Flavor Physics Theory

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Abstract. We review recent progress in measuring and theoretically understanding flavor-changing processes, and the corresponding constraints derived on possible extensions of the Standard Model.

1 introduction

In the last few years there has been a significant experimental progress in quark and lepton flavor physics. In the quark sector, the validity of the Standard Model (SM) has been substantially reinforced by a series of high-precision measurements in the $B_s$ and $B_d$ systems. Most notably, a significant fraction of the parameter space of well-motivated SM extensions has been ruled out by the precise determination of the $B_s$ mixing phase at LHCb [1]. Similarly, severe bounds on several SM extensions have been set by the series of strong bounds on $\mathcal{B}(B_s \to \mu^+\mu^-)$ [2–5], that culminated with the first evidence of this decay reported by LHCb at this conference [6, 7]. Last but not least, in the lepton sector strong bounds on possible SM extensions have been set by the improved bounds on $\mathcal{B}(\mu \to e\gamma)$ obtained the MEG experiment at PSI [8]. Alltogether, the SM works remarkably well: the Cabibbo-Kobayashi-Maskawa (CKM) mechanism of quark-flavor mixing has been tested in various processes (although in many interesting cases the accuracy is still limited) and no flavor-violating effects are observed in the charged-lepton sector.

Despite this progress, the origin of flavor remains a mystery. Our “ignorance” can be summarized by the following two open questions: i) what determines the observed pattern of masses and mixing angles of quarks and leptons? ii) Which are the sources of flavor symmetry breaking accessible at low energies? Is there anything else beside quark and lepton mass matrices? Answering the first of these question is not easy: the energy scale where the flavor structures observed at low energies are originated may well be above any realistic experimental reach. On the other hand, answering the second question is mainly a question of precision, both on the theory and on the experimental side: this is the direction of research along which we can expect significant progress in the near future.

Observing new sources of flavor mixing (i.e. flavor violating couplings not related to quark and lepton mass matrices) is a natural expectation for any extension of the SM with new degrees of freedom not far from the TeV scale. While direct searches of new particles at high energies provide a direct information on the mass spectrum of the possible new degrees of freedom, the indirect information from low-energy flavor-changing processes translates into unique constraints on their couplings.

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The present bounds on possible deviations from the SM in flavor-violating processes already set stringent limits on the flavor structure of physics beyond the SM, and this provides a key information for model-building. However, several options are still open, and the quality of this information could be substantially improved with improved studies of selected flavor-violating observables. In the following I will illustrate this point in three different sectors that have recently seen a significant experimental progress: \( B_s \to \mu^+\mu^- \), CP violation in the charm system, and global CKM fits.

2 \( B_s \to \mu^+\mu^- \)

The purely leptonic decays of \( B_s \) and \( B_d \) s mesons constitute one of the most clean low-energy probes of physics beyond the Standard Model (SM). A first experimental evidence of this type of processes has recently been obtained by the LHCb collaboration in the \( B_s \to \mu^+\mu^- \) channel [6, 7]. The corresponding flavor-averaged time-integrated branching ratio determined by LHCb is [7]

\[
\mathcal{B}^{\text{exp}}(\mu^+\mu^-) = \left(3.2^{+1.5}_{-1.2}\right) \times 10^{-9},
\]

where the error is dominated by the statistical uncertainty and is expected to be improved significantly in the near future.

The theoretical cleaness and effectiveness of \( B_{s,d} \to \ell^+\ell^- \) as a probe of beyond-SM physics is related to a double-suppression mechanism at work within the SM. One the one hand, the \( B_{s,d} \to \ell^+\ell^- \) process is a flavor-changing neutral-current (FCNC) transition and, as such, it does not receive tree-level contributions. On the other hand, the purely leptonic final state and the pseudoscalar nature of the initial state imply a strong helicity suppression and forbid photon-mediated amplitudes at the one-loop level. As a result of this double suppression, up to the one-loop level \( B_{s,d} \to \ell^+\ell^- \) decays receives contributions only from Yukawa and weak interactions.

