

## Theory Question for the Higgs Sector

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**Abstract.** With the discovery of a new particle consistent with the Higgs boson theoretical physics in this field has entered a data driven phase. One of the main task for theorists working on LHC physics is to ask the right questions and provide the physics as well as simulational environment which will allow ATLAS and CMS to answer these questions.

The recent discovery of a boson [1, 2] compatible with the predicted Higgs boson [3] has been an impressive start of the LHC program. With this discovery the LHC with the ATLAS and CMS experiments have established themselves amongst the most important experiments in high energy physics altogether.

The Higgs discovery is a triumph for experimental and theoretical high energy physics. In 1964 which experimentalist would have thought that we would be able to build a super-conducting proton collider with a circumference of 27 km and an collision energy in the 10 TeV range. On the theory side who would have expected this Higgs boson to exist with a mass around 125 GeV, turning the Standard Model into a renormalizable fundamental field theory description of all non-gravitational forces. Unlike the different gauge bosons the Higgs particle is a postulate from the mathematical structure of the theory — assuming that Nature cares about mathematical consistency of fundamental theories. As we will see later, this makes it a more interesting candidate for precision analyses than any other particle in the Standard Model.

After July 4th, 2012 there are a few questions theorists can ask and try to help answer:

1. Where can we help with the analyses?
2. What is the structure of the Higgs Lagrangian?
3. What are the Higgs coupling strengths?
4. What can we expect in the future?
5. What does this all teach us?

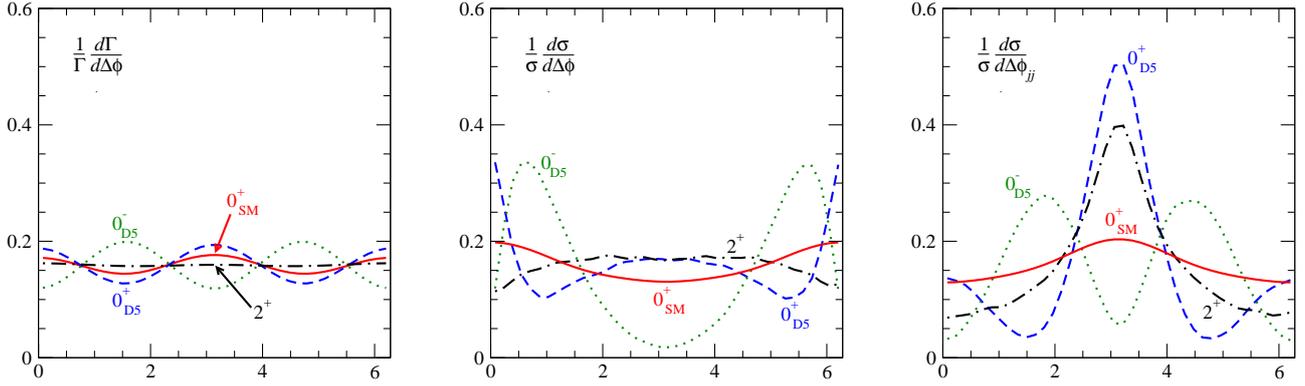
The ordering of the last two questions might have to be exchanged a few times. What is important is that this list of questions implies a change in the standard procedure in high energy physics. Except for the last question, they are not challenging the imagination of theorists. Since the 1970s and 1980s, our field has spent decades being driven

by theory ideas. The Higgs discovery establishes this approach as highly promising; after all, the Higgs boson was postulated by theory and found after a very long hunt. At the same time, for a few years it will motivate theorists to stick close to data, help with its interpretation and eventually define the role of the Higgs boson in a complete Standard Model of particle physics.

