we consider that in all states there are the same d–bosons. To take into account the differences of parameters in upper and lower states in the first variant of calculations the values of $B(E2)$ are found at first with the upper level parameters and then with the lower level ones. These two numbers are averaged and the deviation from the average is considered as an error ($\leq 1\%$) of our method. In the second variant $B(E2)$ were calculated with parameters averaged over all states. The results of the two variants are nearly identical. Therefore, the results of the second variant are only shown in figure 3. Reasonable agreement in $B(E2)$ between the theory and experiment in figure 3 is obtained with the same initial data (single-particle spectrum, effective interaction strengths) which are used in the energy calculations. We would like to especially stress that $B(E2)$ values are found without using effective nucleon charges i.e. $e_p = e$ and $e_n = 0$.

4 Conclusions

The microscopical quasiparticle theory is suggested allowing to calculate the IBM-1 parameters for transitional nuclei. The theory takes into account effects of the interactions of the collective quadrupole phonons with noncollective ones and predicts alterations in superconductive properties of collective states with the excitation energy, whereas the microscopical two-quasiparticle structure of the collective phonon remains practically invariable. Taking into account the wide single-particle basis and the impact of noncollective phonons we obtain the correct theoretical $E2$–probabilities with the natural values of the proton ($e_p = e$) and neutron ($e_n = 0$) charges in $E2$-operator. The numerical calculations have been performed for ^{128}Ce.

5 Appendix

Formulae for the separate components of parameters C_L given below are obtained with factorized forces described in the text: κ_λ, $G^{(0)}$, $G^{(2)}$ are the strength of the partical–hole and partical–partical interactions respectively. $C_L^{(2)}$ includes the Hamiltonian parameter k_1 (Eq. (2)) which is calculated independently of C_L.

$$C_L^{(1)} = \frac{20}{\aleph_L}\left(\sum_{\tau\eta 12} F_{12\tau}^{(1-\eta)} R_{L\tau}^{(1-\eta)}(1,2)\right),$$

$$C_L^{(2)} = \frac{300}{\aleph_L}(2k_1\Omega)R_L,$$

$$\begin{aligned} C_L^{(3)} = \frac{100}{\aleph_L} \sum_{\tau\eta\lambda 1234} & \Bigg(|G_{\tau\tau}^{(\lambda)}| < 1\|\overline{q}^{(\lambda)}\|4 >_\tau \\ \cdot\ & L_{14}^{(1-\eta)} < 3\|\overline{q}^{(\lambda)}\|2 >_\tau L_{32}^{(1-\eta)} \\ -\ & \sum_{\tau'} |\kappa_{\tau\tau'}^{(\lambda)}| < 1\|q^{(\lambda)}\|4 >_\tau M_{14}^{(0)} \\ \cdot\ & < 3\|q^{(\lambda)}\|2 >_{\tau'} M_{32}^{(0)}\delta_{\eta,1}\Bigg) \sum_{68} \begin{Bmatrix} 2 & 2 & \lambda \\ j_1 & j_4 & j_8 \end{Bmatrix} \\ \cdot\ & \begin{Bmatrix} 2 & 2 & \lambda \\ j_3 & j_2 & j_6 \end{Bmatrix} \Bigg(\begin{Bmatrix} 2 & 2 & \lambda \\ 2 & 2 & L \end{Bmatrix} \\ \cdot\ & \Big[(-1)^\lambda (\psi_{63}\psi_{48}\psi_{81}\psi_{26} + \varphi_{63}\varphi_{48}\varphi_{81}\varphi_{26}) \\ +\ & (\psi_{63}\varphi_{48}\varphi_{81}\psi_{26} + \varphi_{63}\psi_{48}\psi_{81}\varphi_{26}) \Big] \\ +\ & \frac{\delta_{L,\lambda}}{2L+1} 2\psi_{63}\varphi_{48}\varphi_{26}\psi_{81} \Bigg), \end{aligned}$$

