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The Measurement of Wall Shear Stress in the Low-Viscosity Liquids

M. Schmirler[1,a], J. Matěcha[1], H. Netřebská[1], J. Ježek[1] and, J. Adamec[1]

[1]CTU in Prague, Faculty of Mechanical Engineering, Department of Fluid Mechanics and Thermodynamics

Abstract. The paper is focused on quantitative evaluation of the value of the wall shear stress in liquids with low viscosity by means of the method of the hot film anemometry in a laminar and turbulent flow. Two systems for calibration of probes are described in the paper. The first of these uses an innovative method of probe calibration using a known flow in a cylindrical gap between two concentric cylinders where the inner cylinder is rotated and a known velocity profile and shear rate, or shear stress profile, is calculated from the Navier-Stokes equations. This method is usable for lower values of the wall shear stress, particularly in the areas of laminar flow. The second method is based on direct calibration of the probes using a floating element. This element, with a size of 120x80 mm, is part of a rectangular channel. This method of calibration enables the probe calibration at higher shear rates and is applicable also to turbulent flow. Values obtained from both calibration methods are also compared with results of measurements of the wall shear stress in a straight smooth channel for a certain range of Reynolds numbers and compared with analytical calculations. The accuracy of the method and the influence of various parasitic phenomena on the accuracy of the measured results were discussed. The paper describes in particular the influence of geometric purity of the probe location, the impact of various transfer phenomena, requirements for the measured liquid and layout of the experiment.

1 Introduction

Wall shear stress (WSS) plays a very important role in the research of fluid flow and transfer of momentum and energy; its determination has been currently discussed quite frequently. As has already been demonstrated in the past, the values of the shear stress near the wall have a decisive impact on various transfer processes [1], on sedimentation rate of particles [2], on physiological changes in blood-vessel walls [3], on investigating loss factors in pipeline [4], on development of velocity profiles after entering the pipeline [5, 6, 7], etc. One of the most progressive is the method of artificial cultivation of human tissue grafts in so-called bio-reactors [8], in which a specific shear stress on walls of the cores (where the grafts grow) is being artificially maintained. It is clearly shown that the wall shear stress has a significant impact on quality of the cultivated grafts (density and quality of the cells, their orientation, etc.). Currently, most experiments focus on investigating the wall shear stress in gases, or alternatively in the air. The works dealing with liquids often satisfy themselves with relative or qualitative results. One problem is to determine an accurate numerical value of the wall shear stress, especially in liquids with low viscosity.

For a quantitative evaluation of the wall shear stress it is necessary to use a appropriate calibration of the measuring device that depends on the chosen method.

[a] michal.schmirler@fs.cvut.cz

Development of the wall shear stress in Newtonian fluids is caused by spatial non-uniformity of the flow when the velocity gradients occur, thus leading to the development of shear velocities. The wall shear stress magnitude is then, according to Newton's law, directly proportional to the rate of shear deformation where the proportionality constant is the dynamic viscosity of the fluid. In case of Newtonian fluids the viscosity is independent of the magnitude of the shear velocity and the issue is thus considerably simplified. It is possible to evaluate, in addition to the standard velocity profile, the profile of the wall shear stress in the fluid stream. The wall shear stress – WSS occurs then near the wall.

One of the first attempts to measure the wall shear stress was made first by Winter (1977) [9], subsequently by Hanratty, Campbell (1983) [10] and Haritonidis (1989) [11]. Measurements were then focused on the research of the aircraft flight performance. We can use several methods for measuring the WSS. Each method has its own specifications and application areas and demands different approach regarding the calibration. This paper focuses on a method that makes use of connection between the intensity of cooling the heating element in body of the probe flowing in the fluid and the magnitude of the shear rate, or alternatively the shear stress in the area of the probe (Reynolds analogy [12]). H. Ludwieg was one of the first to apply this method in anemometry in 1949. The heating element is usually a thin tungsten wire or a nickel film. A more resistant hot-film is used for measurement in liquids and polluted gases.

