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# Vortex migration in protoplanetary discs

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**Abstract.** Vortices embedded in protoplanetary discs can act as obstacles to the unperturbed disc flow. The resulting velocity perturbations propagate away from the vortex in the form of density waves that transport angular momentum. Any asymmetry between the inner and the outer density wave means that the region around the vortex has to change its angular momentum. We find that this leads to orbital migration of the vortex. Asymmetric waves always arise except in the case of a disc with constant pressure, for isothermal as well as non-isothermal discs. Depending on the size and strength of the vortex, the resulting migration time scales can be as short as a few thousand orbits.

## 1 Introduction

Vortices are capable of playing a key role in concentrating solid particles, an essential stage in the formation process of planets [1]. Not only could the enhanced solid concentration inside vortices considerably speed up the formation of planets [2], inside a vortex solids would be safe from radial drift due to a mismatch in gas and solid angular velocity caused by a pressure gradient in the gas disc [3]. Or so it was thought. In this contribution, we show that vortices themselves are subject to radial migration, driven by the same pressure gradient that causes solid particles to drift inward at alarmingly short time scales. For a more detailed description we refer to [4].

## 2 Density wave emission

We will work in cylindrical coordinates $(r, \varphi)$, and use vertically averaged quantities only (surface density $\Sigma$, velocity $\mathbf{v}$ and 2D pressure $p$), and consider a barotropic equation of state with $p \propto \Sigma^{5/3}$. A constant vortensity disc ($\Sigma \propto r^{-3/2}$) then has a constant aspect ratio $c_s/r\Omega$, where $c_s$ is the sound speed and $\Omega$ the Keplerian angular velocity. Linearisation of the Euler equations in a steady state leads to the following relation describing the propagation of density waves in discs [4]:

$$\frac{1}{r\Omega}\frac{d}{dr}(r\Omega)\frac{d}{dr}(r\tilde{u}_\varphi) + \left[\frac{(\omega - m\Omega)^2 - \Omega^2}{c_s^2} - \frac{m^2}{r^2}\right] r\tilde{u}_\varphi = 0, \quad (1)$$

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where we have taken the velocity perturbation to be of the form

$$\mathbf{u}(t, r, \varphi)) = \sum_m \int \tilde{\mathbf{u}}(\omega, r, m) \exp(i\omega t - im\varphi) d\omega. \tag{2}$$

The quantity in square brackets in equation (1) determines whether wave-like solutions exist. Taking $\omega = m\Omega(r_0)$ for a vortex corotating with the gas at $r = r_0$, we see that wave-like solutions exist on either side of the vortex for $r > r_s^+$ and $r < r_s^-$, where

$$r_s^{\pm} = r_0 \left( 1 \pm \sqrt{\frac{1}{m^2} + h^2} \right)^{2/3}, \tag{3}$$

with $h = c_s/r\Omega$ the constant aspect ratio of the disc. In the limit of $m \to \infty$, these radii correspond to the sonic lines of the vortex (hence the subscript $s$), where the disc material moves at the sound speed with respect to the vortex.

The emerging picture is that a vortex acts as an obstacle in the flow, and that the resulting perturbations lead to density wave emission at the sonic lines.

## 3 Wave asymmetries

Two effects conspire to make the density waves launched not of equal strength. First of all, it is easy to see from equation (3) that $|r_s^+ - r_0| < |r_s^- - r_0|$, i.e. the outer sonic line is closer to the vortex than the inner sonic line. This favours wave emission in the outer disc. This is purely a geometric effect, which is also to a large extent responsible for Type I planetary migration in gaseous discs [5, 6]. However, note that in the case of a vortex, the interaction with the disc is purely hydrodynamical, while in the case of a planet, it is gravitational.

A second source of asymmetry results from the fact that the vortex will change its shape in response to background gradients in the disc. This is because the vortex, unlike a planet, is part of the disc and 'feels' any gradients in density and pressure. It turns out that it is the background gradient in vortensity that plays a crucial role here [4]. If the disc has a radial gradient in vortensity, an embedded vortex will be more extended in the direction where the vortensity is highest, thereby favouring wave emission in that direction. This means that for shallow surface density gradients, wave emission in the inner disc will be stronger than in the outer disc, which can compete against the geometric effect discussed above.

Any asymmetry in the strength of the density waves will lead to the region containing the vortex changing its angular momentum. Anticyclonic vortices in isothermal discs are associated with an excess in angular momentum because they have high surface density cores [4]. The vortex can therefore reduce its angular momentum either by shrinking or by migrating inward. We find that the dominant response of the vortex is that of migration.

