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Concluding remarks

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Abstract. Precision asteroseismic observations with the *Kepler* and CoRot satellites enable the internal structure and evolution of pulsating stars to be more exhaustively studied than has hitherto been possible. It is particularly important to study the evolutionary links between white dwarfs and subdwarf-B stars with stars considered to have been their progenitors, those on the Red Giant and Asymptotic Giant Branches. Though observations present challenges for existing stellar evolution and pulsation models, and the data (atomic, molecular and nuclear) on which they are based, excellent prospects for development are identified which will in turn facilitate studies of the Galaxy and extra-solar planets.

1. INTRODUCTION

Ageing low-mass stars ($\leqslant 2M_\odot$) and their evolution were discussed at the 40th Liège International Astrophysics Colloquium (LIAC 40). Objects of interest were red giants, extreme horizontal branch stars and white dwarfs, the meeting being focussed on physical processes affecting structure and what could be learnt from asteroseismology. In particular, the purpose of the meeting was to explore evolutionary links between objects of interest with the view to securing a better understanding of processes such as the helium flash, which defines the end of evolution on the Red Giant Branch (RGB).

High precision asteroseismic observations made with the *Kepler* [1, 2] and CoRot [3] satellites, as well as infrared space observations obtained using Herschel [4] and Spitzer [5, 6], are providing tight constraints on the interior structure of red giants and their evolution. The convening of LIAC 40 from 2012 July 9th–13th to discuss ageing low mass stars was therefore opportune, especially in view of exciting new discoveries anticipated over the next few years following the launch of GAIA [7–9] and the James Webb Space Telescope [10, JWST].

I have summarised here oral and poster presentations made at LIAC 40, and added relevant published topics subsequently discussed at the second part of the 61st Fujihara Seminar, hoping to avoid duplicating what has been written elsewhere in this volume of proceedings. A few other topics such as nuclear reaction rates, not discussed at either meeting, have been included for completeness. My intention was to make an attempt to place our understanding of evolutionary links between red giants, extreme horizontal branch stars and white dwarfs in a context which inspires further development of the field.

2. MAIN SEQUENCE EVOLUTION

2.1 Lower main sequence

Chabrier & Baraffe [11] show non-grey effects in stellar atmospheres to have significant consequences for the evolution of low mass ($0.07 \leqslant M/M_\odot \leqslant 0.8$) stars near the Main Sequence (MS) over a

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range of metallicities $-2.0 \leqslant [M/H] \leqslant 0.0$. Molecular opacities are increasingly important in stellar atmospheres and envelopes as stars become cooler [12–14], and efforts to calculate necessary molecular data [15] are underway. It is clear that the absence of molecular data currently limits the reliability of low-mass MS stellar evolution calculations.

Detached double-lined eclipsing binaries provide a direct determination of accurate fundamental stellar parameters (mass, radius and luminosity). Theoretical stellar models compare favourably with observed properties for MS stars more massive than the Sun but lower mass MS stars have radii slightly larger, and effective temperatures slightly lower, than those predicted by models. An excellent example is provided by IM Vir [16]. While Morales *et al.* [17] show spots concentrated near stellar poles yield radii and effective temperatures for low-mass MS stars from the analysis of eclipsing binary light curves which are in better agreement with theoretical models, those models are limited.

2.2 Solar abundance problem

Images of the Sun [18, fig. 1] on 5 Mm squares show a time-sequence of intensity and velocity variations known as granulation. The existence of granulation shows there is no uniform temperature at any depth in the solar atmosphere, and it is evidently not static. Abundances in the solar atmosphere obtained by modelling it in one dimension (1D), as static layers of uniform temperature, can at best be approximate; they depend on the choice of spectral lines and are inconsistent with meteoritic abundances.

Asplund *et al.* [19] discuss revisions to solar abundances obtained using realistic three-dimensional (3D) models of the solar photosphere which include a proper treatment of radiation hydrodynamics. Difficulties listed above then disappear: all spectral lines give consistent abundances in agreement with those found in meteorites, without obvious line or species dependent systematic errors. As a consequence it is recommended by Grevesse *et al.* [20], and elsewhere in this volume, that solar abundances be adopted from [19] and the use of older 1D abundances [21] be discontinued.

