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Comparison of several models of the laminar/turbulent transition

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Abstract. The contribution deals with modelling of the laminar/turbulent transition using several transition models. Transition models of various types were tested: a) the model with the algebraic equation for the intermittency coefficient according to Straka and Příhoda; b) the three-equation transition model with the transport equation for the energy of non-turbulent fluctuations proposed by Walters and Cokljat; c) the γ-Re_θ model with the transport equation for the intermittency coefficient of Langtry and Menter. The transition models were compared by means of test cases covering both flat-plate boundary-layer flows with various free stream turbulence and the flow over an airfoil including the effect of foregoing wake on the transition. The agreement of numerical results with experimental data is in all cases quite satisfactory.

1 Introduction

The adequate modelling of the laminar/turbulent transition is very important for the correct prediction of wall-bounded flows as the transition substantially influences the skin friction and therefore the energy losses equally as the heat transfer.

The laminar/turbulent transition in the internal and external aerodynamics is usually modelled either by models based on the equation for the intermittency coefficient or by the three-equation model with the equation for the energy of non-turbulent fluctuations proposed by Walters and Leylek [1].

The transition models with the algebraic and/or transport equation for the intermittency coefficient need empirical relations for the onset and the length of the transition region and should be modified for local variables only for the application in complex boundary conditions using unstructured grids. The three-equation model seems to be more general as can be used without any restrictions.

2 Mathematical model

The compressible flow is described by the continuity equation, the averaged Navier-Stokes equations, the energy equation, and by the constitutive relations. The system of governing equations is closed by the turbulence model connected with a transition model. The turbulent heat transfer is approximated by the constant turbulent Prandtl number.

The numerical simulation of transitional flows was accomplished by means of various types of the transition models. Calculations were carried out partly by the two-equation k-ω turbulence model according to Kok [4] with the algebraic transition model by Straka and Příhoda [2] implemented into the in-house numerical code and partly by the three-equation k-k_L-ω model of Walters and Cokljat [3] implemented into the open-source code OpenFOAM. For comparison, some calculations were carried out by the SST turbulence model with the γ-Re_θ transition model proposed by Langtry and Menter [4] accessible in the commercial code ANSYS Fluent.

The algebraic model was implemented into the in-house numerical code. The code is based on the finite volume method of the cell-centered type with the Osher's-Solomon's approximation of the Riemann solver and a two-dimensional linear reconstruction with the Van Albada's limiter. The governing equations are discretized using a multi-block quadrilateral structured grid with a block overlapping implementation. The own implementation of the k-k_L-ω model into the OpenFOAM package was used. The numerical solution was obtained with the finite volume method using the SIMPLE scheme for the incompressible flow.

While the k-k_L-ω model of Walters and Cokljat [3] can be used for the simulation of complex shear flows using unstructured grids, transition models based on the equation for the intermittency coefficient should be adapted for local variables only because the onset and length of the transition region are usually expressed by means of the momentum Reynolds number.

2.1 Algebraic transition model

The algebraic transition model is connected with two-equation *k-ω* turbulence model proposed by Kok [5]. The turbulence model is given by the transport equations for the turbulent energy and the specific dissipation rate

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho u_j k)}{\partial x_j}=\tilde{P}_k-\tilde{D}_k + \frac{\partial}{\partial x_j}\left[(\mu+\sigma_k\mu_t)\frac{\partial k}{\partial x_j}\right], \tag{1}$$

$$\frac{\partial(\rho\omega)}{\partial t}+\frac{\partial(\rho u_j\omega)}{\partial x_j}=\alpha_\omega\frac{\omega}{k}P_k-\beta_\omega\rho\,\omega^2 + \frac{\partial}{\partial x_j}\left[(\mu+\sigma_\omega\mu_t)\frac{\partial\omega}{\partial x_j}\right]+\sigma_d\frac{\rho}{\omega}\max\left(\frac{\partial k}{\partial x_j}\frac{\partial\omega}{\partial x_j},0\right), \tag{2}$$

where the production $\tilde{P}_k$ and destruction $\tilde{D}_k$ are modified in the laminar and transitional part of the shear layer. The turbulent viscosity is given by the relation

$$\mu_t=\max\left(\frac{\rho k}{\omega};6\sqrt{\frac{\mu}{\beta^*\omega k}}\right), \tag{3}$$

with the Kolmogorov time scale used near the wall according to Durbin [6]. Further, a restriction of the turbulent viscosity proposed by Medic and Durbin [7] is applied for the reduction of the energy production near the leading edge.