The list above defines the basic steps to start a dedicated Higgs research program, first at the LHC and eventually at a Higgs factory. The general answer to the first question is known: theorists provide simulation tools for the LHC experiments and precision predictions for a set of particular observables. Both aspects become challenging at a QCD machine like the LHC. The particular problem of QCD at hadron colliders are collinear (and soft) divergences. Massless quarks and gluons can split and get radiated at essentially no energy. For LHC simulations this means that they need to describe energetic QCD radiation going everywhere. Precision predictions for example for signal and background processes do not converge as well as one would expect from the size of the strong coupling  $\alpha_s \sim 0.12$ , and some observables appear to not converge at all. Moreover, experiments add more and more advanced observables to their analyses. Examples are the structure of the jet recoil against a Higgs boson or pre-jet QCD features. Theoretical LHC physics needs to respond to these challenges if the LHC should turn into a Higgs precision machine.

The second to fifth question I will discuss during this talk. Note that the logic of these questions is not necessarily the ordering in which we will find answers to them. The latter is determined by experimental realities which will become clear during the other talks at this conference. Moreover, in this writeup I will only be able to touch each of these questions very briefly. From the references it should be clear where more information can be found. Some of the basics I just covered and wrote up during a Higgs lecture course in Heidelberg this winter [4].

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**Figure 1.** Normalized  $\Delta\phi$  distributions for  $X \rightarrow ZZ$  events (left), WBF angles in the Breit frame (center), and the azimuthal angle between the tagging jets (right). We show the SM operator  $XW_\mu W^\mu$ , the CP-even dimension-5 operator  $XW_{\mu\nu}W^{\mu\nu}$ , the CP-odd dimension-5 operator  $XW_{\mu\nu}\widetilde{W}^{\mu\nu}$ , and an example spin-2 coupling. Figure from Ref. [5].

## 1 Higgs Lagrangian

As we learn in Theoretical Physics I or II the key to understanding a physical system is its Lagrangian. This is particularly true for quantum field theory. For example the Standard Model is largely defined by its Lagrangian: in addition to the particle or field content and the symmetries governing the fundamental interactions it is crucial that we limit ourselves to renormalizable operators.

The interaction between a Higgs and two  $W$  bosons is a good example how we choose the operators in the Lagrangian. There are two obvious terms which can describe our interaction:  $\mathcal{L} \supset HW^+W^-$  is a term of mass dimension three, so we can multiply it with a coupling proportional to a mass and obtain a renormalizable operator. However, this term is not gauge invariant. In contrast,  $\mathcal{L} \supset HW^{\mu\nu}W_{\mu\nu}$  in terms of the field strength tensor is gauge invariant, but has mass dimension five and is hence not renormalizable. We prefer to describe the coupling which we observe in  $H \rightarrow WW$  decays in terms of  $HW^+W^-$ , linking this coupling to the  $W$  mass term in the Lagrangian.

We see that the question about the Higgs Lagrangian can be phrased in many equivalent ways: What are the Higgs quantum numbers? What do the Higgs operators look like? Is the observed Higgs responsible for electroweak symmetry breaking and particle masses?

There are two very general ways to test the Higgs Lagrangian in terms of angular correlations. The first is based on the decay  $H \rightarrow ZZ \rightarrow 4\ell$  and its full reconstruction. The angular correlation between the leptons can be described by the five Cabibbo–Maksymowicz–Dell’Aquila–Nelson angles [6–8]. They can be exploited to identify a spin-0, spin-1, or spin-2 Higgs-like boson coupling to two  $Z$  bosons [9–11]. For a scalar Higgs up to dimension-5 three operators can mediate this coupling: the before-mentioned CP-even  $HW^+W^-$  and  $HW^{\mu\nu}W_{\mu\nu}$  operators and the CP-odd pseudo-scalar coupling  $HW^{\mu\nu}\widetilde{W}_{\mu\nu}$ . They are best distinguished by the angle between the two  $Z$  decay planes  $\Delta\phi$ . In the left panel of Fig. 1 we show how the different structures predict distinct modulations of  $\Delta\phi$ .

