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Lagrangian velocity distributions in thermal counterflow of superfluid ^{4}He

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Abstract. Quantum turbulence in thermal counterflow of superfluid ^{4}He is visualized and studied at length scales comparable to the average distance between quantized vortices. The lagrangian velocities of micrometer-sized deuterium particles are obtained in a planar section of the flow field by using the particle tracking velocimetry technique. It is shown that the lagrangian velocity normalized distributions are strongly non-gaussian, with power-law tails, in contrast to the nearly gaussian distributions obtained in classical turbulent flows. The average distance between quantized vortices can therefore be seen as the length scale indicating the transition between classical-like and quantum behavior in thermal counterflow of superfluid ^{4}He.

1 Introduction

Quantum turbulence, a fast developing field of research combining low temperature physics and fluid dynamics, can loosely be defined as the most general form of motion of quantum fluids displaying superfluidity, e.g., see [1] and [2]. Quantum turbulence occurs in quantum fluids, which are so called because some of their properties cannot be described classically but depend on quantum physics.

Liquid ^{4}He, which is commonly used in cryogenic research, is one of such quantum fluids. It is called normal helium or He I at temperatures larger than 2.17 K (so called lambda-point), at the saturated vapor pressure. He I exhibits a classical-like behavior and is characterized by low density (about eight times lower than that of water) and very low values of the kinematic viscosity (of the order of 10^{-4} cm^2/s), compared to those of air (of the order of 10^{-1} cm^2/s) and water (of the order of 10^{-2} cm^2/s). If the temperature decreases further, liquid helium undergoes a second order phase transition and changes dramatically its properties. It is then called He II or superfluid ^{4}He and its viscosity can be considered null at 0 K, i.e., the fluid is assumed inviscid in the zero-temperature limit and its behavior cannot be accounted for by using the Navier-Stokes equation, as if it were a classical viscous fluid.

The two-fluid model can instead be used to this end and it describes superfluid ^{4}He as consisting of two fluids. The gas of thermal excitations, that is, the normal component of He II, can be considered as a viscous fluid, carrying the entire entropy content of the liquid, while the superfluid component of He II is assumed inviscid. The total density ρ of the liquid, defined as the sum of the densities of its normal and superfluid components, ρ_n and ρ_s, respectively, depends weakly on temperature, while the densities ρ_n and ρ_s display a much stronger temperature dependence (He II can be often considered entirely superfluid at temperatures below 1 K).

If, at finite temperature, a volume of superfluid ^{4}He is suitably heated, the superfluid component moves towards the heater where it becomes converted into the normal component that flows away from the heater. This phenomenon is called thermal counterflow and has no equivalent in classical fluid mechanics.

The superfluid component of He II can be usefully described by a macroscopic wave function, leading to the result that superflow is irrotational. It follows that for a multiply connected fluid region the circulation of the superfluid velocity is not necessarily null but equal to an integer multiple of the quantum of circulation $\kappa = h/m$, where h is the Planck constant and m denotes the mass of ^{4}He. This result can be seen as a quantum restriction to the superfluid motion. In other words, quantized vortices, i.e., line singularities where the superfluid density is null, can exist in superfluid helium. These vortices usually arrange themselves in a tangle and the dynamical behavior of such a tangle constitutes an essential ingredient of quantum turbulence. The latter is then characterized by two relevant length scales, in comparison to classical turbulence, and these are the size of the vortices' core, of the order of 10^{-10} m, and the average distance between quantized vortices ℓ, which depends on the type of quantum flow and can be phenomenologically estimated, e.g., see [2].

Significant progress in understanding the physics of He II, and its relation to other fields of research, has been recently obtained by using modern visualization methods, such as PIV (particle image velocimetry) and PTV (particle tracking velocimetry), e.g., see [3], [4], and references therein. The use of these techniques at low temperatures already led, for example, to the direct visualization of quantized vortices in He II [5], even though their application is difficult for both technical (optical access to the experimental volume, choice of suitable particles) and fundamental reasons (existence of two velocity fields, interaction of particles with quantized vortices). Important results have been obtained in thermal counterflow and include the observation of vortical structures around a cylinder [6], discovery of non-gaussian velocity statistics, linked to vortex reconnections [7], experimental evidence that the normal fluid velocity field may be turbulent [8], and interesting findings on the mechanisms of particles' trapping into the cores of quantized vortices [9].

Still, in comparison with classical fluid dynamics, the implementation of modern visualization methods to study

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quantum flows is in its infancy, sometimes posing more questions than giving clear answers. For example, the mechanisms of particles trapping into the core of quantized vortices and the vortical structures observed around cylinders in thermal counterflow deserve further attention and study. There is consequently a clear call for more detailed experimental analyzes by flow visualization, which is being proven as a valuable tool to study cryogenic flows. In order to fulfill such a need, a novel experimental apparatus has been devised in our Prague low temperature laboratory [10] and corresponding results, recently obtained in thermal counterflow, are reported and discussed here.