The algebraic model is based on the concept of different values of the intermittency coefficient in the boundary layer (γ_i) and in the free stream (γ_e). The empirical relation for the smooth link-up between both zones is considered by the relation

$$\gamma=\frac{\gamma_i+\gamma_e}{2}+\frac{\gamma_e-\gamma_i}{2}\,\mathrm{tgh}\left[C_\gamma\left(\frac{y}{\delta}-1\right)\right], \tag{4}$$

with the constant $C_\gamma \approx 15$. The intermittency coefficient in the boundary layer is given by the relation

$$\gamma_i=1-\exp\left[-\hat{n}\sigma\left(Re_x-Re_{xt}\right)^2\right], \tag{5}$$

proposed by Narasimha [8]. The transition onset is given by the empirical correlation for the momentum Reynolds number $Re_{\theta t}$ = f(Tu, λ_t) where Tu (%) is the free-stream turbulence level and $\lambda_t = (\theta_t^2/\nu)(\mathrm{d}U_e/\mathrm{d}x)$ is the pressure-gradient parameter.

The length of the transition region given by the spot generation rate $\hat{n}$ and the spot propagation rate σ is expressed using the parameter

$$N=\hat{n}\sigma Re_{\theta t}^3, \tag{6}$$

introduced by Narasimha [8]. The effect of the free-stream turbulence and the pressure gradient on the parameter N is correlated by the relation N = f (Tu, λ_t) proposed for the attached flow by Solomon et al. [9]. The onset of transition in separated flow is given by the correlation proposed by Mayle [10] in the form

$$Re_{xt}=300Re_{\theta s}^{0.7}+Re_{xs}, \tag{7}$$

where $Re_{\theta s}$ is the momentum Reynolds number at the separation and Re_{xs} is the Reynolds number related to the distance of the separation position from the leading edge. The length of the transition region is given according to Roberts and Yaras [11] by the relation

$$N=\frac{0.55H_t-2.2}{1-0.63H_t+0.14H_t^2}, \tag{8}$$

where H_t is the form parameter at the transition onset.

To avoid the application of local variables, the maximum of the vorticity Reynolds number $Re_{v\max}$ is used instead of the momentum Reynolds number Re_θ according to Langtry and Menter [4]. The vorticity Reynolds number is given for complex flows by the relation

$$Re_v=y^2S/\nu, \tag{9}$$

where y is the distance from the nearest wall and $S = (2S_{ij}S_{ij})^{1/2}$ is the absolute value of the strain rate. This link can be expressed by the relation

$$Re_\theta = Re_{v\max}/\mathrm{C}, \tag{10}$$

where the parameter C depends on the pressure gradient only. Using the similar solution of the laminar boundary layer of Falkner and Skan [12], a simple correlation

$$C=\frac{Re_{v\max}}{Re_\theta}=\mathrm{f}(L), \tag{11}$$

can be derived, where L is a modified pressure-gradient parameter given by the relation

$$L=Re_{v\max}^2\frac{\nu}{U_e^2}\frac{\mathrm{d}U_e}{\mathrm{d}x}. \tag{12}$$

Relevant values of free-stream turbulence are taken at the outer boundary of the shear layer estimated by means of the position of the maximum vorticity Reynolds number. The boundary layer thickness δ is given by the relation δ/y_{max} = g(L) where y_{max} is the distance of the maximum vorticity Reynolds number from the wall obtained by means of the similar solutions of Falkner and Skan [12]. The modified algebraic transition model is described in detail by Straka and Příhoda [2].

2.2 *k-k*$_L$*-ω* model

The three-equation model with transport equations for the turbulent kinetic energy k_T, the laminar kinetic energy k_L and the specific dissipation rate ω was proposed by Walters and Leylek [1], and later modified by Walters and Cokljat [3]. The model is based on the assumption that velocity fluctuations in the region before the transition onset can be divided into two parts - partly on small vortices contributing to the turbulence production, and partly on large mainly longitudinal vortices near the wall contributing to the production of non-turbulent fluctuations.