The problem with these angles is that they are only accessible in one Higgs decay channel. We would clearly prefer to probe the Higgs coupling structure in as many Higgs production and decay channels as possible. Weak boson fusion (WBF) describes Higgs production in association with two forward tagging jets. However, even if we fully reconstruct the final state we cannot boost into the rest frames of the space-like  $W$  bosons to reconstruct the usual angles. Instead, we boost into the corresponding Breit frames. The resulting distributions are shown in the central panel of Fig. 1, clearly indicating the close connection to the  $H \rightarrow ZZ$  analysis.

A more convenient re-formulation leads us to the second general way to probe Higgs coupling structures [5, 12–14]: the correlations of the tagging jets and the Higgs decay product in WBF Higgs production. In the right panel of Fig. 1 we show the azimuthal angle between the two tagging jets. It is clearly equivalent to the angle between the decay planes in the  $H \rightarrow ZZ$  decays, as can be easily shown [5].

Going beyond spin-0 operators we can extend the  $H \rightarrow ZZ$  and WBF Higgs analyses to spin-1 and spin-2 resonances. As a side remark, the Landau–Yang theorem [15] implies that a resonance observed in the  $\gamma\gamma$  decay channel cannot be spin-1; therefore, in this fully reconstructed decay channel we only need to distinguish spin-0 from spin-2, for example using the scattering angle [16, 17].

The two general analyses described above can distinguish many more different ‘Higgs’ coupling structures. Moreover, they are sensitive to mixed resonances, either overlapping CP-even and CP-odd states or particles with different couplings to the same Standard Model particles. The only caveat is that we should avoid cutting on variables which are sensitive to the coupling structure for background rejection. One well-known example is the angular correlation between the  $W$  decay products in the  $H \rightarrow WW$  decays. It turns out that in WBF Higgs production a second particularly sensitive variable is the rapidity separation of the two tagging jets, which is usually required to be very large, leaving spin-2 resonances unobserved [5].

## 2 Higgs couplings

Once the structures of the Higgs Lagrangian is fixed we can measure the individual pre-factors, *i.e.* the coupling values. For the ideal mass value around 125 GeV the parameters immediately accessible at the LHC include the couplings to the third-generation fermions  $b, t, \tau$  and to the  $W, Z$  bosons. Assuming a Lagrangian with Standard Model structures they all correspond to renormalizable operators. Higgs physics at the LHC is particularly interesting because two loop-induced operators already govern the discovery channels [1, 2], the Higgs couplings to the gluon and the photon field strength. In the Standard Model they are induced by top and  $W$  loops and do not decouple for example for heavy chiral quarks [4]. Allowing for new physics at the TeV scale they are sensitive to any additional particles coupling to the Higgs sector.

Measuring each of these couplings independently is the obviously right question to ask [18–20] — the problem is to phrase it consistently. For example, electroweak loop corrections require the electroweak theory to be renormalizable, which is only true if the Higgs couplings to the  $W, Z$  bosons are exactly as predicted in the Standard Model. The easy way out is to notice that precision predictions in QCD do not require anything from the Higgs sector, so we can ignore the numerically much less relevant electroweak effects and leave the ultraviolet completion of the model open. Once we reach per-cent level precision on the couplings and the electroweak corrections become relevant we will have to specify an appropriate ultraviolet completion. What is important is that in a data-driven analysis this model should make as few assumptions about the weak-scale Higgs couplings as possible.