Solar models can be constructed with 1D or 3D abundances and evolved to be consistent with the observed mass, radius and luminosity of the Sun. Both predict a sound speed and density dependence on distance from the centre of the Sun which may be compared with the same quantities inferred using helioseismology. In the context of the comparison [22, fig. 11], 3D abundances clearly result in a solar model inferior to the one obtained with 1D abundances. Further revisions to envelope opacities is one of several plausible explanations circulating in the literature, but the crucial point is to appreciate that models of ageing low-mass stars are compromised by uncertain MS star models from which they are evolved.

2.3 Convective cores and main sequence turn-off

During MS evolution, stars with masses higher than $\sim 1.1 M_{\odot}$ develop a convective core which may either expand or contract; both cases leave behind a non-uniform chemical profile leading to higher opacities outside the stellar core, resulting in semi-convection. If semi-convection is maintained throughout the MS evolution, any growth of the homogeneously mixed core will be restricted; as Silva Aguirre *et al.* [23] note, this has important consequences, especially for computed evolution tracks near the MS Turn-Off (TO). Semi-convection is one factor affecting TO morphology. Although the open cluster M67 has solar metallicity and a colour-magnitude diagram (CMD) which shows a clear hook-like feature near the TO, a sign that these stars have convective cores, Magic *et al.* [24] conclude that the M67 CMD may only be used to independently constrain solar abundances once other factors affecting TO morphology are determined.

Asterseismology of solar-like stars obtained with the *Kepler* satellite enable progress to be made. For example, Silva Aguirre *et al.* [25] construct a one solar-mass evolutionary sequence using the large frequency separation, frequency of maximum oscillation power and scaling from solar values (without the need for further information from stellar models). Mathur *et al.* [26] complete a uniform asteroseismic analysis of twenty-two solar-type stars observed by *Kepler* for one month only; in each case they determine the radial extent of the convection zone which is important for the characterisation of dynamo processes. Furthermore, while solar-type stars are generally slow rotators, García *et al.* [27] identify fast rotating solar-like stars in the *Kepler* field and mention the prospect of studying the dependence of their evolution on rotation.

2.4 Helioseismic challenge to solar convection models

The need to explain differential rotation observed in the Sun (and likely to be realised in many other stars) as well as the Sun's evolving magnetism with its cyclic behaviour, provide a strong motivation for modelling the highly turbulent solar convection zone. Miesch *et al.* [28] report 3D simulations of compressible convection carried out in rotating spherical shells extending from the base of the convection zone to within 15 Mm of the photosphere. However, Hanasoge *et al.* [29] analyse observations of the wavefield in the solar photosphere using techniques of time-distance helioseismology to image flows in the solar interior; within the wavenumber band $\ell < 60$, they find convective velocities to be 20–100 times weaker than predicted [28] at depths of 0.92, 0.95 and $0.96 R_{\odot}$. Miesch *et al.* [30] argue that if the [29] results are confirmed, there would be important implications for the maintenance of mean flows in the Sun as inferred from helioseismology. Gizon & Birch [31] provide further discussion, noting that viable theories of convection should be applicable to other stars and that suitable observations for testing these are now becoming available.

3. RED GIANTS

3.1 Red and asymptotic giant branch evolution

As presented in more detail elsewhere in this volume, ageing low mass stars evolve up the RGB [32] and in the absence of an interaction with a companion (if in a binary system) they will reach the RGB tip. A helium flash occurs at the RGB tip lifting the core degeneracy, the energy for which comes from the potential energy (collapse) of the red giant envelope. Helium burning begins in the core with hydrogen shell burning extinguished, or continuing to burn at a reduced rate, depending on what fraction of the hydrogen shell was lost at or before the star reached the RGB tip.

Evolved stars at the RGB tip undergo an initiating off-centre flash in their helium cores (having masses $\approx 0.48 M_{\odot}$). Bildsten *et al.* [33] use the Modules for Experiments in Stellar Astrophysics (MESA) code [34] to follow the evolution of a star through the helium flash; this evolution stage is found to last about 2Myr and they identify acoustic signatures of stars undergoing the core helium flash which should be identifiable in a few stars observed with the *Kepler* spacecraft. The core helium flash has yet to be observationally probed and it is clearly important to search in red giants being observed by *Kepler* for acoustic signatures Bildsten *et al.* identify.

Helium shell burning begins once helium is exhausted in the core and the star begins to evolve up the Asymptotic Giant Branch [35, AGB]. Intense mass-loss on the AGB is understood to lead to a planetary nebula (PN), although it is not clear that all AGB stars become PN. It is therefore important to determine the temporal behaviour of the mass-loss rate on the AGB and how it depends on stellar mass, metallicity and pulsational behaviour. While pulsation and the formation of dust in an AGB atmosphere appear to be responsible for the mass-loss, the process is not understood in detail.