The price to pay for this theoretically-clean amplitude is a strong suppression for \( \ell = \mu \) (and \( \ell = e \)), or the channels with the best experimental signature. Following the recent theoretical analysis in [9], the theoretical branching ratio of the flavor averaged state (equal mixture of \( B_s \) and \( B_d \)) into a muon pair (fully inclusive of soft-photon emission) can be written as

\[
\mathcal{B}(B_s \to \mu^+\mu^-)_{\text{SM}} = 3.235 \times 10^{-9} \times \left( \frac{M_t}{173.2 \text{ GeV}} \right)^{3/7} \left( \frac{F_{B_s}}{227 \text{ MeV}} \right)^2 \left( \frac{V_{tb}^* V_{ts}}{4.05 \times 10^{-2}} \right)^2 \times \mathcal{B}_{\text{exp}}^\mu = \left( 3.23 \pm 0.15 \pm 0.23 \mathcal{B}_{\text{exp}}^\mu \right) \times 10^{-9},
\]

where in the second line we have explicitly separated the present contribution to the error due to the \( B_s \) meson decay constant (\( F_{B_s} \)). The latter, which is the only hadronic quantity relevant to estimate \( \mathcal{B}(B_s \to \mu^+\mu^-) \), is determined quite precisely by several Lattice-QCD calculations [10]. As far as the other leptons are concerned, we get

\[
\frac{\mathcal{B}(B_s \to \tau^+\tau^-)_{\text{SM}}}{\mathcal{B}(B_s \to \mu^+\mu^-)_{\text{SM}}} = 215, \quad \frac{\mathcal{B}(B_s \to e^+e^-)_{\text{SM}}}{\mathcal{B}(B_s \to \mu^+\mu^-)_{\text{SM}}} = 2.4 \times 10^{-5}.
\]

The corresponding \( B_d \) modes are both suppressed by an additional factor \( |V_{td}/V_{ts}|^2 F_{B_d}^2 / F_{B_s}^2 \approx 1/30 \).

As recently pointed out in [11], an important point when comparing the above predictions with experiments is the observation that, at present, experiments extract the \( B_s \) decay rates from a time-integrated distribution. As a result, we cannot access the decay rate of a flavor averaged state (that is what is produced at initial time), but its time-integrated evolution. Due to the non-vanishing width difference \( \Delta \Gamma_s \), this imply a nontrivial correction factor of \( O(10\%) \).
What is presently measured by the LHC experiments is the flavor-averaged time-integrated distribution,

\[
\langle \mathcal{B}(B_s \rightarrow f) \rangle_{[t]} = \frac{1}{2} \int_0^\infty dt' \left[ \Gamma(B_s(t') \rightarrow f) + \Gamma(\bar{B}_s(t') \rightarrow f) \right],
\]

where \( \Gamma(B_s(t') \rightarrow f) \) denotes the decay distribution, as a function of the proper time \( (t') \), of a \( B_s \) flavor eigenstate at initial time (and correspondingly for \( \bar{B}_s \)). Defining \( \Gamma_s = 1/\tau_{B_s} = (\Gamma_{1s}^s + \Gamma_{2s}^s)/2 \) and \( y_s = (\Gamma_{1s}^s - \Gamma_{2s}^s)/(2\Gamma_s) = 0.088 \pm 0.014 \), the time-integrated distribution is related to the flavor-averaged rate at \( t = 0 \) by

\[
\langle \mathcal{B}(B_s \rightarrow f) \rangle_{[t]} = \kappa^f(t,y_s) \langle \mathcal{B}(B_s \rightarrow f) \rangle_{[t=0]} \equiv \kappa^f(t,y_s) \frac{\Gamma(B_s \rightarrow f) + \Gamma(\bar{B}_s \rightarrow f)}{2\Gamma_s},
\]

where \( \kappa^f(t,y_s) \) is a model- and channel-dependent correction factor.

For the \( \mu^+\mu^- \) final state (inclusive of bremsstrahlung radiation) the SM expression of the \( \kappa^f(t,y_s) \) factor is [12]

\[
\kappa_{SM}^{\mu\mu}(t,y_s) = \frac{1 - e^{-t/\tau_{B_s}}}{1 - y_s} \sinh \left( \frac{y_s t}{\tau_{B_s}} \right) - e^{-t/\tau_{B_s}} \cosh \left( \frac{y_s t}{\tau_{B_s}} \right) \rightarrow \frac{1}{1 - y_s}, n^0
\]

\[
\langle \mathcal{B}(B_s \rightarrow \mu^+\mu^-) \rangle_{[t=0]}^{SM} = (3.54 \pm 0.30) \times 10^{-9},
\]

where on the second line we have given the SM prediction for the fully integrated branching ratio, that is what we should compare with the experimental result in Eq. (1). As can be seen, at present there is good agreement between data and SM prediction. However, the error is still large and correspondingly there is still a sizable region of possible new-physics contributions still to be explored.