Given that we assume the Standard Model operator basis we define the free Higgs couplings as

$$g_{xxH} \equiv g_x = (1 + \Delta_x) g_x^{\text{SM}}. \quad (1)$$

For the loop-induced Higgs-photon coupling this means

$$g_{\gamma\gamma H} \equiv g_\gamma = (1 + \Delta_\gamma^{\text{SM}} + \Delta_\gamma) g_\gamma^{\text{SM}}, \quad (2)$$

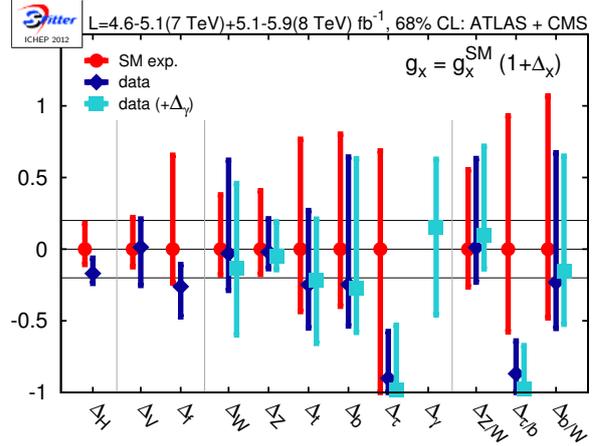
where  $\Delta_\gamma^{\text{SM}}$  contains the measured couplings to all Standard Model particles while  $\Delta_\gamma$  parameterizes additional contributions.

At the LHC the Higgs width cannot be measured independently, but it enters every observable event rate. We consistently assume [18, 21]

$$\Gamma_{\text{tot}} = \sum_{\text{obs}} \Gamma_x(g_x) + 2\text{nd generation} < 2 \text{ GeV}. \quad (3)$$

To avoid offsets in our results we link all second-generation Yukawas to their third-generation counterparts, *e.g.*  $g_c = m_c/m_t \times g_t^{\text{SM}}(1 + \Delta_t)$ .

In Fig. 2 we show the measured Higgs couplings including their error bars from all data available before the HCP conference. They are accompanied by the expected values assuming Standard Model central values for all Higgs event numbers but the observed uncertainties. In



**Figure 2.** Couplings extracted from 2011 and 2012 data ( $m_H = 126$  GeV). In addition we show the form factor  $\Delta_H$  and universal fermion and boson couplings  $\Delta_{V,f}$ . Figure from Ref. [20].

addition to independent couplings we also show an overall form factor  $\Delta_H$ , as it would for example appear in a Higgs portal [22]. While the central value of  $\Delta_H$  is slightly smaller than unity, driven by the Yukawas, it is fully consistent with the Standard Model.

Motivated by electroweak precision data we can assume that nothing unexpected happens in the gauge sector,  $\Delta_Z = \Delta_W \equiv \Delta_V$ . Without a too convincing theory argument, we can also simplify the fermion sector to a universal  $\Delta_f$ , defining a two-dimensional parameter space [23]. Again, the fermion couplings come out a little low, but consistent with the Standard Model.

The question is what happens with the full set of couplings, knowing that both, ATLAS and CMS see a slightly enhanced rate of Higgs decays to photons. If we do not allow for a free  $\Delta_\gamma$  the best fit resides around  $\Delta_t \sim -0.25$ , enhancing the Higgs branching ratio to photons, but reducing the gluon fusion production rate. However, a reduced central value of  $\Delta_b$  compensates this effect through the Higgs width.

Once we include  $\Delta_\gamma$  in the fit the top Yukawa from the Higgs-gluon coupling still comes out slightly low, as does the bottom Yukawa from the total width. These effects of the two Yukawas largely cancel. The  $\Delta_W$  measurement from the observed  $H \rightarrow WW$  decays is fairly close to the Standard Model value, which altogether enhances the rate to the observed values. We find  $\Delta_\gamma = 0.16 + 0.47 - 0.61$  at 68% CL. This means that in a fit with free Higgs couplings we find no evidence for a significantly enhanced Higgs coupling to photons.

It should be added that combining the arguments of Sec. 1 and Sec. 2 leads to a question well known from the electroweak sector: once we are convinced (or assume) that all renormalizable Higgs couplings have Standard Model values we can test additional Lagrangian operators in the Higgs sector and define anomalous Higgs couplings as  $g_{xxH}$  [24]. In that sense, comprehensive Higgs couplings analyses really just have started.

