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The Dead Zones of Protoplanetary Disks are Not Dead

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Abstract. We show that the "dead" zone of a protoplanetary disk fills with robust 3D vortices from a purely hydrodynamic instability. This new instability is not linear and requires a weak finite-amplitude initial perturbation. The instability was not seen previously either due to a lack of numerical spatial resolution, or because many previous simulations either ignored vertical gravity or had initial flows with constant density. Our new finite-amplitude instability is due to a family of previously-unknown critical layers that form in rotating, shearing, vertically stratified flows like those in protoplanetary disks. Initial perturbations of white noise (with Mach numbers much less than unity), waves, or vortices can trigger the instability. A small-volume, small-amplitude initial vortex confined to one part of the disk can fill the disk with vortices by exciting a nearby critical layer. The critical layer produces an intense vortex layer that rolls-up to form vortices with large-amplitudes and volumes. This 1^{st} generation of vortices then sheds waves that excite nearby critical layers, which in turn, create a 2^{nd} generation of vortices with large amplitudes and volumes. The mechanism of exciting nearby critical layers and turning them into large vortices self-similarly, self-replicates until large vortices fill the disk at all radii.

1 Introduction

We have discovered a new, finite-amplitude, purely hydrodynamic instability that allows a small perturbation, such as low-Mach number noise, a wave, or an isolated vortex, to create an array of large-amplitude vortices capable of transporting angular momentum in a protoplanetary disk (PPD). Once triggered, the instability continually excites new vortices that eventually fill all radial locations of the dead zones of a PPD, including locations far from the initial perturbation. The instability is self-sustaining because it draws its energy from the Keplerian shear. This instability is not subtle; it requires no special tuning or unphysical initial conditions. We serendipitously discovered the instability using a variety of codes and in a variety of flows, including vertically-stratified Couette flow and simulations of Jupiter's Great Red Spot. This new instability was the cause of the formation of the off-midplane vortices that we reported, but did not explain, six years ago when we examined vortices in PPDs [1]. In those calculations, a perturbation by a *single* vortex filled the PPD with large-amplitude vortices. The reason that all of these different flows have this instability is that they all have three required ingredients: strong rotation, shear, and vertical stratification.

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2 Why this Instability Was Not Discovered Previously

We need to answer a skeptic's questions: "How can this instability exist, be so ubiquitous, and produce such large vortices, when two decades of numerical simulations of PPDs by others have failed to find it?", and "Doesn't Lord Rayleigh's centrifugal stability theorem [2] show that a flow, such as Keplerian flow, is stable if its angular momentum increases with radius?" One reason that our instability was not discovered previously is that we use a *spectral*, rather than a finite-difference or finite-volume numerical method. Spectral methods can resolve features that are approximately 12–16 times smaller (in each spatial dimension) than a second-order finite-difference calculation with the same number of computational elements. Our instability is 3D, and to be simulated, it requires a 3D code with radial resolution of the PPD that is better than $H/10$, where H is the vertical pressure scale height. Most previous PPD calculations did not have the required resolution. A more fundamental reason that our new instability was not previously discovered is that much of the PPD literature used constant-density fluids, c.f., [3] or ideal gases in which the initial density was uniform in the vertical direction, rather than stably stratified. Rayleigh's criterion for centrifugal stability *applies only to fluids with constant density* and therefore is not relevant to PPDs where the density falls off approximately like a Gaussian away from the midplane. One stability study of PPDs [4] argues that the stability of a PPD is governed by Rayleigh's centrifugal criterion, and another initial-value study of PPDs [5] with a very high spatial resolution did not show our new instability. However, both studies initialized the flow with constant density even though they both used an ideal gas equation of state. We can only speculate on why non-constant density flows have been overlooked in previous stability analyses of PPDs, but there appears to be a belief that if a fluid flow with a constant density is stable, then the same flow with a density that is vertically stably-stratified is even more stable. *This belief is not true*, and our new instability *requires* stable vertical stratification. In the language of fluid dynamicists, these omissions eliminate *baroclinic* instabilities. Finally, we note that much of the classic work on centrifugal stability deals with *linear* instability, rather than finite-amplitude instability, which deals with small, but finite, initial perturbations. PPDs are shear flows, and for many shear flows, such as channel flow, finite-amplitude instabilities are more frequently observed than linear ones. Finite-amplitude instabilities tend to grow to large magnitude and spread throughout the flow (despite the fact that the initial perturbation is often localized in space) because the instability taps into the energy of the differential motion of the shear. We show below that the reason that our new instability is so powerful and invasive is that it derives its energy from the kinetic energy of the Keplerian shear of the PPD.