The turbulent energy k_T near the wall can be similarly divided into the energy of small-scale vortices $k_{T,s}$ contributing to the production of the turbulent energy, and into the energy of large-scale vortices $k_{T,l}$ contributing to the production of laminar kinetic energy. The transport equations for the turbulent kinetic energy k_T, laminar kinetic energy k_L and the specific dissipation rate $\omega = \varepsilon/k_T$ are given by equations

$$\begin{aligned}\frac{\mathrm{D}k_\mathrm{T}}{\mathrm{D}t} &= P_\mathrm{kT} + R_\mathrm{BP} + R_\mathrm{NAT} - D_\mathrm{T} \\ &+ \frac{\partial}{\partial x_\mathrm{j}}\left[\left(\nu + \frac{\alpha_\mathrm{T}}{\sigma_\mathrm{k}}\right)\frac{\partial k_\mathrm{T}}{\partial x_\mathrm{j}}\right] - \omega k_\mathrm{T}\end{aligned}, \tag{13}$$

$$\frac{\mathrm{D}k_\mathrm{L}}{\mathrm{D}t} = P_\mathrm{kL} - R_\mathrm{BP} - R_\mathrm{NAT} - D_\mathrm{L} + \frac{\partial}{\partial x_\mathrm{j}}\left[\nu\frac{\partial k_\mathrm{L}}{\partial x_\mathrm{j}}\right], \tag{14}$$

$$\begin{aligned}\frac{\mathrm{D}\omega}{\mathrm{D}t} &= C_{\omega1}\frac{\omega}{k_\mathrm{T}}P_\mathrm{kT} + \left(\frac{C_{\omega\mathrm{R}}}{f_\mathrm{w}} - 1\right)\frac{\omega}{k_\mathrm{T}}\left(R_\mathrm{BP} + R_\mathrm{NAT}\right) \\ &- C_{\omega2}f_\mathrm{w}^2\omega^2 + C_{\omega3}f_\omega\alpha_\mathrm{T}f_\mathrm{w}^2\frac{\sqrt{k_\mathrm{T}}}{y^3} + \frac{\partial}{\partial x_\mathrm{j}}\left[\left(\nu + \frac{\alpha_\mathrm{T}}{\sigma_\omega}\right)\frac{\partial\omega}{\partial x_\mathrm{j}}\right]\end{aligned}. \tag{15}$$

These equations contain as the standard turbulence model terms representing the advection, production, diffusion, and destruction. First terms on the right side describe the production of turbulent and laminar energy and the production of the specific dissipation rate respectively. In the equation for turbulent energy k_T, there are moreover terms R_{BP} and R_{NAT} expressing the effect of the decay of laminar fluctuations during the natural and bypass transition. These terms are then included with the opposite sign in the equation for the laminar energy k_L. Terms D_T and D_L give the non-isotropic part of the dissipation rate of the turbulent and laminar kinetic energy near the wall. The second term in the equation for the specific dissipation rate ω brings about the decrease of turbulent length scale during the transition. The fourth term decreases the length scale in the outer region of the boundary layer and ensures thus the adequate modelling of the wake region in the turbulent boundary layer.

The transition process is express by the energy transfer from the energy of non-turbulent mostly longitudinal fluctuations k_L to the turbulent energy k_T representing the three-dimensional turbulent fluctuations of various length and time scales. The laminar energy k_L is caused by the interaction of non-turbulent velocity fluctuations with the shear flow in the region before the transition region where the turbulent fluctuations are damped. After the transition onset, this damping is restricted to the viscous sublayer. The model is in detail described by Walters and Cokljat [3].

2.3 γ-Re_θ transition model

The correlation-based transition model in the connection with the SST turbulence model was proposed by Langtry [13]. This transition model with two transport equations for the intermittency coefficient and for the transition criterion expressed by means of the local transition onset momentum thickness Reynolds number is strictly based on local variables. The model can be used for complex three-dimensional shear flows using unstructured grids.