In order to evaluate the absolute values of the wall shear stress, an adequate probe calibration is necessary. At present, there are a few methods for calibrating the film probes used for measuring the WSS in liquids. Each method has obviously its advantages and disadvantages. The calibration with the help of two concentric cylinders designed in this paper is advantageous due to the possibility of precise control of the liquid temperature in the device, the possibility of continuous measurement and the need for only a small sample of the liquid measured. As it turned out, the disadvantage of this method is a relatively narrow application area, for values of the wall shear stress approximately to 0.1 Pa. Although the value is sufficient for many applications, especially in the field of hemodynamics, for other applications it was necessary to develop a second calibration device working on the principle of direct measurement of WSS. The second method has an advantage of direct probe calibration with a known level of force effect on the channel wall and possibility of using both for laminar and for turbulent area of the flow and for non-Newtonian fluids.

2 Materials and methods

Two devices for calibration of the probe of the wall shear stress were designed, built and verified. Both devices are described in the following chapters.

2.1 Calibration using a known flow in cylindrical gap

A method of probe calibration using a known flow in cylindrical gap was designed in order to enable evaluation of very small value of the wall shear stress. A device called WASSCOS 1 (figure 1, figure 2) with two concentric cylinders where the inner cylinder rotates with a known angular velocity ω was constructed and built. The cylinder diameter is 52 mm, gap thickness 2mm.

Fig.1. The calibration device „WASSCOS 1“

The device is primarily determined for calibration of flush mounted hot-film probe. However, its design enables to insert several other types of probes for measuring the wall shear stress (electrodiffusion probes, etc.).

The process of deriving the velocity profile in the cylindrical gap with the help of Navier-Stokes equations (1) is a common task of the fluid mechanics. Derivation

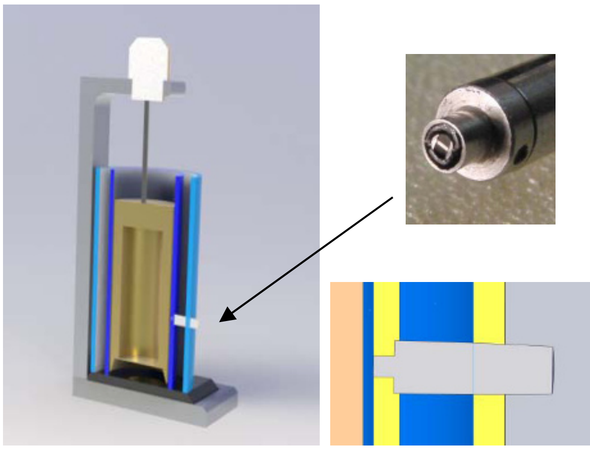

Fig. 2. WASSCOS 1 - schematic wiev.

enabled to obtain a resultant relation for velocity profile in the cylindrical gap between two cylinders with radius R_1 and R_2 with rotation of the inner cylinder with angular velocity ω:

$$u_\varphi(r) = \omega \cdot \frac{R_1^2}{R_2^2 - R_1^2} \cdot \left(\frac{R_2^2}{r} - r \right). \tag{1}$$

It is possible to derive a relation for the course of shear stress in the gap from the velocity profile in the gap with the help of Newton's law (2) of shear friction.

$$\tau_{(r)} = \eta \cdot \frac{du_\varphi}{dr} \tag{2}$$

The velocity u_φ in the direction r in cylindrical coordinates is given by:

$$\frac{du_\varphi}{dr} = \frac{1}{2} \cdot \left[r \cdot \frac{\partial}{\partial r}\left(\frac{u_\varphi}{r} \right) + \frac{1}{r} \cdot \frac{\partial u_r}{\partial \varphi} \right]. \tag{3}$$

The shear stress profile across cylindrical gap can be calculated after incorporation of equation (3) into Newton's law (2):

$$\tau_{(r)} = -2 \cdot \eta \cdot \frac{R_1^2 \cdot R_2^2}{R_2^2 - R_1^2} \cdot \omega \cdot \frac{1}{r^2}. \tag{4}$$

The calibrated probe of the wall shear stress is located in the calibration device in such a way that its active part is flush mounted with the outer cylindrical wall with radius R_2. After substitution of radius R_2 for general radius r in the equation (4) we are able to obtain the value of the wall shear stress τ_W at the probe:

$$\tau_{W(r=R_2)} = -2\cdot\eta\cdot\omega\cdot\frac{R_1^2}{R_2^2-R_1^2}, \qquad (5)$$

Where η is the dynamic viscosity and ω angular velocity of rotation of inner cylinder.The proposed method is generally applicable for smaller magnitudes of the wall shear stress, or alternatively for such values that correspond to subcritical Reynolds numbers, which means the area of laminar flow in the used cylindrical gap. The limit of applicability of the method is given for one thing by the transition to the turbulence in the cylindrical gap and for another by the occurence of so-called Taylor's vortices.