3 Critical Layers and Vortex Layers

Our new instability is due to critical layers [6], which are linear eigenmodes of a fluid that neither grow nor decay in time – they are neutrally stable. In addition, the eigenmode's velocity is singular (infinite) at the critical layer. Traditional critical layers are considered unimportant because they do not grow. However, we have discovered a new family of critical layers that are easy to excite. They occur in rotating, shearing flows *that are vertically stably stratified*, such as PPDs. We call these new layers *baroclinic* critical layers. In traditional critical layers, the singularity occurs in the stream-wise component of the eigenmode's velocity, but in baroclinic critical layers, the singularity is in the vertical or z component (e.g., along the rotation axis of a PPD). This difference makes baroclinic critical layers easy to excite. Baroclinic critical layers create large-amplitude vortex layers because in a fluid rotating with angular velocity Ω, vertical vorticity is created at a rate approximately equal to the *Coriolis term*, or $2\Omega(\partial v_z/\partial z)$, where v_z is the vertical velocity, which is singular (i.e. $\pm\infty$) in a baroclinic critical layer. Note that the *Coriolis term* is what makes an intense vortex appear in a sink of slowly rotating water (with angular velocity Ω) when the sink's drain is opened; the drain creates

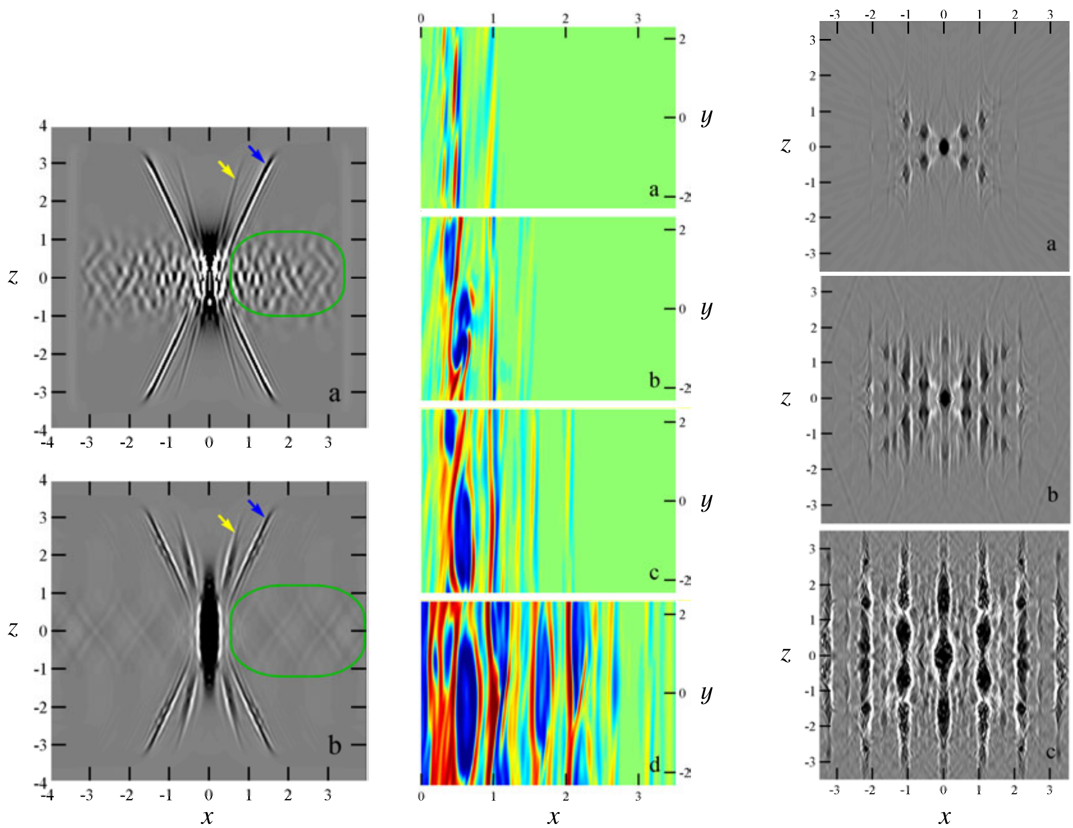

Figure 1. Left Column Bottom Panel: Vertical vorticity ω_z in the PPD like the one used in Barranco & Marcus [1]. The ω_z is shown in the cross-stream (radial) x and vertical z plane. A small perturbing vortex is located at the origin. The two X patterns are the vortex layers and they coincide with the locations of the critical layers. The blue [yellow] arrow points to the fundamental $m = 1$ [first harmonic $m = 2$] critical layer. See text for details. The green balloon surrounds Poincaré waves produced by the perturbing vortex, and are not critical layers. Top Panel: Same as bottom but with a perturbation from a transient wave at the origin. **Middle Column** Cross-stream (radial) x and stream-wise (azimuthal) y half-plane showing ω_z with red as the most cyclonic vorticity; blue as the most anticyclonic; and green with $\omega_z = 0$. The unperturbed flow is Boussinesq plane Couette flow with $\bar{V} = -\sigma x$ and linearly stratified in the vertical direction so N is constant. The computational domain is larger than shown. The initial perturbing vortex is at the origin (which is beneath the planes shown in the figure). a) $t = 64/N$, showing the 1^{st} generation of critical layers excited by the initial perturbation. b) $t = 256/N$, showing the rolled-up vortices from the 1^{st} generation of critical layers and vortex layers. c) $t = 576/N$, showing a 2^{nd} generation. d) $t = 2240/N$, showing a 3^{rd} generation and the filling of the domain with large-amplitude vortices. **Right Column** The same flow as in the Middle Column. A grey scale shows ω_z in the x–z plane at $y = 0$ at different times with black anticyclonic and white cyclonic vorticity. a) $t = 480/N$. 1^{st}-generation vortices. b) $t = 1632/N$, 1^{st} and 2^{nd}-generation vortices. c) $t = 3072/N$, the large-amplitude vortices fill the domain.