### 2.1 \( B_s \rightarrow \mu^+\mu^- \) and Supersymmetry

The strong helicity suppression and the theoretical cleanliness make these modes excellent probes of several new-physics models and, particularly, of scalar FCNC amplitudes. Scalar FCNC operators, such as \( B\bar{q}q_H \), are present within the SM but are negligible because of the smallness of down-type Yukawa couplings. On the other hand, these amplitudes could be non-negligible in models with an extended Higgs sector. In particular, within the Minimal Supersymmetric extension of the SM (MSSM), where two Higgs doublets are coupled separately to up- and down-type quarks, a sizable enhancement of scalar FCNCs can occur at large \( \tan \beta = v_u/v_d \). This effect is very small in non-helicity-suppressed \( B \) decays (because of the small Yukawa couplings), but could have easily enhanced the \( B_{s,d} \rightarrow t^+t^- \) rates by one order of magnitude or more. This possibility is ruled out by the present data on \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) \), resulting in a significant constraint on such class of models.

Despite being "minimal" from the particle point of view, the MSSM contains a large number of free parameters (especially in the flavor sector) and we cannot discuss its implications in flavor physics in generality (namely without specifying in more detail the flavor structure of the model). On general grounds, the \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) \) constraint (as well as most FCNC constraints) is very stringent if we include non-minimal sources of flavor symmetry breaking in the model (see e.g. Ref. [13]). However, the peculiar structure of \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) \) makes it a relevant constraint also in constrained versions of MSSM without new sources of flavor symmetry breaking (see e.g. Ref. [14, 15]).

In Fig. 1, were we show the prediction of \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) \) in the \( M_A-\tan \beta \) plane of the so-called constrained MSSM (CMSSM) and the NUHM framework (a minimal variation of the CMSSM with one extra free parameter allowing non-universal soft masses for the Higgs fields, compared to squark and lepton, at the unification scale). As can be seen, the present result on \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) \) strongly
disfavor the region of parameter space with large $\tan\beta$ and low $M_A$ values. It is also worth to stress that this constraint is fully complementary to the strong limits already sets on such class of models by the direct searches at ATLAS and CMS. On the other hand, it is fair to say that a large fraction of the parameter of these models can be explored only with higher experimental precision on $B_s \rightarrow \mu^+\mu^-$. In a long-term perspective, the discovery and the precise measurement of all the accessible $B \rightarrow \ell^+\ell^-$ channels is one of the most interesting items of the $B$-physics program at hadron colliders.

### 3 CP violation in the charm system

On general grounds, long-distance contributions are usually largely dominant with respect to the short-distant ones in charm mixing and decay amplitudes. This happens is because SM short-distance contributions are not top-mass enhanced as in the $B$ and $K$ systems, and are strongly disfavored by the CKM hierarchy with respect to the dominant transition amplitudes into light quarks. Within the SM the genuine short-distance contributions are suppressed by five powers of the Cabibbo angle. Other contrary, long-distance amplitudes into light quarks can be Cabibbo allowed (i.e. not suppressed by any power of $\lambda$) for partonic transitions of the type $c \rightarrow us\bar{d}$, Cabibbo suppressed [$c \rightarrow ud\bar{d}(s\bar{s})$], or at most doubly Cabibbo suppressed [$c \rightarrow ud\bar{s}$].

Given this hierarchy of amplitudes, within the SM charm physics does not provide interesting precisions tests of the CKM mechanism. However, the charm system offers a unique opportunity to explore up-type FCNC amplitudes that maybe significantly enhanced over the SM level in possible extensions of the SM. For instance, very stringent constrains on generic $|\Delta C| = 2$ operators can be derived by the experimental constraints on $D-\bar{D}$ mixing [16].

The neutral $D$ system is the latest system of neutral mesons where mixing between the particles and anti-particles has been established. The observation of a non-vanishing amplitude at more than 5$\sigma$ has been reported for the first time by LHCb at this conference [17] and turns out to be consistent with the (long-distance dominated) SM expectation.