The result of the measurement is the dependence of bridge voltage E on the temperature of the measured liquid and on the rotational speed of the inner cylinder. Graphically, this dependence is shown in Figure 3.

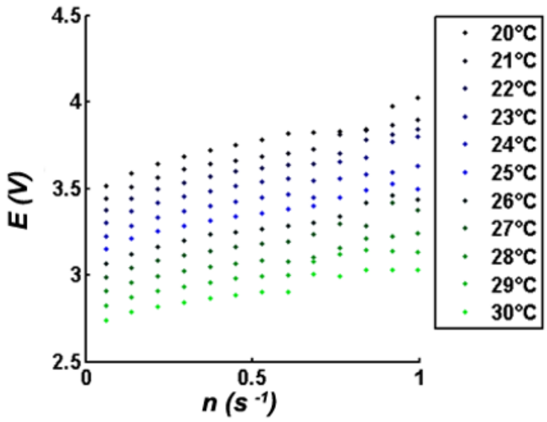

Fig. 3. WASSCOS 1 – bridge voltage measurement.

The resultant calibration plane (Figure 4) was obtained by supplementing the values of analytical calculation (5).

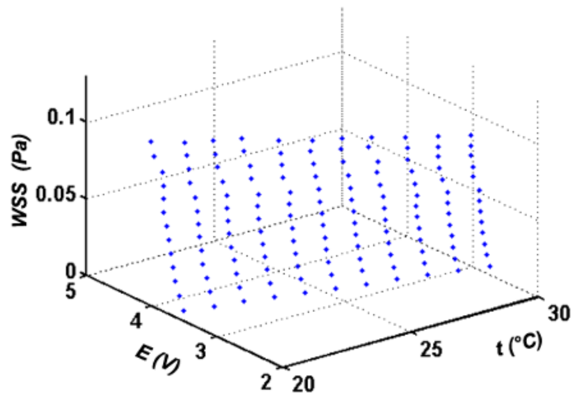

Fig. 4. Display of measured values in the resultant calibration area.

At this phase, although the calibration plane of the used probe was known, it was necessary to calculate the final value τ_W for practical application of the method during measurement when the bridge voltage E and temperature of the liquid t is known. Similarly to George's using polynomic function for calibration of the speed probe in 1981 [13], the function used here for mathematical description of the probe calibration plane for measurement of the wall shear stress is following:

$$\tau_W = (A_1) + (A_2)E + (A_3)t + (A_4)Et + (A_5)E^2 + (A_6)t^2 + (A_7)E^2t^2. \qquad (6)$$

For this purpose, a function in MATLAB was written, which enables to substitute the measured calibration plane for the required equation and to calculate the coefficients $A_1 - A_7$ for particular members. The result of the substitution of the calibration plane is shown in figure 5.

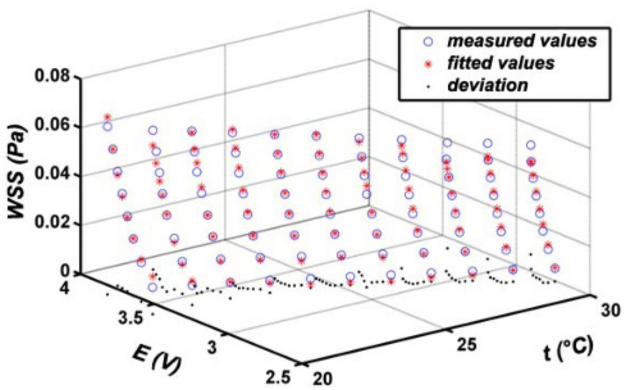

Fig. 5. Replacement of values in the resultant calibration area.

In practice, calibration is carried out in the cylinders in the following way. The values of bridge voltage E are measured in the cylinders at different rotational speed and different temperatures for required liquid and used probe.This process takes approximately six to eight hours, mainly depending on the temperature range of the calibration. The obtained calibration matrix is loaded into the program created in the MATLAB language.

