

Relationship between Luminosity, Irradiance and Temperature of star on the orbital parameters of exoplanets

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Abstract. For 759 exoplanets detected by radial velocities method we found that distances of exoplanets from central star comply in general Schmidt law and these distances depend on the stellar surface temperature. Every stellar spectral class has a little different distribution. The Luminosity and the Irradiance has not effect on the distribution of distances of exoplanets. We have found the new formulas for calculation of effective temperature of exoplanets for spectral classes F, G, and K. These new formulas we can use for future calculation of habitable planets.

1 Introduction

Planets are extremely faint objects compared to their parent stars. At visible wavelengths, they usually have less than a millionth of their parent star's brightness. It is very difficult to detect these faint light sources and it is necessary to block the light from the parent star in order to reduce the glare. Detection of light from planet is great challenge [1]. Most confirmed extrasolar planets have been found with ground-based telescopes but now many exoplanets have been found also with space-based telescopes. COROT [2] and KEPLER [3], [4] are the two currently active space missions dedicated to searching for extrasolar planets. We know many methods, how to detect the exoplanets. We will use extrasolar planets detected by these methods in our work:

- Radial velocity method,
- Transit method,
- Astrometry method.

The radial velocity method or Doppler method is most productive method of discovering exoplanets [5]. Variations in the star's radial velocity can be detected from displacements in the star's spectral lines due to the Doppler effect. According [6] we cannot determine a planet's true mass only we can set a lower limit of mass.

The transit method has been the second most productive method for the detection of exoplanets. If a planet transits in front of its parent star's disk, then the observed brightness of the star drops by a small amount. The amount depends on its size and on the size of the planet. This method reveals the radius of a planet.

The astrometry method is not well productive method today, because we need precisely measuring a star's position in the sky and observing the changes in that position over time.

The best idea is to confirm exoplanets with a combination of these methods together. A total of 759 planets in 609 extrasolar systems have been identified as of February 11, 2012 [7]. We know also 99 multi-planet extrasolar systems with more than 2 planets in one system [8], [9] and

[10]. In [11] it is argued that the physical quantities of the exoplanet are functions of astrophysical parameters of the central (host) stars, such as the stellar luminosity L , stellar irradiance J or stellar effective temperature T_{eff} . These main physical parameters we will use in our study.

2 Definition of parameters and regression analyse

We have calculated the luminosity of a star according [12]

$$L = 4\pi R_{\text{Sun}}^2 \sigma T_{\text{eff}}^4, \quad (1)$$

where R_{Sun} is the stellar radius in radii of the Sun, σ is the Stefan-Boltzmann constant.

We have used for stellar irradiance the Stephan-Boltzmann law in form

$$J = \sigma T_{\text{eff}}^4. \quad (2)$$

We have used for the statistical study of detected extrasolar systems the regression methods. A regression model relates Y to a function of X and β in the form

$$Y = f(X, \beta), \quad (3)$$

where β are unknown parameters, X are independent variables, and Y is a dependent variable. The observed data (x_i, y_i) , $i = 1, 2, \dots, n$, with n the number of observations, satisfy equations

$$y_i = f(x_i, \beta) + \epsilon_i, \quad (4)$$

where ϵ_i is an error term.

For a linear regression, we get two parameters β_0 and β_1 and it is valid that

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i. \quad (5)$$

The residual

$$\epsilon_i = y_i - f_i \quad (6)$$

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is the difference between the true value of the dependent variable y_i and the value of the dependent variable predicted by the model f_i . We measure the goodness of fit by the coefficient of determination R^2 ,

$$R^2 = \frac{SS_{\text{reg}}}{SS_{\text{tot}}}, \quad (7)$$

where SS_{reg} is the regression sum of squares,

$$SS_{\text{reg}} = \sum_{i=1}^n (f_i - y_{\text{avg}})^2, \quad (8)$$

and SS_{tot} is the total sum of squares,

$$SS_{\text{tot}} = \sum_{i=1}^n (y_i - y_{\text{avg}})^2. \quad (9)$$

In (8) and (9), y_{avg} is the mean of observed values y_i . When the model function is not linear in the parameters, the sum $\sum_{i=1}^n \epsilon_i^2$ must be minimized by an iterative procedure.

In our recent work [13] we have studied the dependence of orbital parameters of exoplanets. We have studied the distance of planets r_p from the central star on the parameter $v_p r_p$, where the parameter v_p is orbital velocity of planets. In this work we will study the parameters $r_p T_{\text{eff}}$, $r_p L$ and $r_p J$ on the orbital parameter $v_p r_p$ for the stellar spectral classes.