The intermittency coefficient γ is determined from the transport equation

$$\begin{aligned}\frac{\partial(\rho\gamma)}{\partial t} &+ \frac{\partial\left(\rho U_\mathrm{j}\gamma\right)}{\partial x_\mathrm{j}} = P_\gamma - E_\gamma \\ &+ \frac{\partial}{\partial x_\mathrm{j}}\left[\left(\mu + \frac{\mu_\mathrm{t}}{\sigma_\gamma}\right)\frac{\partial\gamma}{\partial x_\mathrm{j}}\right]\end{aligned}, \tag{16}$$

where

$$P_\gamma = F_\mathrm{length}\mathrm{C}_\mathrm{a1}\rho S\left(\gamma F_\mathrm{onset}\right)^{1/2}\left(1 - \mathrm{C}_\mathrm{e1}\gamma\right) \tag{17}$$

is the production term with empirical correlations F_{onset} for the transition onset and F_{length} for the length of the transition region. The destruction term E_γ enabling the relaminarization of the boundary layer is given by the relation

$$E_\gamma = \mathrm{C}_\mathrm{a2}\rho\Omega\gamma F_\mathrm{turb}\left(\mathrm{C}_\mathrm{e2}\gamma - 1\right), \tag{18}$$

where $\Omega = (2\Omega_{ij}\Omega_{ij})^{1/2}$ is the absolute value of the vorticity. The application of local variables is accomplished by the relation between the momentum Reynolds number and the maximum vorticity Reynolds number used in the form $Re_\theta = Re_{vmax}/2.193$ valid for the Blasius boundary layer.

The empirical relation for the transition onset F_{onset} is dependent on the critical momentum Reynolds number $Re_{\theta c}$ giving the location where turbulence starts to grow, see Langtry [13]. This Reynolds number $Re_{\theta c}$ is expressed by means of the parameter $\tilde{R}e_{\theta t}$ called the local transition onset momentum thickness Reynolds number obtained from the transport equation

$$\begin{aligned}\frac{\partial\left(\rho\tilde{R}e_{\theta\mathrm{t}}\right)}{\partial t} &+ \frac{\partial\left(\rho U_\mathrm{j}\tilde{R}e_{\theta\mathrm{t}}\right)}{\partial x_\mathrm{j}} = P_{\theta\mathrm{t}} \\ &+ \frac{\partial}{\partial x_\mathrm{j}}\left[\sigma_{\theta\mathrm{t}}\left(\mu + \mu_\mathrm{t}\right)\frac{\partial\tilde{R}e_{\theta\mathrm{t}}}{\partial x_\mathrm{j}}\right]\end{aligned}. \tag{19}$$

The onset and the length of the transition region, i.e. parameters $Re_{\theta c}$ and L_{length} are correlated by means of the local transition onset momentum thickness Reynolds number. The transition in separated flow is determined using a simple algebraic equation for the intermittency coefficient. The complete γ-Re_θ model with all necessary empirical relations was firstly published by Langtry and Menter [4].

3 Results

The chosen transition models were compared by means of classical ERCOFTAC test cases concerning to boundary-layer flow on the flat plate with various free-stream turbulence (T3A, T3AM, T3B). Further, the incompressible flow over the NACA 0012 aerofoil in a tandem configuration measured by Lee and Kang [14] was used for testing of transition models.

The rectangular computational domain starts in the distance x = 0.05 m upstream the leading edge of the flat plate. The quadrilateral structured grid refined near the leading edge and the wall was used in all transition models. At the inlet boundary the mean velocity U_e and the turbulent energy k and the specific dissipation rate ω are prescribed. The laminar kinetic energy $k_L = 0$ was used for the k-k_L-ω model. The pressure was calculated using the homogeneous Neumann condition $\partial p/\partial n = 0$. The inlet values of k and ω were prescribed according to Langtry [14] by means of the free-stream turbulence level Tu (%) = $100(3k/2)^{1/2}/U_e$ and by the viscosity ratio μ_t/μ. Used inlet parameters corresponding to the leading edge are given in the table 1.

Table 1. Inlet condition for the flat plate test cases

Test case	U_e (m/s)	Tu (%)	μ_t/μ
T3A	5.4	3.3	12
T3B	9.4	6.5	100
T3AM	19.8	0.874	8.72

At the outlet plane, the static pressure was prescribed and the homogeneous Neumann condition was applied for the other parameters. The symmetry condition is assumed at the upper boundary and at the lower boundary upstream the leading edge.