While CP-conserving observables in $D$ decays are largely dominated by long-distance effects, CP-violating observables are typically strongly suppressed within the SM and offer a potentially deeper
probe of short-distance dynamics. One of the most interesting recent developments in flavor physics has been the experimental evidence of direct CP violation in two-body Cabibbo-suppressed $D$ decays. An asymmetry close to the 1% level has been announced by LHCb about one year ago [18], and soon after it has been confirmed both by CDF [19] and by Belle, although none of the experiments has reached yet the 5σ level. Such a large direct CP asymmetry was not expected within the SM according to pre-LHCb theoretical predictions (see e.g. Ref. [20]), and the theoretical interpretation of this result has open an interesting debate that is still in progress.

3.1 Standard Model vs. New Physics in $\Delta a_{CP}^{\text{dir}}$

The current experimental world average for the direct CP-violating asymmetry in two-body Cabibbo-suppressed $D$ decays can be summarized as follows

$$\Delta a_{CP}^{\text{dir}} \equiv a_{CP}^{\text{dir}}(D \rightarrow K^+K^-) - a_{CP}^{\text{dir}}(D \rightarrow \pi^+\pi^-) = (-0.67 \pm 0.16) \%,$$  

where

$$a_{CP}^{\text{dir}}(D \rightarrow f) \equiv \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow f)}.$$  

The separate determinations of $a_{CP}^{\text{dir}}(D \rightarrow K^+K^-)$ and $a_{CP}^{\text{dir}}(D \rightarrow \pi^+\pi^-)$ are affected by larger relative uncertainties and, at present, do not allow to establish a clear evidence of CP-violation in one of the two channels.

In order to be non zero, $\Delta a_{CP}^{\text{dir}}$ requires the interference of two amplitudes with different weak and strong phases. Within the SM, taking into account that one of the two amplitudes is necessarily generated at the one-loop level, this implies the following naive expectation $\Delta a_{CP}^{\text{dir}} = O\left(|V_{cb}^* V_{ub}/V_{cs} V_{us}|^2 \alpha_s/\pi\right) \sim 10^{-4}$ [20], well below the experimental result in Eq. (8). This has led to extensive speculations in the literature that the measurement of $\Delta a_{CP}^{\text{dir}}$ is a signal of NP. This is a particularly exciting possibility, given that reasonable NP models can be constructed in which all related flavor changing neutral current constraints from $D$ meson mixing are satisfied.

The naive expectation for the SM value of $\Delta a_{CP}^{\text{dir}}$ is based on a perturbative (short-distance) estimate of the loop amplitude with suppressed CKM factors. In fact, there is consensus that a SM explanation for $\Delta a_{CP}^{\text{dir}}$ would have to proceed via a dynamical (long-distance) enhancement of specific hadronic matrix elements, the so-called penguin contractions. The latter are nothing but penguin-type matrix elements that vanish at the tree level, with internal light-quark loops ($s$ and $d$): they cannot be estimated reliably in perturbation theory [21]. The enhancement necessary to explain the observed result is quite large compared to the typical size of non-perturbative effects at the charm scale (the naively suppressed penguin contractions should exceed by a factor 3 to 5 the naively dominant tree-level contractions of the same operators [22]). However, such possibility cannot be excluded from first principles and could even lead to a more coherent picture of available CP data on two-body Cabibbo-suppressed $D$ decays [23].

On the other hand, a value of $\Delta a_{CP}^{\text{dir}}$ of $O(1\%)$ can naturally be accommodated in well-motivated extensions of the SM. In particular, it fits well in models generating at short distances a sizable CP violating phase for the effective $\Delta C = 1$ chromomagnetic operators (see e.g. [20, 22, 24]). Given this situation, it is important to identify possible future experimental tests able to distinguish standard vs. non-standard explanations of $\Delta a_{CP}^{\text{dir}}$.

A general prediction of this class of models, that could be used to test this hypothesis from data, are enhanced direct CP violating (DCPV) asymmetries in radiative decay modes [25] (see also [26, 27]). The first key observation to estimate DCPV asymmetries in radiative decay modes is the strong link
between the $\Delta C = 1$ chromomagnetic operator ($Q_8 g$) and the $\Delta C = 1$ electromagnetic-dipole operator ($Q_7 \gamma$). In most explicit NP models, the short-distance Wilson coefficients of these two operators are expected to be similar. Moreover, the two operators undergo a strong model-independent mixing (from QCD) in running down from the electroweak scale to the charm scale. Thus if $\Delta a_{CP}$ is dominated by NP contributions generated by $Q_8 g$, we can infer that sizable CP asymmetries should occur also in radiative decays, given the presence of a CP-violating electromagnetic-dipole operator.