3.2 Asteroseismic observations of red Giants

A direct probe of red giant core regions is provided by mixed modes which propagate as pressure waves in the convective envelope, and gravity waves in the radiative core. The increase in core density expected as a star evolves on the RGB results in a decreased gravity-mode period spacing. When helium ignition occurs in the core, the energy released should result in a dramatic decrease in density and an increased gravity-mode period spacing.

Red clump (RC) stars, which are core helium burning, have radii and luminosities essentially indistinguishable from hydrogen shell burning stars approaching the RGB tip. RC and RGB stars near the RGB tip therefore have similar envelope structures and the pressure mode large frequency spacing is essentially unaffected by the onset of core helium burning. Mosser *et al.* [36] use the changed gravity-mode period spacing, and unchanged pressure-mode large frequency spacing, to distinguish RC and RGB stars (near the RGB tip) observed by the *Kepler* satellite; this is one of the many significant successes of the mission.

As stars evolve up the RGB, an increasing fraction of the stellar mass becomes concentrated in the contracting helium core which, by conservation of angular momentum, would then be expected to rotate faster. Nonradial modes identified in *Kepler* observations of KIC 7341231 are split by rotation and Deheuvels *et al.* [37] deduce the internal rotation profile of this star on the lower RGB. As might be expected, the core rotation speed exceeds the envelope rotation speed by roughly a factor of four.

Ensemble asteroseismology by Mosser *et al.* [38] shows mean core rotation to slow down significantly during the last stages of the RGB phase, as angular momentum is transferred from the core to the envelope. Despite core angular momentum loss at the RGB tip, RC star cores rotate more rapidly than expected from their radii. Marques *et al.* [39] demonstrate that if horizontal turbulent viscosity is responsible for angular momentum transfer, it is seriously underestimated; the alternative they propose is that angular momentum is transferred by an unknown process. In either case, current stellar evolution models fail to reproduce RGB star core rotation speeds.

3.3 Red giant branch mixing length

The location of the theoretical RGB in the Hertzsprung-Russell Diagram at a chosen metallicity is uncertain because convection is modelled using the mixing length theory; the mixing length parameter (α) is adjusted so that the theoretical and observed RGB coincide. But as Harris & Lynas-Gray [40] and VandenBerg *et al.* [41] note, the theoretical RGB at a chosen α also depends on whether the RGB stellar evolution includes non-grey radiative transfer in the red giant atmosphere. The choice of α therefore also depends on how radiation transport was calculated during the RGB stellar evolution calculation. A non-grey calculation is itself limited by the available molecular data. As a consequence, the structure of a red giant at the RGB tip, without the aid of asteroseismology, can at best only be known approximately.

4. HOT SUBDWARFS

4.1 Formation channels for subdwarf-B stars

Stellar evolution on the horizontal and extreme horizontal branches is determined by progenitor evolution on the RGB. Han *el al.* [42, 43] discuss the loss of red giant envelopes at or just before the helium-flash at the RGB-tip to form extreme horizontal branch or subdwarf-B [44, sdB] stars, either as single stars or as a consequence of binary interaction. The merger of two helium-core white dwarfs is also proposed [42, 43] as a channel by which single sdB stars may be formed. Modelling of the ultraviolet upturn in elliptical galaxies [45] is a notable success of the [42, 43] models.

4.2 Asteroseismic analyses

Asteroseismic studies of pulsating sdB stars provide estimates of the stellar mass, as well as the mass of the thin hydrogen envelope. Mass estimates are also available for hot subdwarfs found in eclipsing binary systems. Fontaine *et al.* [46] use the available twenty-two sdB mass determinations to construct an empirical mass distribution which they compare with predictions [43, 47]. Good agreement with both theoretical predictions is found [46, fig. 6] although Fontaine *et al.* stress that there are currently insufficient mass determinations to allow the double star scenario of Han *et al.* [42, 43] to be adopted in favour of the single star scenario of Dorman *et al.* [47].

4.3 Rotation

A further test of proposed sdB star formation channels is obtained [48] through the study of projected rotational velocities obtained for 105 sdB stars. The sample includes sdB stars which appear to be single as well as those in wide binaries where tidal effects become negligible. As helium cores of stars on the lower RGB are thought to spin faster as evolution up the RGB proceeds, resulting sdB stars might also be expected to be fast rotators. But all sdB stars in the sample are slow rotators having $v_{rot} \sin i \leqslant 10\,\mathrm{km/s}$. The implication must be that before envelope loss, a red giant helium core transfers much of its angular momentum to the envelope as Mosser *et al.* [38] find using asteroseismology. Some of the angular momentum of a red giant is then lost with the envelope, or transferred to an orbit for red giants in binary systems.