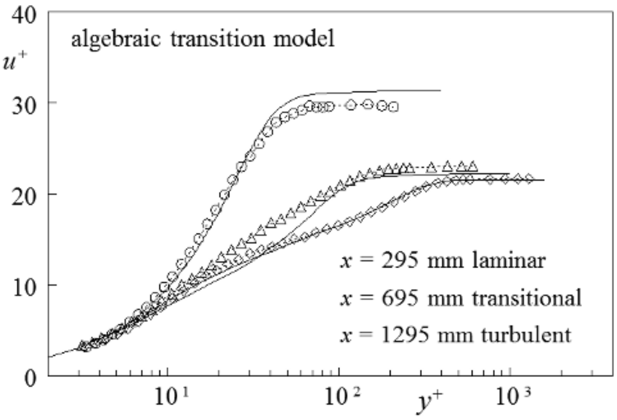

Fig. 1. Mean velocity profiles for the test case T3A

Mean velocity profiles typical for the laminar, transitional and turbulent part of the boundary layer for the test case T3A are demonstrated in figure 1. The agreement of velocity profiles predicted by the algebraic transition model with experiments is quite good.

The distribution of the skin friction coefficient $C_f = 2\tau_w/\rho U_e^2$ predicted for the ERCOFTAC test cases by used transition models is compared with experimental data in figures 2, 3 and 4.

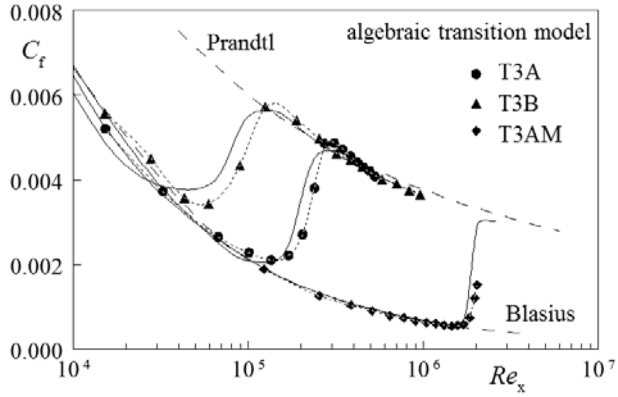

Fig. 2. Distribution of the skin friction coefficient for flat-plate flows (algebraic transition model)

The algebraic transition model and the k-k_L-ω model give a very good agreement with experiments for all test cases. Numerical results predicted adequately the onset and the length of the transition region for flows both with low free-stream turbulence (Tu < 1%), and with high free-stream turbulence ($Tu \approx 6\%$).

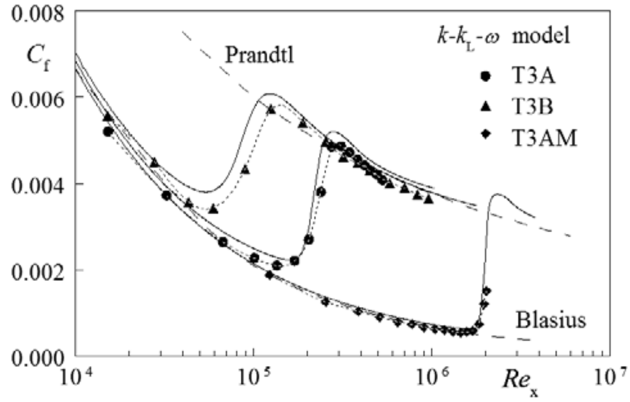

Fig. 3. Distribution of the skin friction coefficient for flat-plate flows (k-k_L-ω model)