### 3 Future

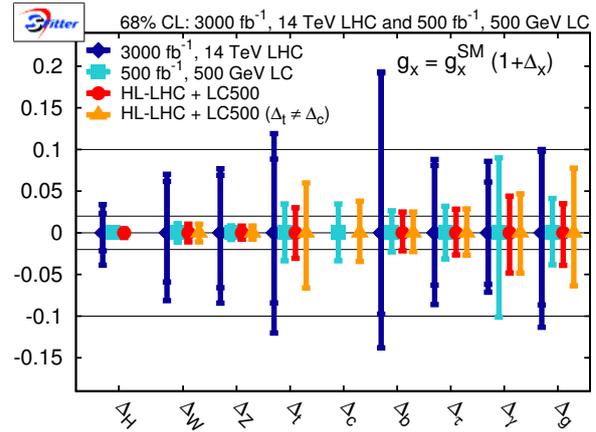
The discovery of a Higgs boson is only the first step in a comprehensive study of electroweak symmetry breaking. In the coming years one of the main tasks of ATLAS and CMS will be to study this Higgs boson and to determine its properties in detail. As discussed in Sec. 2 we already have first results on the coupling strength. While it might be disappointing to (some of) us theorists that there appears to be no deviation from the Standard Model predictions, we need to keep in mind that in the usual models for physics beyond the Standard Model we would have hardly expected such  $O(1)$  deviations in any of the Higgs couplings [25]. Moreover, many of the Higgs couplings are not yet accessible in the 7 TeV and 8 TeV data.

The 2011 and 2012 running of the LHC has led to a Higgs discovery much earlier than expected. However, due to the reduced collider energy the Higgs searches had to entirely rely on the inclusive production processes, dominated by Higgs production in gluon fusion. For Higgs decays to photons and to  $Z$  bosons this clearly is sufficient, for the decay to  $W$  bosons the situation is already less clear. The couplings we directly probe are therefore the effective Higgs couplings to gluons and to photons, and the tree-level coupling to  $Z$  bosons. Everything else needs conceptual improvement.

The situation will automatically change once the LHC energy is increased to 13 TeV. First of all, weak boson fusion as a Higgs production mechanism will allow for a proper extraction of the first fermionic Higgs decay  $H \rightarrow \tau\tau$  [26]. Using the same production channel the decay  $H \rightarrow WW$  will be visible with a signal-to-background ratio of order one [27]. Maybe, at some point we will even be able to observe a Higgs decay to second-generation fermions  $H \rightarrow \mu\mu$  in this production channel [28].

The second sub-dominant Higgs production mode which should be observable at higher energies is associated  $WH$  and  $ZH$  production, with a leptonic  $W$  or  $Z$  decay. The Higgs decay we want to target with this production process is  $H \rightarrow b\bar{b}$ , giving us direct access to the bottom Yukawa. The large continuum backgrounds in this channel force us to extract the signal from phase space regions with a boosted Higgs. This can best be achieved using a Higgs tagger [29]. Similarly, we can directly extract the top Yukawa from  $t\bar{t}H$  production, again combined with a Higgs decay to bottoms. In that case the large QCD backgrounds together with overwhelming combinatorics point towards using top and Higgs taggers [30]. All of these subdominant production and decay channels will be thoroughly probed after the current LHC shutdown, determining the individual Higgs couplings  $g_{xxH}$  at a typical  $\pm 20\%$  level [19].

Precision studies of electroweak gauge bosons at SLC and LEP have been an overwhelming success. They showed that systematic tests of particle properties at the level of quantum corrections can be a key tool to understanding the structure of the underlying field theory. If we want to improve the precision of many Higgs-related measurements to the per-cent level the best way is to build a



**Figure 3.** Expected precision for Higgs couplings from a high-luminosity LHC, a 500 GeV ILC, and their combination. For the LHC we also show a scenario with improved systematic uncertainties. Figure from Ref. [31].