2 Experimental set-up

A low-loss custom-built cryostat is equipped with 25 mm diameter windows, which enable optical access to the experimental volume. The latter is of square cross section (sides of 50 mm) and about 300 mm long (some experiments are performed in a glass square channel, of 25 mm sides, inserted in the larger volume, i.e., the tail of the optical cryostat). The windows are placed about 100 mm above the bottom of the experimental volume (50 mm above the bottom of the glass channel).

A purpose-made seeding system supplies micrometer-sized solid deuterium particles, obtained by mixing at room temperature deuterium and helium gases and injecting the mixture into the helium bath. Solid deuterium is slightly heavier than our working fluid, i.e., liquid helium. Our apparatus allows for preparing also solid hydrogen particles, with the density slightly less than that of liquid helium [10].

A continuous-wave solid-state laser and suitable lenses are used to obtain a laser sheet of less than 1 mm thickness and height of about 10 mm. A digital camera is situated perpendicularly to the laser sheet and its CMOS sensor focused on a 12.8 mm width and 8 mm height field of view by using an appropriate macro lens (a slightly larger field of view is obtained in some cases). Such a set-up enables us to use the PTV technique for the measurement of lagrangian quantities in a plane parallel to the counterflow direction, in the middle of the experimental volume, i.e., as far as possible from its boundaries.

The vertical thermal counterflow is generated by a flat square heater of sides slightly smaller than 50 mm, placed on the bottom of the optical tail (a flat square heater of about 25 mm sides is instead placed at the bottom of the smaller glass channel).

Our experimental protocol is typically as follows. Once liquid helium is transferred into the cryostat, a pumping unit is used to lower its temperature. The gaseous mixture is then injected and the helium bath stabilized at a chosen temperature. As the particles are not neutrally buoyant, images are recorded before switching on the heater, in order to estimate their settling velocities and dimensions. The heater is then switched on, images collected at different heat fluxes, at 50 fps (some movies are recorded at 100 fps), and the particles' tracks calculated [11]. The corresponding particles' velocities in the horizontal and vertical direction are estimated by purpose-made computer codes, developed by us; for further details see [10].

3 Results and discussion

The probability density function PDF of the horizontal velocity u of micrometer-sized deuterium particles is plotted in Fig. 1 as a percentage of the total number of points of the tracks having at least five points, at different values of temperature T and applied heat flux q (the instantaneous velocity is divided by the corresponding standard deviation u_{sd} in order to obtain a more meaningful comparison).

The key point here, which emphasizes the difference between classical and quantum turbulence, is that the normalized velocity distribution is strongly non-gaussian, with $|u/u_{sd}|^{-3}$ power-law tails. It is therefore consistent with the law reported to be valid in decaying thermal counterflow [7]. Our results thus confirm that, as suggested in [10] and [12], the power-law velocity distribution is a characteristic feature of steady-state thermal counterflow, too.

This behavior has been linked to vortex reconnections [7], even though it can be obtained also when vortex reconnections do not occur, such as in two-dimensional classical vortex points systems [13]. The non-gaussian velocity distribution can however be seen as a fundamental property of quantum turbulence, supporting the quantum mechanical description of superfluid ^{4}He as containing a tangle of quantized vortices. It clearly distinguishes quantum flows from classical turbulent flows, which are usually characterized by nearly gaussian velocity distributions. A quasi classical behavior can indeed be recovered at length scales larger than the average distance between quantized vortices ℓ, as reported in [12] and [14].

The particles' sizes are here of few micrometers, i.e., about one order of magnitude smaller than ℓ. On the other hand, the average distance traveled by the particles between frames is of the order of ℓ, the average particles' velocity being of few mm/s, and this means that the flow is studied at length scales comparable to the average distance between quantized vortices. It is indeed evident from Fig. 1 that most particles follow a classical-like behavior as the distributions' core has a gaussian shape, in contrast to the power-law tails. It could then be said that ℓ is the length scale indicating the transition from classical-like to quantum behavior in thermal counterflow of superfluid ^{4}He. It follows that, at smaller length scales, more pronounced non-gaussian tails, with a less evident classical core, should be obtained, as shown in computer simulations [14].

Note that the magnitude of the experimentally obtained maximum velocity, of about 20 mm/s, is also consistent with the quantum mechanical description of He II. Such a velocity can indeed be calculated as the ratio of the quantum of circulation $\kappa \simeq 10^{-3}$ cm^2/s and average particles' size $r \simeq 5$ μm, that is, as the fluid velocity at a distance r from the vortex core. This can be seen as a further confirmation of the soundness of the presented results.