The second important ingredient is the observation that in the Cabibbo-suppressed $D \to V \gamma$ decays, where $V$ is a light vector meson ($V = \phi, \rho, \omega$), $Q_7 \gamma$ has a sizable hadronic matrix element. More explicitly, the short-distance contribution induced by $Q_7 \gamma$, relative to the total (long-distance) amplitude, is substantially larger with respect to the corresponding relative weight of $Q_8 g$ in $D \to P^+ P^-$ decays. As a result, DCPV asymmetries in these modes could easily reach the few×% level in presence of NP. An observation of $|\alpha_{V \gamma}| > 3\%$ would be a clear signal of physics beyond the SM, and a clean indication of new CP-violating dynamics associated to dipole operators.

4 CKM fits and $B$ meson mixing

An overall picture showing the good consistency of the SM expectations for flavor-changing processes and the experimental data is provided by the so-called CKM fits. As usual, the results of such fits are expressed by the corresponding projection in the $\bar{\rho}$–$\bar{\eta}$ plane, where $\bar{\rho}$ and $\bar{\eta}$ are the less known CKM parameters in the modified Wolfenstein parameterization of this matrix [28, 29]. This projection is shown in Figure 2. As can be seen, the overall result is a good consistency of the SM predictions. Note that the overall quality of the fit has significantly improved since a few months, after the new measurement of $\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau)$ announced by Belle this summer [30].

In order to estimate the possible room for New Physics (NP) in $B$ meson mixing, one can perform an independent global fit reinterpreting the experimental observables including possible model-independent NP contributions to both modulo and phase of $\Delta F = 2$ processes. The latest fit of this
Figure 3. Model independent fit [33] in the scenario where NP affects $B_d$ and $B_s$ mixing amplitudes separately. The coloured areas represent regions with $\text{CL} < 68.3\%$ for the individual constraints. The red area shows the region with $\text{CL} < 68.3\%$ for the combined fit, with the two additional contours delimiting the regions with $\text{CL} < 95.45\%$ and $\text{CL} < 99.73\%$.

...type has been presented in Ref. [33], where the mixing amplitudes are expressed in terms of two complex parameters, $\Delta_s,d$, describing the normalization with respect to the corresponding SM case (the SM is recovered for $\Delta_s = \Delta_d = 1$). The results of the fit thus obtained, shown in Fig. 3, indicate that a mild tension persist in the $B_d$ case (a similar conclusion is obtained also by the UTfit collaboration [31]). This could be the first hint of a non-standard contribution in the $B_d$ mixing amplitude, although the statistical significance of this discrepancy is still quite low. The precise determination of the $B_s$ mixing phase at LHCb [1] has ruled to the possibility of sizable deviations from the SM in this observable; however, a relative deviation of the same size to that hinted by $B_d$ mixing data is still allowed. Interestingly, this is what is predicted in well-motivated extensions of the SM, such as supersymmetric models with heavy first two generations of squarks and with a minimally broken $U(2)_3$ flavor symmetry [34]. While it will be very difficult to test this hypothesis in the $B_d$ sector, due to large irreducible theoretical uncertainties, there is still significant room for improvements in the $B_s$ system, where the experimental error is still about one order of magnitude large with respect to the theoretical one.

5 Conclusions

As we have shown with a few explicit examples, a clear message emerges from present data: if physics beyond the SM is not far from the TeV scale (hence it is directly accessible with present and future high-energy facilities), it must have a highly non-trivial flavor structure in order to satisfy the existing low-energy flavor-physics bounds. However, this structure has not been clearly identified yet and its investigation is the main purpose of future experiments in flavor physics.

The recent discovery of a new state with mass around 125 GeV, compatible with properties of the SM Higgs boson (and pointing toward the existence of a a fundamental Higgs field), makes the...
case of future high-precision studies in flavor physics even more motivated: all the key properties of low-energy flavor physics are determined by the Yukawa couplings, or by the couplings of the Higgs field to the fermions. A deeper investigation of flavor physics is therefore a necessary element for a deeper understanding of the properties of the Higgs field. Indeed our knowledge of the Yukawa sector is still quite limited (often not exceeding the 30% relative accuracy for amplitudes forbidden at the tree level). Making progress in this field is mainly a question of precision, both on the theory and on the experimental side, and visible deviations from the SM may simply be around the corner in terms of statistical precision.

Acknowledgments

It is a pleasure to thank the organizers for the perfect organization and the partial support that allowed me to attend this interesting conference.

References