Higgs factory, *i.e.* a linear collider (ILC) which produces  $e^+e^- \rightarrow ZH$  events in large numbers.

A linear collider has a few major advantages: first, it can measure the Higgs width from a combination of cross sections times branching ratios and a total  $ZH$  cross section measurement. Closely linked, it can study Higgs decays without ever reconstructing the decay products, simply using a fully reconstructed  $Z$  decay and the known initial state parameters. Compared to the LHC coupling extraction described in Sec. 2 this means that we can extract invisible or exotic Higgs decays without having to define a specific search for an assumed final state.

Apart from these conceptual advantages an ILC can in general achieve higher precision than a hadron collider, simply because the hadron collider analysis machinery is eventually limited by our knowledge and the poor convergence of perturbative QCD. In Fig. 3 we first show projected error bars on all available Higgs couplings for a luminosity-upgraded LHC. Because modern Higgs analyses for example depend on jet observables which violate collinear factorization we do not postulate significantly improved theory errors. Similarly, effects like pile-up turn constant systematic uncertainties into a challenge once we postulate an integrated luminosity a factor 100 above the 2011/2012 data. However, to test the effect of reduced experimental systematics we also show results for an improved 2% error on the luminosity and neglecting all other systematics. The effect is relatively sobering.

For a linear collider running at a high enough energy to also probe  $t\bar{t}H$  production we show the same uncertainties. Figure 3 shows the results based on the assumed ILC measurements are of Refs. [32]. The result significantly exceeds the expected LHC results. Moreover, in comparison to the individual results a joint analysis translates the improved ILC measurement of  $\Delta_W$  into an improvement of  $\Delta_\gamma^{\text{SM}}$  at the LHC, so eventually  $\Delta_\gamma$  can be determined at the 5% level. The case for a Higgs factory should be the community's next step in worldwide facilities.

## 4 Interpretation

Up to this point our entire discussion has focused on very phenomenological aspects of the Higgs sector. Determining the Higgs operator basis and the numerical coupling values is an absolutely necessary first step, but it does not tell us anything about the fundamental structure of electroweak symmetry breaking. What we need is patterns in weak-scale parameters and possibly an extrapolation of the theory to large energy scales.

Higgs properties measured at the LHC are sensitive to new physics effects for two reasons: first, renormalizable Higgs couplings can be significantly modified when we expand the minimal Higgs sector of the Standard Model. One such modification is the so-called Higgs portal [33, 34], coupling an additional scalar  $\phi_h$  to the Standard Model Higgs potential in terms of  $\phi_s$ ,

$$V = \mu_s^2 |\phi_s|^2 + \lambda_s |\phi_s|^4 + \mu_h^2 |\phi_h|^2 + \lambda_h |\phi_h|^4 + \eta_\chi |\phi_s|^2 |\phi_h|^2. \quad (4)$$

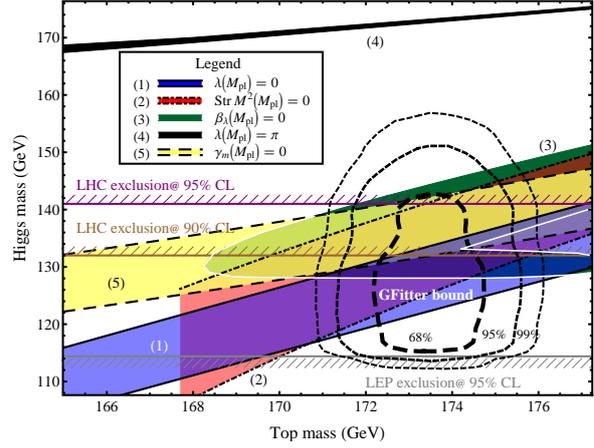
Such a portal has two effects: all Higgs couplings get universally reduced by a mixing angle  $\cos\chi$ , and depending on the size of the mixing we might be able to observe invisible Higgs decays at the LHC. The latter would be able to distinguish a Higgs portal from other scenarios introducing a universal mixing angle or form factor.