Assuming rigid rotation, the same inclination angle for the rotation axis and the same mass of $\approx 0.5 M_{\odot}$, rotation angular momentum scales as $\Omega = v_{rot} \sin i \times g^{-1/2}$. Geier & Heber [48, fig. 7] plot $\log_{10} \Omega$ against $\log_{10} T_{\rm eff}$ and find a linear dependence with some scatter attributable to necessary assumptions stated above. Along with the 105 sdB stars, Geier & Heber [48, fig. 7] include data for blue horizontal branch (BHB) stars; these fall on the same linear relation as the sdB stars.

As Geier & Heber [48] point out, single sdB stars formed through the helium white dwarf merger channel are unlikely to have the same $\log \Omega - \log T_{\rm eff}$ distribution as sdB and HB stars formed as a result of red giant envelope loss; their conclusion is that theory [42, 43] overestimates the fraction of sdB stars formed from helium white dwarf mergers. Clausen & Wade [49] propose singleton sdB stars may result from the merger of an helium white dwarf and a very low mass hydrogen burning star. Injecting an already formed helium core into a low-mass star yields an object effectively at an advanced stage of stellar evolution; it can evolve up the RGB and undergo a helium flash, expel the envelope and lead to a sdB star. Provided the orbital angular momentum is lost with the envelope, the [49] channel would result in slowly rotating sdB stars consistent with those which Geier & Heber [48] analyse.

4.4 Pulsation amplitude changes

Fourier transforms, and therefore the standard power spectrum analysis technique, are only strictly applicable to signals comprising a sum of sinusoids having (by definition) constant frequencies and amplitudes. Pulsation amplitude changes in sdB stars, in some cases on a timescale of days or even hours, therefore complicate asteroseismic studies. Kilkenny [50] presents a sample of interesting cases and notes that every pulsating sdB star photometrically monitored more than once, shows amplitude variations in one or more frequencies.

In the case of the pulsating subdwarf-O (sdO) star SDSS J160043.6+074802.9, Lynas-Gray *et al.* [51] note the presence of the first harmonic of the dominant pulsation frequency (f_1) in the power spectrum. The f_1 frequency is therefore non-sinusoidal, indicative of non-linear effects; pulsation amplitude changes in SDSS J160043.6+074802.9 could [51] then be a consequence of non-linear mode coupling although beating between closely spaced (and unresolved) frequencies, as Kilkenny [50] proposes for sdB pulsators, remains a possibility. Van Hoolst [52] considers resonances between two stellar oscillation modes of nearly equal frequency and this may prove useful for modelling pulsation amplitude variations in sdB stars.

Baade's [53] pulsation test shows [54] that the sdB star CS 1246 is a radial pulsator; given its $T_{\rm eff}$, $\log g$ and period, tables by Charpinet *et al.* [55] do not exclude pulsation in the fundamental mode. The secular decrease in the CS 1246 pulsation amplitude which Barlow *et al.* [56] report is understood by them to be a consequence of structural changes as a consequence of stellar evolution.

If CS 1246 is confirmed as a fundamental mode radial pulsator, beating between closely spaced unresolved frequencies or non-linear mode coupling may be safely excluded as explanations of pulsation amplitude changes in this case. Stellar structure changes induced by stellar evolution would then need to be considered as contributing to pulsation amplitude changes in sdB nonradial pulsators which have many pulsation frequencies.

5. WHITE DWARFS

5.1 Progenitors

Althaus *et al.* [57] and Fontaine *et al.* [58] review evolutionary and pulsational properties of white dwarfs. The immediate progenitors of DA white dwarfs with C/O cores are thought to be PN central stars, mass-loss which removes almost all the hydrogen envelope having occurred earlier on the AGB. Core helium burning must have occurred before the AGB stage to form the C/O core and these white dwarfs therefore have masses of $\sim 0.5M_{\odot}$ or more.

A significant fraction of white dwarfs are hydrogen deficient non-DA white dwarfs; these are understood to form as a result of a very late thermal pulse during the early stages of white dwarf evolution when hydrogen burning has not quite ceased. The helium burning shell drives the convection zone outwards; when it reaches the hydrogen envelope this is violently burnt and the star is forced to evolve rapidly back towards the AGB, becoming a PG 1159 star before rejoining a white dwarf cooling track.