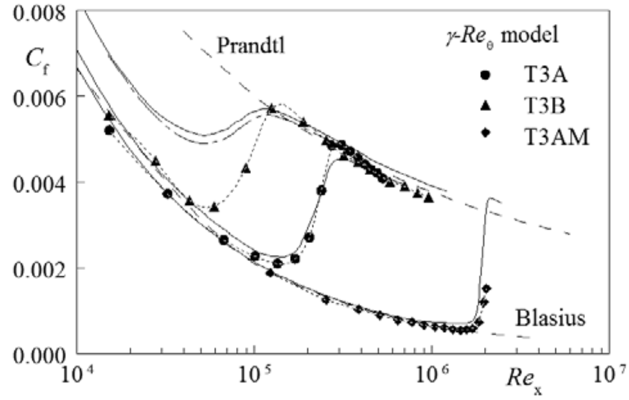

Fig. 4. Distribution of the skin friction coefficient for flat-plate flows (γ-Re_θ model)

Similar results given in figure 4 were obtained by the γ-Re_θ model except the test case T3B where the transition onset begins slightly upstream and so the effect of free-stream turbulence is overestimated. The corresponding distribution of the skin friction coefficient given in figure 4 by the dash-and-dot line was obtained by

Langtry [13]. This can be caused by the relatively high inlet free-stream turbulence artificially increased by the grid upstream of the flat plate. It leads to a rapid decay of turbulence level and thus to the extreme inlet dissipation rate.

The incompressible flow over two NACA0012 aerofoils in a tandem according to Lee and Kang [14] was simulated by the algebraic transition model and by the k-k_L-ω model. Presented simulations are concentrated on the effect of the Reynolds number. The relevant measurements of the flow around aerofoils with the chord c = 0.3 m was completed for the distance between airfoils t/c = 1 and the Reynolds numbers $Re_c = 2\times10^5$ (CASE 3); 4×10^5 (CASE 2) and 6×10^5 (CASE 0 and CASE 1).

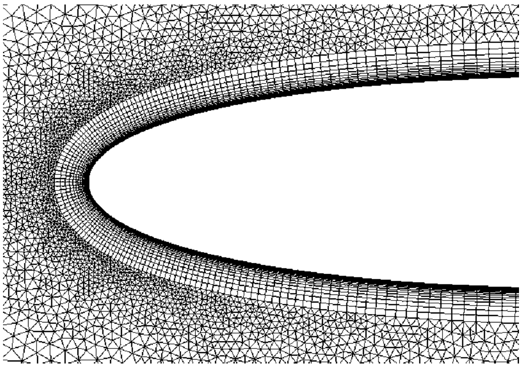

Fig. 5. Detail of the computational grid (k-k_L-ω model)

The rectangular computational domain corresponding to the experiment starts 1.5 c upstream the leading edge of the first aerofoil. The prescription of the inlet quantities is the same as in the case of the flat plate. The inlet turbulence characteristics Tu = 0.3% and the viscosity ratio $\mu_t/\mu = 8$ were chosen. The algebraic transition model uses a multi-block quadrilateral structured grid with the block overlapping while the hyperbolic structured O-grid near airfoils and the triangular grid in the rest of the domain was used for the k-k_L-ω model. The grid was refined near the leading edge and near the wall in the both cases and so the wall nearest node was about $y^+ \approx 1$. The detail of the computational grid near the leading edge for the k-k_L-ω model is shown in figure 5.

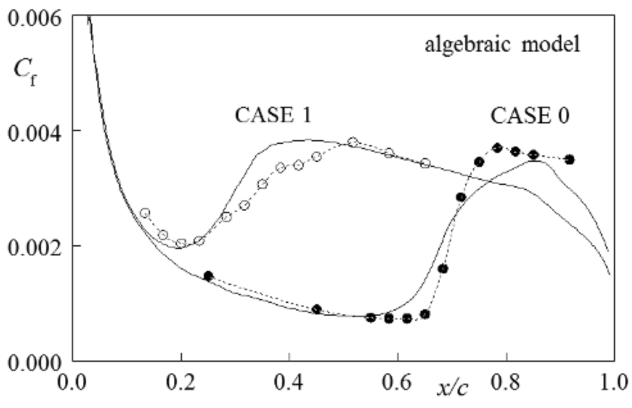

Fig. 6. Distribution of the skin friction coefficient for NACA 0012 aerofoils (algebraic transition model)

The distribution of the skin friction coefficient on aerofoils obtained by the algebraic transition model for the Reynolds number $Re_c = 6\times10^5$ is compared with experimental data in figure 6. The transition onset was moved upstream due to the incoming wake from x/c = 0.62 (CASE 0) to x/c = 0.20 (CASE 1). The agreement of the prediction with experiment is quite good.