Here, the matrix is processed. The result is the mathematically described calibration plane where the program for entering the values of bridge voltage E and liquid temperature t returns the value of the wall shear stress τ_W. Now it is possible to move the probe into the experimental setup and to initiate the measurement.

2.1.1 Advantages and disadvantages of the WASSCOS 1 device

Advantages:

- Continuous operation of the device – advantage over devices with pulse current generated by the piston in the cylinder, etc.

- Need for only a small sample of the fluid – this feature plays a very significant role when using expensive chemicals as substitutes of some special liquid (i.e. using a solution of sodium iodide in order to improve the optical properties of water).
- Possibility of using the device for very small values of shear stress – device can be operated practically from zero values of wall shear stress (with respect to free convection around the probe).
- Very precise control of the device temperature – due to the construction of the device (where tempering water surrounds almost all measurement area), it is possible to control the temperature with an accuracy of 0.05 °C. Such precision is very difficult to obtain by the channel-type device.
- Measurement of non-Newtonian liquids – it is possible to evaluate the instant viscosity of the used material at certain shear rate (function of rotational viscometer) if the device is complemented by drive with simultaneous measurement of the torque.
- Possibility of using the device for several types of probes – it is possible to use both probes with hot-film and electrodiffusion probes or pressure probes.

Disadvantages**:**

- Indirect method – anemometer data for calibration are compared with theoretical calculation of values of the wall shear stress at the probe.
- Limited applicability at higher values of shear rates – limitations due to the Taylor's vortices and subsequent transition to turbulence at limit speed. This disadvantage could be overcome by adding device for measuring the torque of the inner cylinder.
- Cylindrical surface at the probe – a change in curvature near the active part of probes comes to pass for probes with a flat face. The sudden change in shape can lead to faulty calibration results.

2.2 Probe calibration with the help of direct wall forces measurement

Despite the functionality of WASSCOS 1 system, verified by practical measurement in the models of vascular connection at laminar flow [3, 14], it was necessary to develop a device enabling measurement of the wall shear stress at higher values of shear rates and at the turbulent flow. This device was called WASSCOS 2. Despite known problems emerging when using direct methods (elements' oscillation, measurability of small force effects on the element, etc.) a decision was made to construct a calibration device based on direct measurement of WSS. The reason for that decision was both the distinct advantage of direct method in the possibility to gain direct value of the wall shear stress via friction force effect on the floating element, and the availability of modern measuring devices capable of measuring forces with an accuracy of 10^{-3} N and distance with an accuracy to 1 µm. Another reason was the occurence of innovative ideas for device construction that lead to limitation or complete elimination of some problems in using direct methods.

The floating element flush mounted with one wall of narrow rectangular channel is the main part of the WASSCOS 2 device. A required flow, or Reynolds number, can be set up by overflow system in the channel. During the measurement, the location of the element and force acting on the element is scanned. The calibration probe with a hot-film is located at the element, on the opposite side of the channel.

The water channel has a rectangular cross-section measuring 200x10 mm and a total length of 1500 mm, the floating element is placed at a distance of 1100 mm from the inlet edge of the channel (Figure 6).

Constant flow rate is ensured by the overflow systemsThe channel and tanks are made from cast Plexiglas 10 mm thick. The channel is, for increased stability, stiffened by lateral braces separated by a distance of 10 cm in the longitudinal direction of the channel. The channel as well as the supporting structure are also designed in such a way as to allow the use of optical anemometry methods (PIV - Particle Image Velocimetry) [15] for measurement of velocity profiles. Liquid circulation is provided by a centrifugal pump with a speed control. The liquid passes the inlet to the front calming chamber through the exchanger and chamber with a heating unit, helping to keep the desired temperature of the liquid.

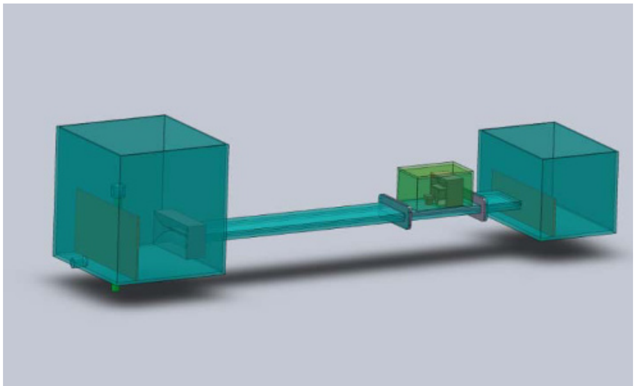

Fig. 6. Schematic image of the calibration setup with floating element.