A small proportion of white dwarfs ($\sim$10%) are clearly the remnant cores of red giants from the RGB; they have masses which are too low for helium burning to have ever started. Such low mass DA white dwarfs have helium cores and must have formed as a result of a binary interaction on the RGB. Before it could have arrived at the RGB tip; the red giant progenitor loses enough of its hydrogen envelope to almost extinguish the hydrogen burning shell and it then contracts, evolving to higher effective temperatures before joining a white dwarf cooling track.

Only a very small proportion of C/O white dwarfs have sdB stars as progenitors. Once burning has exhausted helium in the core, a sdB star burns helium in a shell around the C/O core and the star passes through the sdO star domain before joining a white dwarf cooling track. With a hydrogen envelope which has insufficient mass to support hydrogen shell burning, the star cannot ascend the AGB.

5.2 Cooling

Fontaine *et al.* [58, fig. 2] give the main cooling stages for H/He/C stratified DA models having masses between $0.4M_{\odot}$ and $1.2M_{\odot}$. Immediately following the PN phase, white dwarf interiors are hot enough to produce large numbers of neutrinos through electroweak interactions, and neutrino luminosities may exceed photon luminosities by two orders of magnitude. Neutrino loses therefore dominate the early stages of white dwarf cooling.

At lower luminosities, the base of the superficial hydrogen convection zone reaches the upper boundary of the degenerate core. Convection now determines the rate of cooling, since it efficiently transports energy through the outer insulating layer. The onset of crystallisation at the white dwarf centre releases energy and reduces the cooling rate; this is eventually more than offset by the specific heat falling to zero for the solid degenerate matter which loses all ability to store thermal energy. A relatively rapid cooling phase often referred to as “Debye Cooling” then follows.

An increasing number of DA and DB white dwarfs observed with Spitzer are found to have infrared excesses due to circumstellar disks [59]. At the same time, high resolution spectroscopy by Dufour *et al.* [60] for example reveals the presence of accreted heavy elements. Deal *et al.* [61] show that when thermohaline mixing is taken into account, inferred accretion rates are not only consistent with photospheric abundances but accreted material is mixed in a much deeper part of the star than the convection zone and could significantly modify the opacity and cooling time.

Given the physics necessary for constructing white dwarf models, white dwarf cooling rate calculations are limited by an uncertain knowledge of how much thermal energy is stored in the interior and how rapidly that thermal energy can be transferred to the surface. Both are dependent on the progenitor evolution. Specifically, as most white dwarfs have AGB progenitors; uncertainties in AGB evolution discussed above translate directly into an uncertain chemical stratification in white dwarfs.

5.3 Pulsation and rotation

Four distinct types of pulsating white dwarfs are known as Fontaine *et al.* [58, fig. 5] discuss. By period matching it is possible, to some extent, to compensate for a limited understanding of AGB evolution and determine the chemical stratification in pulsating white dwarfs. Period change rates would establish the core composition. Knowing the core composition and envelope stratification allows a white dwarf cooling time to be calculated, and it becomes a useful cosmochronometer.

Montgomery [62] assumes pulsating white dwarfs have sinusoidal flux perturbations below the convection zone. The non-linear response of the convection zone results in the pulsating white dwarf having a non-sinusoidal light curve. Montgomery *et al.* [63] extend the analysis in [62] to multiperiodic stars and develop a non-linear technique for fitting non-sinusoidal light curves. An empirical determination of convection parameters is then possible as Provencal *et al.* [64] demonstrate and future applications to pulsating white dwarfs should enable changes to white dwarf convection zones to be mapped as cooling proceeds. The technique could usefully be applied to other ageing low mass stars such as RR Lyraes [62].

Charpinet *et al.* [65] use period matching and rotational splitting to establish that the post-AGB white dwarf PG 1159–035 rotates as a solid body with a remarkably low rotation rate. As with sdB stars discussed above, considerable progenitor angular momentum is lost when it loses its envelope and the AGB and RGB would appear no different in this regard. RC stars which Mosser *et al.* [38] study are found to have cores which rotate faster than expected from their radii and this suggests a further core to envelope transfer of angular momentum during the AGB evolution.