a large ($\partial v_z/\partial z$), and this term couples with Ω to produce vertical vorticity. Thus, despite the fact that the eigenmodes of baroclinic critical layers are neutrally stable, they are easily excited and create intense vortex layers. Baroclinic critical layers were responsible for the vortex layers and vortices observed in Barranco & Marcus [1]. We verified this by noting that the locations of critical layers and vortex layers coincided. For an isothermal PPD, the Brunt-Väisälä frequency $N(z)$ (i.e, the frequency that a convectively stable parcel of gas oscillates when it is displaced vertically) is linear in z, and the locations of the critical layers form X-patterns in the x–z planes, where x is the radial direction. The two panels in the left column of Fig. 1 show the X-patterns of the vertical vorticity layers in numerically-computed PPDs. The layers in the upper (lower) panel of the left column of Fig. 1 were excited by a small initial perturbation consisting of a wave (vortex) at the origin. In both cases the critical layers and vortex layers coincide.

To understand how vortex layers roll up into vortices and fill the dead zone with vortices, we simplified our calculation by "Cartesianizing" our PPD using the shearing box approximation. The unperturbed sub-Keplerian velocity $\bar{V}(x)$ of the PPD in the y direction was approximated as $\bar{V}(x) = -\sigma x$ where σ is the shear. We simplified further by making the flow Boussinesq and N constant. This flow is linearly stable. The locations in x of these critical layers are independent of z (not X patterns), and the dimensionless distance in x between a steady perturbation and the critical layer it excites is $x^* = \pm 1/m$, where the unit of length is $(NL)/(2\pi\sigma)$, L is the local circumference of the PPD, and m is a non-zero integer such that the wavelength of the eigenmode of the critical layer is $L/|m|$. The middle column of Fig. 1 shows the vorticity due to the $m = 1$, 2, and 3 critical layers that were excited by a perturbation at the origin. Panels a and b show that the critical layers are located at $x = 1/m \leq 1$, as predicted. The layers are initially parallel to the y-axis. Vortex layers can become linearly unstable [7, 8], and when they do, they "roll-up" into compact vortices with large circulations because all of the vorticity of the layer becomes concentrated in the vortices. Panel b in the middle column of Fig. 1 shows rolled up vortices. Panels c and d appear to show that our analysis is incorrect because there are critical layers at $x > 1$. However, the layers at $1 < x \leq 2$ (panel c) were excited by the 1^{st}-generation vortices at $x \leq 1$, not the initial vortex at the origin. A vortex at *any* location can excite critical layers. The 2^{nd}-generation vortices at $1 < x \leq 2$ spawn 3^{rd}-generation (panel d) critical layers at $2 < x \leq 3$. The self-replication of critical layers and vortices continues until the entire domain fills with vortices. The right column of Fig. 1 shows the same flow as in the middle column, but viewed in the radial x and vertical z plane. These vortices have large amplitudes. The magnitude of their vorticity is of the same order as the Coriolis parameter (i.e., their Rossby numbers are of order unity). The kinetic energy of the vortices in the bottom panel of the right column of Fig. 1 has more than 400 times the energy of the initial perturbing vortex. The energy of the vortices is derived from the kinetic energy of the unperturbed sub-Keplerian velocity.

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