3 Results for stellar spectral classes

The spectral class of a star is a designated class of a star describing the ionization of its photosphere, giving an objective measure of the photosphere's temperature. Most stars are currently classified using letters O, B, A, F, G, K and M, where O stars are the hottest and M stars are the coolest. We have identified more than 80 exoplanets for the statistical study only in these spectral classes:

- F class with T_{eff} between 6000 – 7500 K,
- G class with T_{eff} between 5200 – 6000 K,
- K class with T_{eff} between 3700 – 5200 K.

We can find for stars of the spectral type F from figure 1 that the parameter $r_p T_{\text{eff}}$ depends on the parameter $v_p r_p$ with the coefficient of determination $R^2 = 0.997$. The predicted equation of regression is in the form

$$r_p T_{\text{eff}} \approx K x^{1.9963}, \quad (10)$$

where $K = 4 \times 10^{-17}$ in unit $[sK m^{-1}]$ and $x = v_p r_p$.

It is valid for the stellar spectral type F that the parameter $r_p L$ has big scattering with the coefficient of determination $R^2 = 0.923$. We get better results for the parameter $r_p J$ with the value of reliability $R^2 = 0.995$. It is obvious from figure 1 that the parameter $r_p T_{\text{eff}}$ affects mainly the orbits of exoplanets for the stellar spectral type F.

We can find for stars belonging to the spectral type G from figure 2 that the parameter $r_p T_{\text{eff}}$ depends on the parameter $v_p r_p$ with the coefficient of determination $R^2 = 0.995$. The obtained equation of regression is in the form

$$r_p T_{\text{eff}} \approx K x^{1.976}, \quad (11)$$

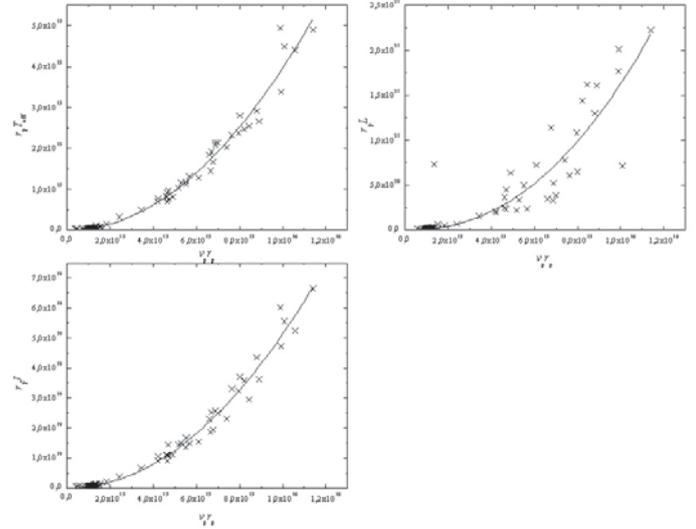


Fig. 1. Dependence of the parameters $r_p T_{\text{eff}}$, $r_p L$, and $r_p J$ on $v_p r_p$ for the stellar spectral class F. According to the regression analysis, the best is the power interpolation with the coefficient of determination $R^2 = 0.997$ for the parameter $r_p T_{\text{eff}}$.

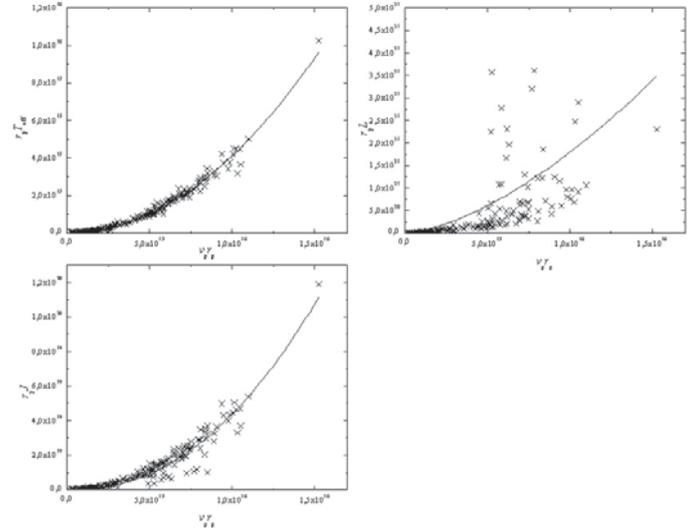


Fig. 2. Dependence of the parameters $r_p T_{\text{eff}}$, $r_p L$, and $r_p J$ on the parameter $v_p r_p$ for the stellar spectral class G. According to the regression analysis, the best is the power interpolation with the coefficient of determination $R^2 = 0.983$ for the parameter $r_p T_{\text{eff}}$.

where $K = 1 \times 10^{-16}$ in unit $[sK m^{-1}]$ and $x = v_p r_p$.