5.4 Hot DQ stars

Dufour *et al.* [66] present an abundance analysis of SDSS J133710.19+002643.7 and find it to be a white dwarf, the first of the so called Hot DQ Stars, with a largely carbon atmosphere. Hydrogen and helium are present in the atmosphere of Hot DQ Stars ($T_{\rm eff} \approx 20000$K) as a minor species. Channels by which Hot DQ Stars may be formed have yet to be unambiguously identified.

5.5 New white dwarf catalogue

Kleinman *et al.* [67] catalogue 19712 white dwarfs, increasing the known number of such objects by an order of magnitude. There are now over 100 ZZ Ceti, V777 Her and GW Vir confirmed pulsators. The larger sample allows class properties and unique aspects of individual pulsators to be studied in more detail. It particular, it is now possible extend the ZZ Ceti instability strip to lower masses.

Of the 923 stars classified [67] as DB, nine have $T_{\rm eff} \geqslant 45000K$, thirty have $30000K \leqslant T_{\rm eff} \leqslant 45000K$ and 231 have $20000K \leqslant T_{\rm eff} \leqslant 30000K$. The Kleinman *et al.* numbers suggest a decrease in numbers, although not a gap, of DBs in $30000K \leqslant T_{\rm eff} \leqslant 45000K$ in relation to the hotter DO range. Eisenstein *et al.* [68] reach a similar conclusion, based on fewer stars, and deduce that some as yet unexplained atmospheric transformation takes place in roughly 10% of DA white dwarfs as they cool from $30000K$ to $20000K$.

6. FUTURE PROSPECTS

6.1 Atomic and molecular opacities

A model solar spectrum computed in 3D with a proper treatment of hydrodynamics, abundances by Asplund *et al.* [19] and high quality oscillator strengths [69–89] results in model and observed spectra in excellent agreement. The correction for the solar gravitational redshift is the only necessary adjustment. The case for using new abundances [19] as fiducial "solar abundances" is therefore almost overwhelming. Discrepant sound speed and density profiles inferred from helioseismology when new abundances are used has prompted [90] a reexamination of Opacity Project [91, OP] and OPAL [92] opacities on which contemporary stellar structure, pulsation and evolution calculations depend.

Pradhan & Nahar [93] note that to recover a good agreement between solar models and the predictions of helioseismology, an opacity increase of at least 10% would be needed; this is more than any discrepancy between OPAL and OP. Using a much larger wavefunction expansion than used for OP work, it is found [93] that the Rosseland mean opacity is occasionally underestimated by 50% in OP opacities. It is therefore clear that a further revision to opacities is needed although this will not necessarily resolve the solar abundance problem.

High energy laser and z-pinch facilities, discussed elsewhere in this volume and in [90], produce plasma at conditions comparable with those found in stellar envelopes and a direct measurement of monochromatic opacity may be made. Measurements reported by Turck-Chièze *et al.* [90] for chromium, iron and nickel are compared with theoretical calculations. As a result, it is clear a new theoretical calculation is needed as Pradhan & Nahar [93] also conclude.

In the case of white dwarfs, Fontaine *et al.* [58] comment on the need to extrapolate OPAL opacities at high densities. Iglesias & Rogers [92] point out that the high densities and temperatures not included in their tables correspond to degenerate plasmas, which are beyond the range of validity of their calculations. Avoiding exptrapolation by extending OPAL [92] opacity tables to any high density region not currently included, where the plasma is not degenerate, would remove a critical uncertainty in calculating models of white dwarfs in particular.

Molecular opacities are needed for modelling cool star atmospheres and envelopes but following the discovery of planets orbiting nearby stars [94], and the need to model their atmospheres and evolution along with those of brown dwarfs, the lack of suitable molecular line strength data with which to calculate molecular opacities has become acute. For any molecular species, accurate molecular opacities would require good quality line strengths for about 10^9 lines. Only an insignificant fraction of the required line strengths could be measured in a laboratory. Essentially all required data have to be calculated and the ExoMol Project [15] has been established for this purpose.

6.2 Equation of state

Baturin [95] summarises the SAHA-S equation-of-state (EOS) and its successful application to solar modelling though for MS stars having masses $\geqslant 0.1 M_{\odot}$, the definitive source of EOS tables are those by Rogers & Nayfonov [96] and constitute what has become known as the Updated OPAL Equation-of-State. Users of any EOS need to be aware that the treatment of molecules is necessarily incomplete, as is made clear in the previous section. But for evolved stars, where central temperatures exceed 10^8K the EOS by Pols *et al.* [97] would appear to be a reasonable choice. In the case of white dwarfs Fontaine *et al.* [58] note that the weakest link in structure and cooling calculations is the absence of an adequate envelope EOS for a partially ionised, partially degenerate and non-ideal plasmas.