The main part of the calibration device consists of hydrodynamic scales (Figure 7) with a floating element of 120x80 mm. Duralumin floating element is hung on three flexible hinges. The hinges are in the middle part stiff, the elastic planchets are only at ends of the hinges.

The stiffening of the middle parts enabled the placement of the scale sensors at the half of the hinges height and thus doubling of the force effect measured by force meter. The rigidity of planchets was tested experimentally and tuned in such a way as to maintain the sensitivity of the scales while avoiding undesired element vibration at the fluid flow rate.

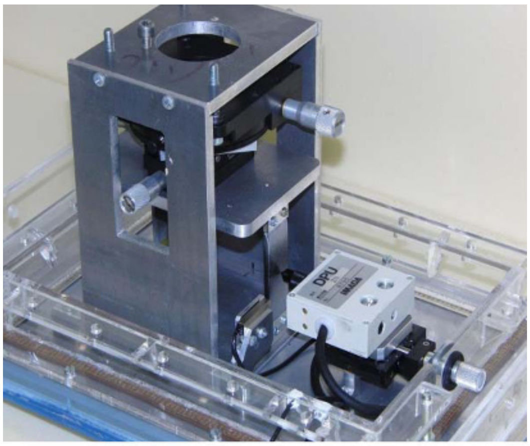

Fig. 7. Photo of functional part of the hydrodynamic scales.

Given the relatively large area of the floating element (120x80 mm) and the fact that the measured force is an area average value of the force effects in particular element locations, it was necessary to ensure that the values of the wall shear stress on the element area were constant. Fulfilment of this requirement is possible provided that a developed velocity profile is ensured over the channel height h (smaller size) and vice versa, if the boundary layer growing from the walls over the channel width b does not reach the floating element (Figure 8).

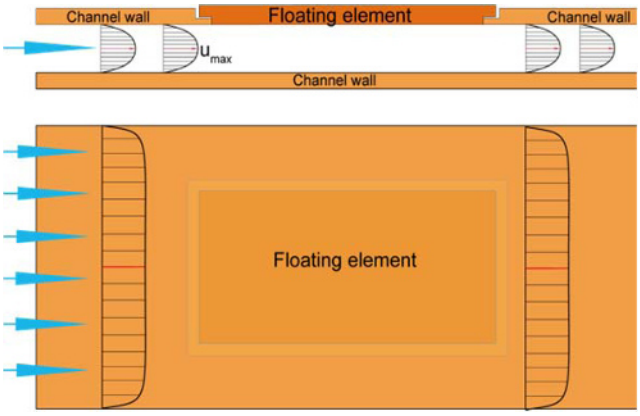

Fig. 8. Verified velocity profiles at the floating element.

The size and location of the floating element was chosen based on the requirements mentioned above. A suitable place for the element location was chosen after initial numerical calculation of velocity profiles in the channel. The actual velocity fields close to the element were subsequently verified experimentally using the PIV method.

Elimination of forces caused by pressure gradient

The flow in the channel causes, due to friction losses, a pressure gradient that generates an additional force influence on the floating element. After the elimination of those forces it is necessary to use a complicated scale structure when using direct methods of the wall shear stress measurement. These scales then tend to have a large number of precision-made parts, which greatly complicates the maintenance possibilities of scales and also the measurement. When using a direct method of measurement with the help of floating element for needs of probe calibration the situation is simplified mainly through the use of stationary flow, and thus the time-independent pressure gradient on the element. In the WASSCOS 2 device, the elimination of forces caused by pressure gradient was carried out by determination of the gradient and by subsequent subtraction of additional forces F_P from total forces on the element F_T. The friction force for evaluation of WSS is then given by:

$$F_{T_E} = F_T - F_P \quad . \tag{7}$$

The process was following. First, a pressure drop was measured at a distance equal to three times the element length, then $(3 \cdot L_E)$. Its measured dependence on Re is plotted in graph in figure 9.