$$
\begin{aligned}
R^{(1-\eta)}_{L\tau}(1,2) = & \sum_{34}\Bigg[\begin{Bmatrix} L & 2 & 2 \\ 2 & j_1 & j_2 \\ 2 & j_3 & j_4 \end{Bmatrix} \\
\cdot \quad & (\psi_{34}\psi_{13}\psi_{24} - (-1)^{1-\eta}\varphi_{34}\varphi_{13}\varphi_{24}) \\
+ \quad & 2\begin{Bmatrix} L & j_4 & j_1 \\ j_2 & 2 & 2 \end{Bmatrix}\begin{Bmatrix} L & j_4 & j_1 \\ j_3 & 2 & 2 \end{Bmatrix} \\
\cdot \quad & (\varphi_{24}\psi_{13}\varphi_{34} - (-1)^{1-\eta}\psi_{24}\varphi_{13}\psi_{34})\Bigg]_\tau ,
\end{aligned}
$$

$$
\begin{aligned}
R_L = & \sum_{\tau 1234}\Bigg[\begin{Bmatrix} L & 2 & 2 \\ 2 & j_1 & j_2 \\ 2 & j_3 & j_4 \end{Bmatrix} + \begin{Bmatrix} L & j_4 & j_1 \\ j_3 & 2 & 2 \end{Bmatrix} \\
\cdot & \begin{Bmatrix} L & j_4 & j_1 \\ j_2 & 2 & 2 \end{Bmatrix}\Bigg](\psi_{13}\psi_{12}\psi_{34}\varphi_{24} \\
- & \varphi_{13}\varphi_{12}\varphi_{34}\psi_{24})_\tau ,
\end{aligned}
$$

$$
\begin{aligned}
\aleph_L = & < |DD(D^+D^+)^{(L)}| > \\
= & 2 - 100\sum_{\tau 1234}\begin{Bmatrix} L & 2 & 2 \\ 2 & j_1 & j_2 \\ 2 & j_3 & j_4 \end{Bmatrix} \\
\cdot & (\psi_{12}\psi_{13}\psi_{24}\psi_{34} - \varphi_{12}\varphi_{13}\varphi_{24}\varphi_{34})_\tau ,
\end{aligned}
$$

$$
F^{(1-\eta)}_{12\tau} = (\widetilde{f}^{(1-\eta)}_{12} + f_{12}\delta_{\eta,1})_\tau ,
$$

$$
\begin{aligned}
f_{12\tau} &= \sum_{\tau'}(\kappa_{\tau\tau'}Q_{\tau'}) < 1\|q\|2 >_\tau (-1)^{l_2}L^{(0)}_{12}, \\
\widetilde{f}^{(1-\eta)}_{12\tau} &= G^{(2)}_\tau P^{(1-\eta)}_\tau < 1\|\widetilde{q}\|2 >_\tau (-1)^{l_2}M^{(1-\eta)}_{12}, \\
Q_\tau &= \sum_{12} < 1\|q\|2 > (-1)^{l_2}L^{(0)}_{12}z^{(1)}_{12\tau}, \\
P^{(\eta)}_\tau &= \sum_{12} < 1\|\widetilde{q}\|2 > (-1)^{l_2}M^{(\eta)}_{12}z^{(1-\eta)}_{12\tau}, \\
M^{(\eta)}_{12} &= u_1u_2 + (-1)^{1-\eta}v_1v_2, \\
L^{(\eta)}_{12} &= u_1v_2 + (-1)^{\eta}u_2v_1, \\
z^{(\eta)} &= \psi + (-1)^{\eta}\varphi, \\
q &= \frac{\partial V(r)}{\partial r}Y_{\lambda\mu}(\Omega), \\
V(r) &= V_0(1 + exp(\frac{r - R_0}{\alpha}))^{-1}.
\end{aligned}
$$

References

1. F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge Univ. Press, Cambridge, 1987)
2. F. Donau, D. Janssen, R.V. Jolos, Nucl. Phys. A **224**, 93 (1974)
3. A. Arima and F. Iachello, Phys. Rev. Lett. **35**, 1069 (1975)
4. A.D. Efimov and V.M. Mikhajlov, Bull. RAS., Physics, **74**, 547 (2010); ibid **75**, 890 (2011); ibid **76**, 857 (2012)
5. A. Bohr and B. Mottelson, *Nuclear Structure*, Vol. 2 (Benjamin, New York, Amsterdam, 1974)
6. B. Saha, *et al.*, Phys. Rev. C **70**, 034313 (2008)