6.3 Nuclear reaction rates

Accurate nuclear reaction rates are an essential component of stellar evolution calculations. As with atomic and molecular data, the astrophysicist is dependent on numerical calculations to obtain the necessary rates. Guzik [98] and Wiescher *et al.* [99] summarise available sources of nuclear reaction rates and existing uncertainties. In addition Goriely *et al.* [100] point out that reaction rates for medium or heavy nuclei, used in astrophysical applications, are traditionally calculated using a statistical model [101] with simplified methods to estimate capture reaction cross-sections which in some cases have never been tested. Reaction rate revisions [102] result in considerable modifications to the composition of post-MS stars with, for example, the whole population of grains produced in AGB stars (and having $^{18}O/^{16}O \leqslant 0.0015$) now being explained.

6.4 Atomic diffusion, mixing and element abundances

Element segregation in the outer layers of stars is reviewed by Vauclair & Vauclair [103]. Depth dependent radiative acceleration for specific elements may, in the case of iron and nickel, lead to their accumulation or depletion in different layers and drive pulsation through the $\kappa-$mechanism [104]. Heavy element accummulation induced by atomic diffusion [105] leads to an inverse molecular weight gradient, resulting in thermohaline convection [106] which moderates the atomic diffusion process. Thermohaline convection and its consequences therefore need including in atomic diffusion calculations; these processes both compete with rotation-induced mixing, which is more complex than previously thought as a consequence of mutual interaction. Vauclair & Théado [106] suggest that the neglect of thermohaline convection could be responsible for pure atomic diffusion predicting lower photospheric abundances than those observed.

6.5 Magnetic fields

Magnetic fields are known to impede the flow of energy by convection, and so alter stellar evolution. Lithium depletion is also convection dependent and affected by the presence of a magnetic field. There are often significant differences among ages of individual stars estimated from lithium and MS fitting, when the presence of magnetic fields is neglected. Hertzsprung-Russell Diagram and lithium depletion isochrones which MacDonald & Mullan [107, fig. 1–2] calculate, with convection impeded by a magnetic field, leads to consistent ages for individual stars.

6.6 Development of non-linear theory for nonradial pulsation

High quality power spectra are now routinely obtained for pulsating stars by the *Kepler* [1, 2] and CoRot [3] satellites. Linear theory for nonradial pulsators provides accurate predictions of stellar pulsation frequencies. Non-linear theory is needed for amplitude prediction in the general case. However, for pulsating red giant (mostly RC) and MS stars observed by CoRoT, Baudin *et al.* [108, 109] use heights (amplitudes) and widths of peaks in power spectra to derive scaling relations indirectly relating these quantities to the luminosity-to-mass ratio and effective temperature.

Christy [110] introduces non-linear hydrodynamical calculations for radial pulsation which may be applied to cepheids and RR Lyraes. Deupree [111] carries out numerical calculations of nonlinear axisymmetric oscillations in the adiabatic regime, the non-adiabatic case being tackled by Deupree [112]. Däppen & Perdang [113] use a variational principle of fluid mechanics to extend the Hamiltonian particle formalism for non-linear radial oscillations to the nonradial case; the oscillations are assumed to be adiabatic. A further development is provided by Buchler & Goupil [114, 115] who present a non-adiabatic perturbation formalism, which reduces the study of nonlinear pulsations of a given stellar model to the solution of a set of coupled ordinary differential (amplitude) equations.

As a method of calculating the amplitudes of oscillation in rotating stars, Lee [116] develops a weakly non-linear theory of oscillation in which coupling coefficients between low frequency modes are essential. Non-linear effects are identified in some pulsating hot subdwarfs, for example the sdO pulsator SDSS J160043.6+074802.9 [51, 117–119], where the first harmonic of the highest amplitude mode is also present in the power spectrum. Accordingly, attempts to apply Lee's [116] method to pulsating hot subdwarfs could provide useful estimates of coupling coefficients between pulsation modes.

Reviews of modelling and interpreting stellar pulsation are given by Gautschy & Saio [120, 121] and Shibahashi [122, 123]. Lee [124] notes that a non-linear hydrodynamical calculation of nonradial modes would require enormous computational resources, particularly for high radial order low frequency modes in rotating stars. In the absence of necessary computational resources, amplitude equations [114, 115] appear to be the method of choice for interpreting pulsation amplitudes in nonradial pulsators.