We observe for the stellar spectral type G that the parameter $r_p L$ has big scattering with the coefficient of determination $R^2 = 0.895$. We get better results for the parameter $r_p J$ with the coefficient of determination $R^2 = 0.979$. From this it follows that the parameter $r_p T_{\text{eff}}$ affects mainly the orbits of exoplanets.

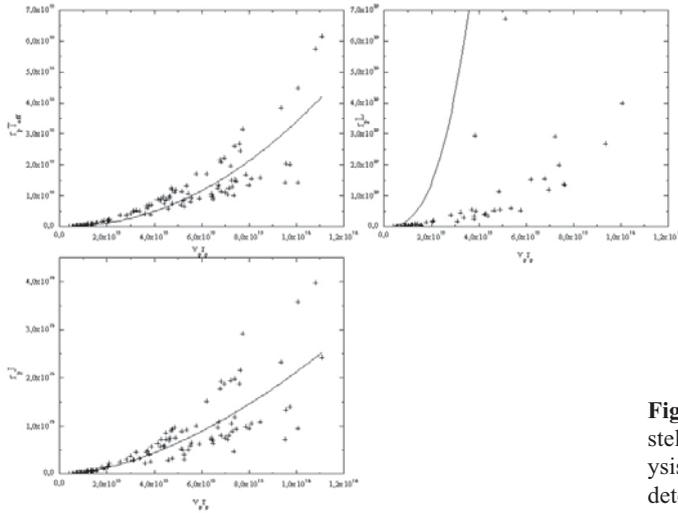


Fig. 3. Dependence of the parameters $r_p T_{\text{eff}}$, $r_p L$, and $r_p J$ on $v_p r_p$ for the stellar spectral class K. According to the regression analysis, the best is the power interpolation with the coefficient of determination $R^2 = 0.776$ for the parameter $r_p T_{\text{eff}}$.

We can find for stars of the spectral type K from figure 3 that the parameter $r_p T_{\text{eff}}$ depends on the parameter $v_p r_p$ with the coefficient of determination $R^2 = 0.776$. The predicted equation of regression is in the form

$$r_p T_{\text{eff}} \approx Kx^{1.807}, \quad (12)$$

where $K = 4 \times 10^{-14}$ in unit $[sKm^{-1}]$ and $x = v_p r_p$.

It is valid for stars of the stellar spectral type K that the parameters $r_p L$ and $r_p T_{\text{eff}}$ have the same regressions with the coefficient of determination $R^2 = 0.974$. This is the main difference from stars of the spectral type G. The parameter $r_p T_{\text{eff}}$ affects mainly the orbits of exoplanets for stars of the spectral type K. We can see two types of distributions in figure 3. Many exoplanets are described by the power regression, but some exoplanets deviate from this function.

4 Temperature of exoplanets

We can calculate the effective temperature of the exoplanets T_{eq} by equating the energy received from the star and the energy radiated by the exoplanet, under the black-body approximation in the form

$$T_{\text{eq}}^4 = \frac{R_{\text{Star}}^2 T_{\text{eff}}^4}{4r_p^2}, \quad (13)$$

where R_{Star} is radius of star. In [14] they have calculated T_{eq} for Kepler exoplanet candidates according the formula

$$T_{\text{eq}} = T_{\text{eff}} \sqrt{\frac{R_{\text{Star}}}{2r_p} [f(1 - A_B)]^{1/4}}, \quad (14)$$

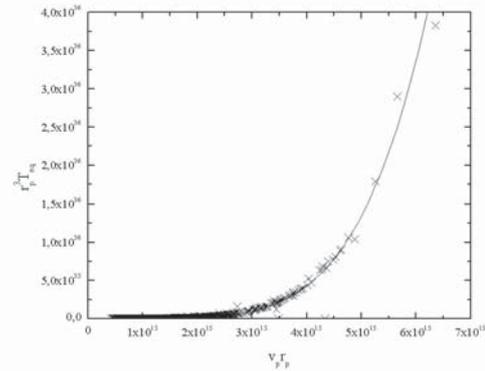


Fig. 4. Dependence of the parameters $r_p^3 T_{\text{eq}}$ on the $v_p r_p$ for the stellar spectral type F and G. According to the regression analysis, the best is the power interpolation with the coefficient of determination $R^2 = 0.9987$ for the parameter $r_p^3 T_{\text{eq}}$.

where $f = 1$ is factor of full atmospheric thermal circulation and A_B is the Bond albedo. In [14] they have assumed $A_B = 0.3$.