Another pattern of modified Higgs couplings could be traced back to a second Higgs doublet. Supersymmetry is only one example for such a modification [35]. At tree level a two-Higgs doublet model would modify all Yukawa couplings independently, but  $g_{WWH}$  and  $g_{ZZH}$  will change in phase. This way the  $\rho$  or  $T$  parameter is protected and the model is not in fundamental conflict with electroweak precision data. Nevertheless, an additional charged Higgs state or pseudo-scalar Higgs state are seriously challenged by many flavor physics results [36]. A second scalar even with a reduced Higgs coupling to gauge bosons is constrained by LHC searches. Taking into account all experimental constraints we would not expect deviations of  $\Delta_\chi = \mathcal{O}(1)$  for most of the Higgs couplings in these models [25].

The second powerful feature of Higgs measurements at the LHC are loop-induced couplings to photons and gluons. A fourth generation of chiral fermions, for example, does not decouple in  $\Delta_g$  and can be ruled out relatively easily based on LHC data [37]. Strictly speaking, not even the sign of the induced couplings is fixed by LHC measurements [19, 38]. What is interesting is that we can link the changes in  $\Delta_g$  and  $\Delta_\gamma$  for example for a new colored state with charge  $Q$  and a quadratic Casimir  $C_2$  of the  $CU(3)_c$  representation. We find

$$1 + \Delta_\gamma = \left[ 1 + 0.28\xi \left( 1 \mp \sqrt{1 + \Delta_g} \right) \right]^2, \quad (5)$$

with  $\xi = 3Q^2/C_2$  [38]. Again, supersymmetry is one of the prime examples where we can change both loop-induced couplings via a light top squark or just induce a sizeable  $\Delta_\gamma$  value via a light slepton [35].



**Figure 4.** Higgs and top masses from different boundary conditions at the Planck scale. The width of each band indicates the theory error. The black dashed lines show the electroweak precision fit from ZFitter/GFitter. Figure from Ref. [40].

Following the above argument we can use LHC or ILC measurements in the Higgs sector to probe physics beyond the Standard Model. Independently of the answer, but of particular importance when we do not find new particle at the TeV scale, we can test how consistent the Standard Model is at large energy. The key tool for such an analysis is the renormalization group equation for the Higgs self coupling in the presence of a top Yukawa,

$$\frac{d\lambda}{d \log Q^2} = \frac{1}{16\pi^2} \left( 12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 \right). \quad (6)$$

Using  $m_H = \sqrt{2}\lambda v$  and  $m_t = y_t v / \sqrt{2}$  it links the Higgs and top masses to the ultraviolet behavior of the Higgs potential. This evolution gives us the triviality bound as an upper limit on the Higgs mass and the stability bound as a lower limit on the Higgs mass [39–41]. The latter is very close to the measured Higgs mass around 126 GeV.

In the absence of any new physics the top and Higgs masses at the weak scale depend on the boundary condition for  $\lambda$  at the Planck scale. In Fig. 4 we show different scenarios [40].

If we combine the running of  $\lambda$  with the running of  $y_t$  we find another structure: a finite infrared fixed point for the ratio  $\lambda/y_t^2$  [41]. It can be combined to a set of infrared and ultraviolet fixed points to predict a Higgs mass in the observed range. Of course, such fixed point arguments might be the worst outcome when we want to understand the fundamental nature of the Higgs sector: they tell us that renormalization group running to high scales makes sense and that something there sets all parameters. However, the infrared fixed point perfectly hides these structures from our measurements.

In any case, these renormalization group arguments give a clear hint that a precision analysis of the Higgs sector does not only include a precise measurement of Higgs parameters, but that we need to include the top mass. In other words, an ILC Higgs factory should have sufficiently large energy to probe top pairs or even  $t\bar{t}H$  production.

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