6.7 Binary star evolution

Binaries in which the two component stars are well separated may be evolved as two isolated stars as neither interacts with its companion to any significant degree. If the two stars in a binary system are close enough to interact, there are consequences for the evolution and appearance of the stars and their orbits. Hurley *et al.* [125] summarise the important interactions and provide a fast algorithm for accounting for them in the context of single star evolution.

As already noted, non-grey radiative transfer in the atmospheres of red giants and low mass MS stars has consequences for their evolution [11, 40, 41]. In close binaries, one component irradiates the atmosphere of the other and this cannot be neglected if the treatment is to be realistic [126]. The inevitable conclusion must be that a low mass star irradiated in a close binary system will have a different evolution from an identical star which is a singleton; as far as I am aware, irradiated atmospheres are not taken into account in modelling binary star evolution.

6.8 Stellar evolution in three dimensions

Mixing-Length Theory [127, MLT] is the most commonly used approximation for convection in 1D models and it has served the community well for more than half a century. But the need to choose a suitable α is a severe limitation, as discussed above. As Tanner *et al.* [128] point out, having α as a free parameter in MLT means that the stellar radius is also in effect a free parameter. MLT accurately models stellar structure where convection is efficient and the temperature gradient is almost adiabatic, but near the surface in the superadiabatic layer (SAL) MLT breaks down because energy transport becomes radiative. In order to obtain a more realistic stratification in the SAL where convection is inefficient, Tanner *et al.* investigate the use of 3D radiation hydrodynamic calculations.

Meridional circulation and differential rotation in stars actively contribute to chemical mixing by forcing turbulent motions. Prat & Lignières [129] discuss turbulent mixing in stellar radiative zones, pointing out that turbulent motions involved in rotating stars are full 3D motions, and cannot be easily resolved in models having fewer dimensions. Kitiashvili *et al.* [130] use numerical radiation transfer simulations in 3D to study convective and oscillation properties of MS stars observed by *Kepler* and having masses of $1M_{\odot}$ and above; they find supersonic granular-type convection significantly larger than solar granulation, with strong overshooting plumes penetrating the stable radiative zone, potentially affecting oscillation properties.

Guzik [98] presents a comprehensive summary of two dimensional and 3D stellar interior models and of what may be achieved through their use. A further success of 3D simulations may be noted. Guided by their 3D hydrodynamic helium-shell flash convection simulations in spherical geometry, Herwig *et al.* [131] assume that the ingestion process of hydrogen into the helium-shell convection zone leads to it being split into the original (driven by helium burning) and a new one (driven by the rapid burning of ingested hydrogen). Neutron densities obtained are high enough to reproduce an overproduction of rubidium, strontium and yttrium, by about two orders of magnitude more than the overproduction of barium and lanthanum; this is a peculiar nucleosynthesis which is impossible to obtain with mixing predictions made by 1D stellar models.

6.9 Application to Galactic structure and evolution

Elsewhere in this volume, Chiappini and Miglio *et al.* [132] show how ageing low mass stars can be used to constrain the Milky Way chemical evolution once precise ages and distances are available. Excellent distance estimates may be expected once GAIA [7–9] parallax measurements are published. Luminosities should follow from distance estimates and the source of age uncertainty would then be the stellar evolution tracks and stellar isochrones.

7. SUMMARY

In this paper, I have presented a threadbare summary of the evolution of low-mass ageing stars as I understand it. I have only mentioned PN and post-AGB stars [133, 134] in passing as they were not discussed in detail at LIAC 40. Difficulties and uncertainties have been identified with modelling evolution from the MS to white dwarf stages. I have presented the challenges, as I believe them to be, confident that present and future generations of astronomers will overcome them and manage to advance the subject.

There is much to inspire progress, above all curiosity aroused by the discovery of planets outside the Solar System and the need to learn about their stellar hosts. New molecular data from ExoMol should stimulate improvements to model stellar atmosphere, synthetic stellar spectrum, stellar structure, stellar pulsation and stellar evolution calculations. The JWST should provide higher quality observations of faint cool objects, which in the LIAC 40 context are the low effective temperature white dwarfs. GAIA may be expected to provide accurate luminosities for 10^6 or more stars, constraining age estimates from stellar evolution models. Moreover, GAIA has the potential to astrometrically identify companions (planetary and stellar) of stars that have hitherto been regarded as single objects.

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