We have applied our regression methods on the 2321 Kepler exoplanet candidates for the calculation of the parameter T_{eq} . We have found according the regression methods the dependence of parameters $r_p^3 T_{\text{eq}}$ on the $v_p r_p$.

The obtained equation is from figure 4 with the coefficient of regression $R^2 = 0.9987$ for spectral class F and G in the form

$$y = Cx^5, \quad (15)$$

where $C = 5 \times 10^{-43}$ in unit $[sKm]$ or

$$r_p^3 T_{\text{eq}} = C(v_p r_p)^5 \quad (16)$$

and after the modification we can get new formula in the form

$$T_{\text{eq}} = C v_p^5 r_p^2. \quad (17)$$

The obtained equation from figure 5 with the coefficient of regression $R^2 = 0.9688$ for spectral class K is in the form

$$y = Cx^{4.52}, \quad (18)$$

where $C = 1 \times 10^{-35}$ in unit $[sKm]$ and after modification we get the new formula

$$T_{\text{eq}} = C v_p^{4.52} r_p^{1.52}. \quad (19)$$

We can use these new formulas for determination of habitable planets [15] and [16] in the future. Habitable zone is the region around a star within which it is theoretically possible for a planet with sufficient atmospheric pressure to maintain liquid water on its surface. The habitable planets should have surface temperature T_{eq} between 273 K - 373 K. But it is very important to calculate also the green house effect of exoplanet atmosphere and also the factor f of thermal circulation.

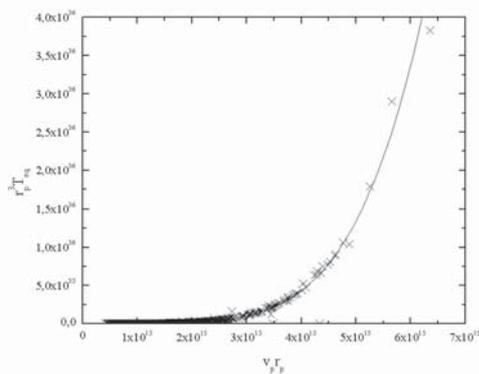


Fig. 5. Dependence of the parameters $r_p^3 T_{eq}$ on the $v_p r_p$ for the stellar spectral type K. According to the regression analysis, the best is the power interpolation with the coefficient of determination $R^2 = 0.9688$ for the parameter $r_p^3 T_{eq}$.

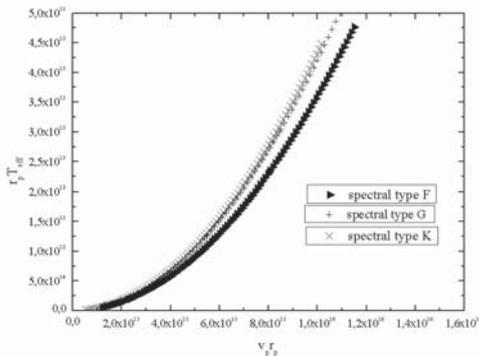


Fig. 6. Dependence of the parameter $r_p T_{eff}$ on $r_p v_p$ for different stellar spectral classes.

5 Conclusion

We have found for 759 exoplanets detected by the method of radial velocities that the distances of exoplanets from the central star obey, in general, the Schmidt law and these distances r_p depend on the stellar surface temperature T_{eff} . Each stellar spectral class has a little different regression of $r_p T_{eff}$ on the $r_p v_p$. The parameter $r_p L$ has big scattering than the parameter $r_p J$. From this it follows that the parameter $r_p T_{eff}$ affects mainly the distribution of exoplanets (see figure 6).

We have found for the 2321 candidates from the Kepler mission new formulas (17) and (19) for the calculation of the effective temperature T_{eq} of exoplanets for the spectral classes F, G and K. We can use these formulas for future calculation of habitable planets.

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