Results of the k-k_L-ω model are shown in figure 7 together with prediction by the γ-Re_θ model, see Straka et al. [15]. In the both models, the laminar boundary layer separates before the transition and so a laminar separation bubble was predicted for the CASE 0. It can be caused by a relatively low free-stream turbulence of the incoming flow. A good agreement was achieved by the k-k_L-ω model for the CASE 1 where the turbulence level is increased by the wake of the foregoing aerofoil. On the other hand, this effect of increased turbulence level on the transition onset is overestimated by the γ-Re_θ model.

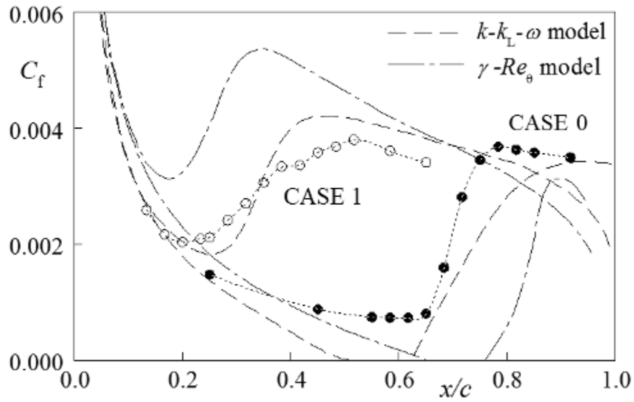

Fig. 7. Distribution of the skin friction coefficient for NACA 0012 aerofoils (k-k_L-ω model and γ-Re_θ model)

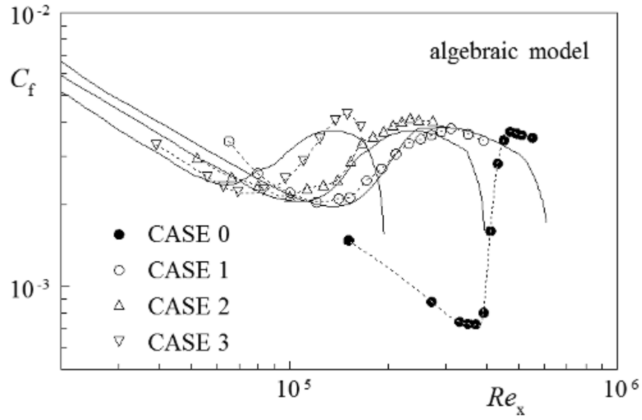

Fig. 8. Effect of the Reynolds number on the skin friction coefficient (algebraic transition model)

The effect of the Reynolds number Re_c on the transition onset affected by the wake of the foregoing aerofoil was tested by the algebraic transition model and by the k-k_L-ω model. The dependence of the skin friction coefficient C_f on the Reynolds number Re_x is shown in figure 8 for the algebraic model and in figure 9 for the k-k_L-ω model.

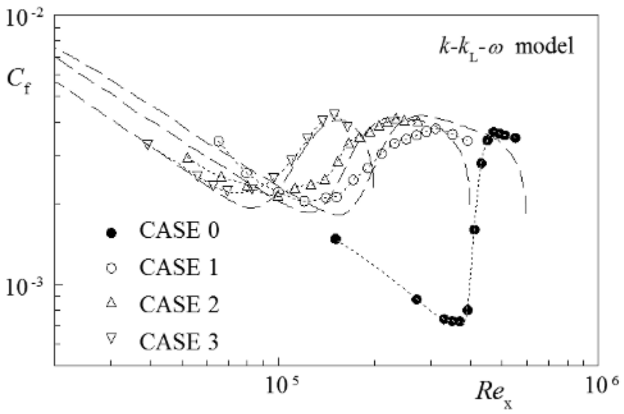

Fig. 9. Effect of the Reynolds number on the skin friction coefficient (k-k_L-ω model)

The transition onset on the second aerofoil moves upstream with the decreasing Reynolds number. The transition begins at the Reynolds number $Re_x \approx 1.2\times10^5$ for $Re_c = 6\times10^5$ and moves upstream to $Re_x \approx 0.7\times10^5$ for $Re_c = 2\times10^5$. The agreement of numerical simulations by the both transition models with experimental data is quite good for all three cases except the prediction by the algebraic transition model for the CASE 3 where the shift of the transition onset is somewhat greater.