## 4 Results

We have performed two-dimensional isothermal hydrodynamic simulations of embedded vortices using the RODEO code [7] to study their migration behaviour. We put a vortex in by hand of a specified strength and size, and let it evolve for up to 100 orbits. Since our isothermal simulations do not include any mechanism to create and grow vortices, the vortices slowly decay because of numerical diffusion. Before the decay sets in, a well-defined migration rate can be measured [4].

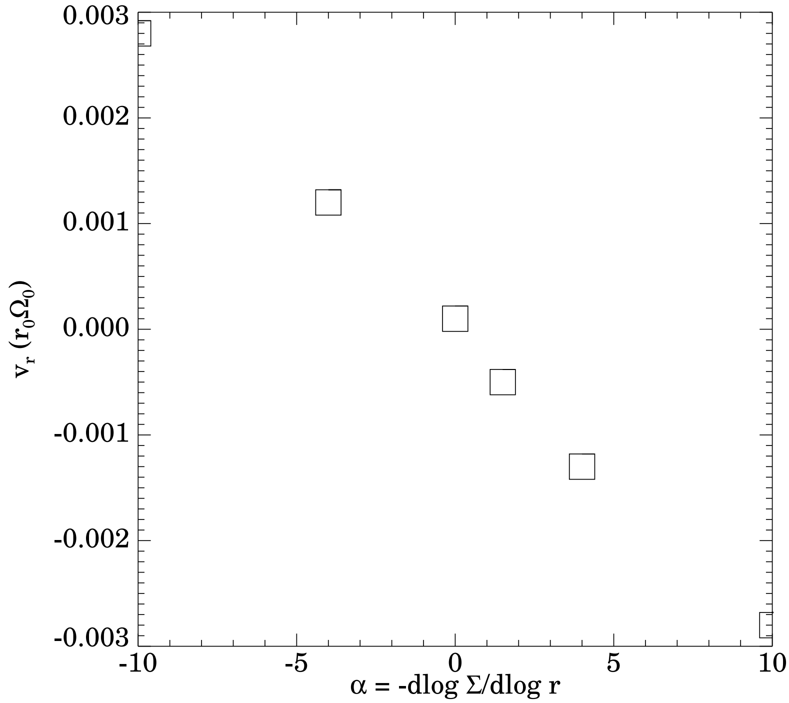


**Figure 1.** Radial migration speed of a vortex of size $hr/2$ in an isothermal disc with $hr = 0.1$, as a function of surface density gradient.

In Fig. 1 we show the resulting migration speeds in a disc with $h = 0.1$ for vortices of size $hr/2$, as a function of the background surface density gradient. Since these are isothermal simulations, a surface density gradient corresponds to a pressure gradient. The migration speed is found to be proportional to the pressure gradient, with inward migration found for negative pressure gradients. Non-isothermal runs confirm that it is the pressure gradient that determines the migration direction. Note that negative pressure gradients are expected in discs around young stars, where the hottest and densest regions exist close to the star. The migration time scale can be as short as 1000 orbital time scales for these big vortices in thick discs. Thinner discs and smaller vortices show slower migration, but migration is always very significant on time scales comparable to the disc life time as soon as vortices form that are comparable in size to the disc thickness.

## 5 Consequences

Vortex migration may seriously affect the ability of vortices to collect and save dust. While their mobility may increase their potential for sweeping up dust in the disc, it becomes much harder for vortices to save solids from drifting into the central star, because the same pressure gradient that drives solids inward, also makes vortices migrate inward. Since typical migration time scales are much less than the disc life time, we still expect all solids to end up very close to the central star.

A second problem arises in the Subcritical Baroclinic Instability (SBI) [8–10]. While regions of the disc can be unstable to the SBI and create large vortices, as soon as the vortices produced get large enough they will quickly migrate away from the unstable region. If this instability is to play a role in the evolution of the disc, a mechanism will have to be found to generate new vortices on which the SBI can feed. Note that it is the subcritical nature of the SBI that is at the heart of the problem: it needs a finite amplitude perturbation to get started. If all large perturbations migrate away, it is difficult to see how to keep the SBI going.

## 6 Conclusions

Vortices embedded in protoplanetary discs act as obstacles to the unperturbed disc flow. The resulting perturbations propagate away from the vortex as density waves, which may be asymmetric leading to vortex migration in the case of a background pressure gradient in the disc. Since migration time scales are typically much shorter than the disc life time, vortex migration can have serious consequenses for any mechanism invoking them.

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