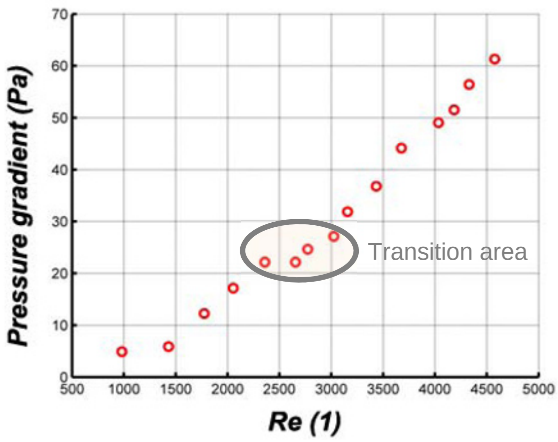

Fig. 9. Measured pressure drop dependence on the Reynolds number.

A following assumption was made to determine the additional forces on the element. A pressure difference $\Delta p = p_1 - p_2$ occurs at liquid flowing between the front and rear edge of the floating element. A narrow gap between the element and channel wall 1 mm long and approximately 0.2 mm wide (the gap width depends on the current position of the element) suddenly widens to approximately ten times higher value. The element is wholly flooded which means that the pressure in the place above the gap is equal to hydrostatic pressure p_E. According to estimates, a small amount of fluid is flowing between the place in front of the element and the place behind the element. This amount has to pass over the gap whose edge is the place of balancing the pressures $p_1 \to p_E$, or $p_E \to p_2$. In conclusion of this consideration, we can consider the assumption that an area $A_{E_{\Delta p}}$, which is influenced by the pressure gradient, is maximum equal to the product of gap length δ_S and gap width of the element b_E:

$$A_{E_{\Delta p}} = \delta_S \cdot b_E = 0{,}001 \cdot 0{,}08 = 8 \cdot 10^{-5} m^2. \tag{8}$$

The additional force on the floating element caused by the pressure gradient is being calculated:

$$F_P = \Delta p \cdot A_{E_{\Delta p}} . \qquad (9)$$

The caused additional force depending on Re is plotted in figure 10. This force is automatically subtracted at the creation of calibration planes.

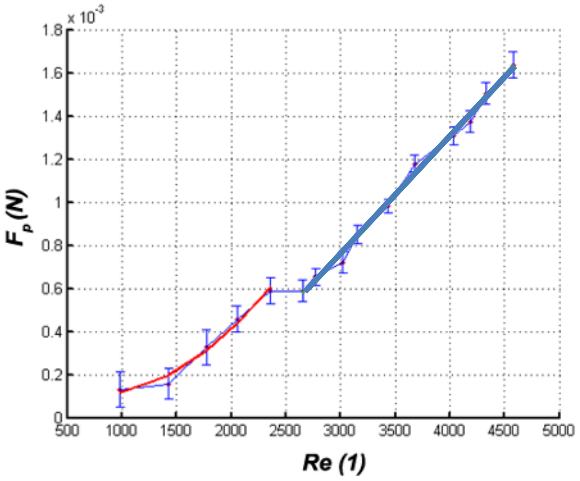

Fig. 10. The additional force on the floating element caused by the pressure gradient on the element.

Calibration range

In the channel, a constant flow rate at Reynolds numbers from 0 to approximately 10 000 can be achieved during the operation. Due to the small force effects on the floating element at small values of wall shear stress and distinction of the force meter, this method can be used from force F_{MIN} = 0.005 N. When converting to the value of wall shear stress the value will be $\tau_{W_{MIN}} = 0.26\ Pa$. The upper boundary at maximum flow corresponds to the force $F_{MAX} \doteq 0.11\ N$ and the wall shear stress $\tau_W \doteq 6\ Pa$. These values apply to water. The calibration ranges for liquids of different viscosities are different.

Calibration results

During the probe calibration using a floating element, friction force on the floating element F_T was compared with output voltage of anemometer E at liquid temperatures from 20°C to 30°C. The measurement was carried out for Reynolds numbers from 900 to 5500. The additional force caused by the pressure gradient F_P was subtracted from the friction force measured by the force meter F_C for relevant Reynolds numbers.

As a next step, it was necessary to combine the results of CTA and force measurement and to eliminate the Re value from the results. By this step, a resultant calibration plane for the probe was obtained (Figure 11).