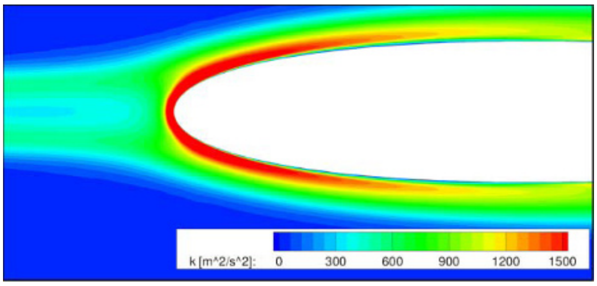

Fig. 10. Detail of the turbulent energy near the leading edge of the second aerofoil (CASE 3, $Re_c = 2\times10^5$)

The effect of the foregoing wake on the transition can be demonstrated by the field of turbulent energy near the leading edge of second aerofoil given for $Re_c = 2\times10^5$ in figure 10. With the decreasing Reynolds number, the turbulent energy in the wake increases and so the transition onset on the second aerofoil moves upstream.

Conclusions

Transition models bases on the algebraic and/or transport equation for the intermittency coefficient and on the transport equation for the energy of non-turbulent fluctuations were tested using test cases covering the flat-plate flows with various free-stream turbulence levels and the flow over two aerofoils in a tandem configuration.

The algebraic model of Straka and Příhoda [2] and the three-equation model of Walters and Cokljat [3] give a quite good agreement with flat-plate test cases whereas the γ-Re_θ model of Langtry and Menter [4] somewhat overestimates the effect of free-stream turbulence for the case T3B with the high turbulence level $Tu \approx 6\%$.

The algebraic model gives as well good results for the flow over NACA0012 aerofoils while the k-k_L-ω model and the γ-Re_θ model predict the separated-induced transition for first aerofoil in low free-stream turbulence. The ability of the algebraic model and the k-k_L-ω model to predict the wake-induced transition is similar, but the estimation of the boundary for the relevant free-stream turbulence level in the algebraic model can be somewhat questionable in some cases.

The next study will be focused on the more general estimation of relevant free-stream turbulence and on the effect of the wall roughness.

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References

1. D. K. Walters, J. H. Leylek, ASME, J. Turbomachinery, **126**, 193–202 (2004)
2. P. Straka, J. Příhoda, Proc. Conf. Experimental Fluid Mechanics 2010, Liberec, 636-641 (2010)
3. D. K. Walters, D. Cokljat, ASME, J. Fluids Engineering, **130**, 121401/1-121401/14 (2008)
4. R. Langtry, F. R. Menter, AIAA J., **47**, 2894-2906 (2009)
5. J. C. Kok, AIAA J., **38**, 1292-1295 (2000)
6. P. A. Durbin, Theor. Comput. Fluid Dyn., **3**, 1-13 (1991)
7. G. Medic, P. A. Durbin, ASME, J. Turbomachinery, **124**, 187-192 (2002)
8. R. Narasimha, J. Aerospace Science, **24**, 711-712 (1957)
9. W. J. Solomon, G. J. Walker, J. P. Gostelow, ASME, J. Turbomachinery, **118**, 744-751 (1996)
10. R. E. Mayle, ASME, J. Turbomachinery, **113**, 509-537 (1991)
11. S. K. Roberts, M. I. Yaras, ASME, J. Turbomachinery, **128**, 349-356 (2006)
12. V. M. Falkner, S. W. Skan, Phil. Mag., **12**, 865-896 (1930)
13. R. Langtry, *A correlation-based transition model using local variables for un-structured parallelized CFD codes,* PhD dissertation, Univ. Stuttgart (2006)
14. H. Lee, S. - H. Kang, ASME, J. Fluids Engineering, **9**, 122, 522-532 (2000)
15. P. Straka, J. Příhoda, D. Šimurda, Proc. Conf. Experimental Fluid Mechanics 2011, Liberec, 971-978 (2011)