The distribution of the instantaneous vertical velocity v of the solid deuterium particles displays a similar power-law behavior, at large enough values of velocity. It is however less clear than the corresponding horizontal velocity distribution, depicted in Fig. 1, even though it appears to be definitely non-gaussian, as already shown in [10]. This can be explained by considering that the vertical component of the velocity is dominated by the imposed counterflow velocities (the normal fluid component of He II flows upwards, away from the heater, while the superfluid compo-

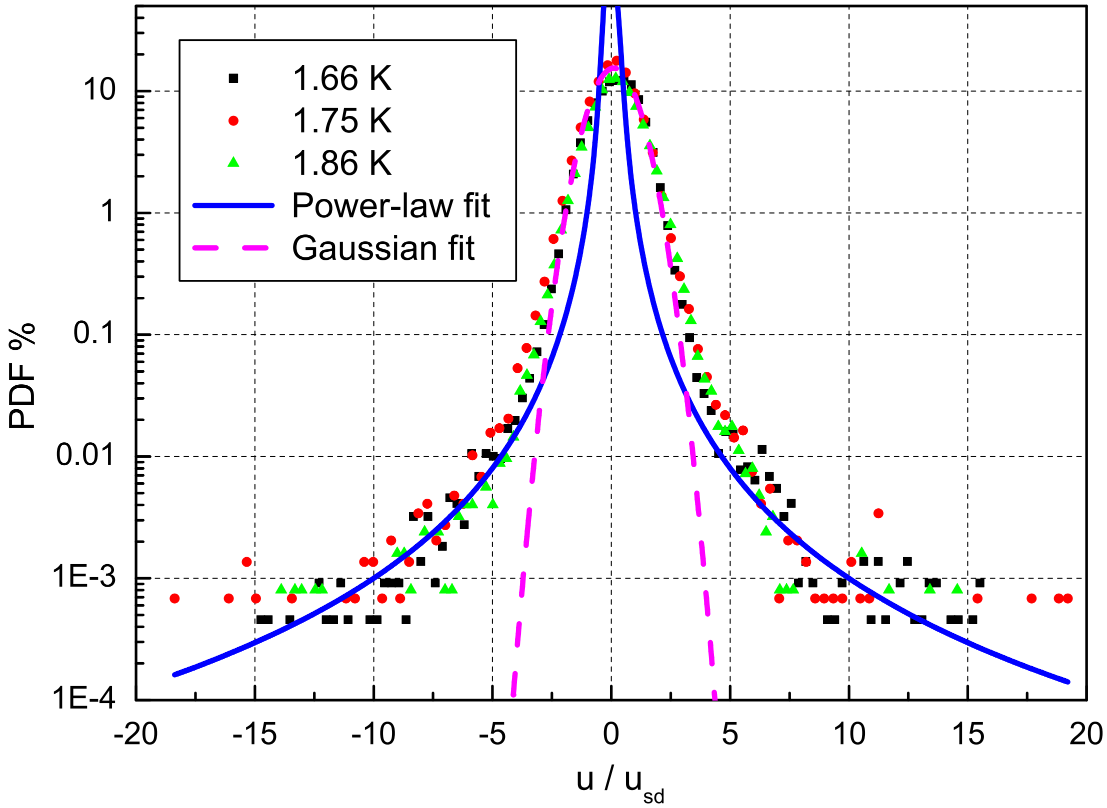

Fig. 1. (Color online) Probability density function PDF of the normalized horizontal velocity u/u_{sd}, where u_{sd} is the standard deviation of u, shown as a percentage of the total number of points of the trajectories having at least five points. Black squares: temperature $T = 1.66$ K, heat flux $q = 492$ W/m^2, average distance between quantized vortices $\ell = 74$ μm, 218260 points and 8127 tracks; red circles: $T = 1.75$ K, $q = 590$ W/m^2, $\ell = 72$ μm, 146571 points and 4417 tracks; green triangles: $T = 1.86$ K, $q = 595$ W/m^2, $\ell = 73$ μm, 124579 points and 3065 tracks; blue line: power-law fit $|u/u_{sd}|^{-3}$; dashed magenta line: gaussian fit.

nent flows downwards, towards the heater). It is therefore expected that clearer $|v/v_{sd}|^{-3}$ power-law tails, where v_{sd} is the standard deviation of v, should be obtained for larger data sets, in steady-state thermal counterflow.

4 Conclusions and future work

It was confirmed that the quantum nature of thermal counterflow of superfluid ^{4}He, at length scales comparable to the average distance between quantized vortices, is evident from the corresponding velocities' distributions. These are strongly non-gaussian, with power-law tails, in contrast to those obtained in classical turbulent flows, which are know to be of a nearly gaussian shape.

The results show, however, that larger data sets should be collected at larger frame rates in order to probe quantum turbulence at length scales significantly smaller than the average distance between quantized vortices, where classical and quantum flows are expected to be fundamentally different. The outcome could also be supported by computing particles' accelerations – in addition to the velocities' statistics presented here – and the corresponding lagrangian structure functions of velocity and position (these studies are in progress and will be published elsewhere).

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