The calibration in the WASSCOS 2 device usually takes place the following way. A calibration probe is placed at the floating element on the opposite side of the

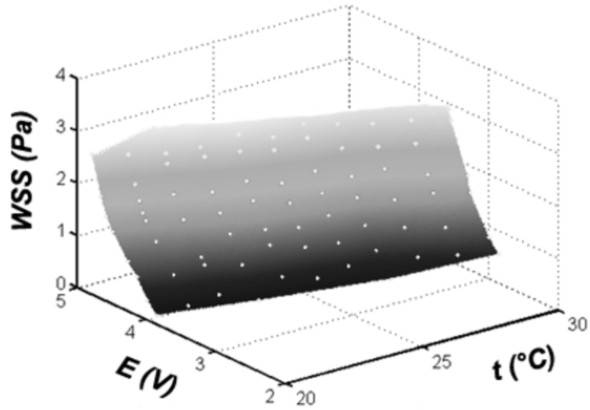

Fig. 11. The resultant probe calibration area.

channel. It is theoretically possible to use any liquid in the channel without necessary knowledge of its viscosity. For different viscosities, it is only necessary to provide relevant shapes of the velocity profiles in front of and behind the element. The values of the forces on the element, pressure drop and bridge voltage CTA are measured in the channel for the required range of temperatures and Reynolds numbers. The measured values are subsequently saved in files. Data are evaluated with the help of programs prepared in MATLAB. The result is a mathematical description of the probe calibration plane and subsequent possibility of determining the value of the wall shear stress τ_W from the liquid temperature **t** and bridge voltage **E**. It is now possible to move the probe to the measuring point and start the measurement.

2.2.1 Advantages and disadvantages of the WASSCOS 2 device

Advantages**:**

- Direct method of measurement – value of the wall shear stress is deducted directly through the force acting on floating element. The method can be also used for research of the influences of different surfaces on the friction resistance of the solids.
- Possibility of using in laminar and turbulent flow – it is not necessary to fulfil the condition of laminar flow for the function, because the calibration is not based on the analytical calculations.
- Possibility of using for non-Newtonian liquids – there is no need to have an explicit knowledge of the instant viscosity, because it is a direct method.
- Vertical channel wall – at place of active part of probes with a flat face there is no change in the curvature, as in the case of rotating cylinder.
- Simplicity – hydrodynamic scales are constructed from a relatively small number of parts leading to ensuring the maintenance of the whole device, lowering the element vibration caused by clearances and better repeatability of the measurement.

- Elimination of forces caused by pressure gradient – the floating element structure allows for precise quantification of the area where the pressure gradient acts, making it possible to determine the additional force caused by measured difference of pressure in front of and behind the element.

Disadvantages**:**

- Need for large quantities of fluid – the capacity of the entire experimental setup is approximately 100 liters. Such large capacity allows for easy working only with water or weak solutions of other chemicals. Using of some special substances is difficult for health, environmental and also the financial reasons.
- Temperature control – monitoring the temperature in the whole volume is more complicated as opposed to the device with rotating cylinder. The temperature in the setup with rectangular channel can be controlled with an accuracy of 0.1 °C.

3. Conclusion

After performing a more detailed analysis it became clear that the question of probe calibration of the shear stress in the liquids of low viscosity was solved by various authors in the past but the devices developed by them were often very complex and difficult to apply for common experiment. At the same time, having solved this problem successfully, we can formulate our own contribution of our work as follows:

1) Two devices for calibration were designed, constructed and verified. The devices are built in a way that enables a relatively simple usability at common laboratory experiments. The construction is designed so that the device could be easily serviced and maintained, which proved to be a basic condition for successful repeatability of the experiments. WASSCOS 2 device with a floating element extends the use of the WASSCOS 1 device to turbulent flow and to possibility of using it in non-Newtonian liquids.

2) Software equipment was created in MATLAB, which, after loading the data from calibration devices, evaluates data and calculates a mathematical formulation of calibration plane. The wall shear stress $\tau_w\ [Pa]$ at the calibration planes depends on the value of the anemometer voltage $E\,[V]$ and the liquid temperature $t\,[°C]$. Subsequently, the program is ready for the measurement and the wall shear stress is calculated after entering the value of the anemometer voltage and liquid temperature.

4. Acknowledgement

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