\(Q_T\) and \(\phi^*\) observables in Drell-Yan processes

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Abstract. We describe results on the phenomenology of electro-weak bosons produced in hadron-hadron collisions. In particular, we discuss the \(\phi^*\) variable, which enables us to probe the low-transverse-momentum region of lepton pairs produced via the Drell-Yan mechanism with unprecedented experimental accuracy. We describe state-of-the-art theoretical calculations for \(\phi^*\) and their comparison to Tevatron and LHC data.

1 Introduction

The transverse momentum spectrum of vector bosons produced via the Drell-Yan mechanism has been extensively studied by the Tevatron experiments [1–3] and, more recently, by the LHC collaborations as well [4, 5]. State-of-the-art theoretical predictions have been compared to data in order to test the ability of the theory to describe them. The interest in transverse momentum spectra, and related distributions, is twofold. On the one hand, one can use precise calculations to extract electro-weak fundamental parameters such as, for instance, the mass of the W boson. On the other hand, one can see these studies as an interesting testing ground for theoretical predictions that can be then applied to Higgs physics.

In the framework of collinear factorisation, the vector boson acquires a non-zero transverse momentum by recoiling against QCD radiation off the initial states. The transverse momentum distribution is therefore sensitive to multi-gluon emission, while having a particularly simple final state, and hence provides a powerful tool to test our understanding of QCD dynamics at low scales. The observable we are considering is a typical example of a multi-scale problem and the correct treatment of multi-gluon emissions goes beyond fixed-order perturbation theory. Let us introduce the invariant mass of the leptons \(M\), and the let us call \(Q_T\) the magnitude of the vector boson transverse momentum. We can encounter three different regimes. When \(Q_T \sim M\) we expect fixed order perturbation theory to work well. In the region \(\Lambda_{\text{QCD}} \ll Q_T \ll M\), we can still rely on perturbation theory but large logarithms of the ratio \(Q_T/M\) may spoil the convergence of the perturbative expansion and must be resummed to all orders [6–17]. Finally, in the region \(Q_T \sim \Lambda_{\text{QCD}}\) we expect non-perturbative effects to play a significant role, signalling the necessity to go beyond standard collinear factorisation. Therefore, it is important to compute a solid perturbative prediction, so that we can compare it to precise data coming from the experiments and be able to assess the role of non-perturbative (NP) contributions.

2 The \(\phi^*\) variable and its resummation

Despite the theoretical and experimental effort, no clear conclusions about the ability of perturbative QCD to describe the low-\(Q_T\) data, and hence about the size of NP effects, have been drawn by the comparison of the theory to data from the Tevatron experiments. On the theoretical side, one finds different ways of estimating the uncertainty as well as different NP models, e.g. [16, 18, 19]. On the experimental side, one would like to reduce the uncertainty associated with the measurement. To this purpose new variables, labelled \(a_T\) and \(\phi^*\), were introduced in Refs [20, 21]; the \(\phi^*\) distribution was then measured by the DØ collaboration [22].

The variable \(a_T\) is component of \(Q_T\) orthogonal to the dilepton thrust axis [20]:

\[
\vec{a}_T = \frac{\vec{Q}_T \times (\vec{l}_1 - \vec{l}_2)}{|\vec{l}_1 - \vec{l}_2|},
\]

where all the above three-vectors are defined by \(\vec{p} = (p_T, 0)\) and \(\vec{l}_{1,2}\) are the leptons’ transverse momenta. One of the main experimental uncertainties comes from the resolution in the measurement of these momenta. We note that at low \(Q_T\) we have:

\[
a_T \approx \frac{2l_{1T}l_{2T}}{l_{1T} + l_{2T}} \sin \Delta \phi,
\]

where \(\Delta \phi\) is the dilepton azimuthal separation, which is close to \(\pi\) in the limit we are considering. Thus, at low-\(Q_T\), the uncertainty for \(a_T\) is reduced with respect to the one for \(l_{1T}\) (and hence \(Q_T\)) by the presence of the small factor \(\sin \Delta \phi\) [20]. Moreover, it was noticed that this uncertainty can be further reduced if we rescale \(a_T\) by the dilepton invariant mass \(M\), essentially because resolution effects partly cancel in the ratio [21]. Ideally, one would...
like to define a variable that depends only on very well-measured angles and, at the same time, maintains a close relation to $Q_T$. The observable $\phi^*$

$$\phi^* = \tan \left( \frac{\pi - \Delta \phi}{2} \right) \sin \theta^*$$  \hspace{1cm} (3)

satisfies these properties [21]. In Eq. (3), $\theta^*$ is the scattering angle with respect to the beam, in the boosted frame where the leptons are aligned. This angle can be related to the pseudo-rapidities of the leptons: $\sin \theta^* = 1/\cosh (\eta^{\ell 1} - \eta^{\ell 2})$, hence $\phi^*$ is fully determined by angular measurements. Moreover, it can be shown that in the low-$Q_T$ limit $\phi^*$ reduces precisely to $a_T/M$ [21].

In order to fully exploit the advantage of the variable $\phi^*$, we would like to derive a resummed expression for its distribution, so that we can compare it to the DØ data [22]. We have already mentioned that, in the relevant $Q_T \rightarrow 0$ limit, this variable reduces to $a_T/M$, i.e. one component of the transverse momentum vector. The resummation for $a_T$ was discussed in [23] and it can be related to the traditional $Q_T$ resummation. However, in the present case, we are only interested in one component of $Q_T^*$ rather than its magnitude. An explicit formula for the resummation of the $\phi^*$ distribution was obtained in [24]:

$$\frac{d\sigma}{d\phi^*} = \frac{1}{\pi} \int_0^{\infty} db \, M \cos (b M \phi^*) e^{-R(b,M) \Sigma(x_1, x_2, b, M)},$$  \hspace{1cm} (4)

where $\Sigma$ contains the convolution between the hard matrix elements squared and the parton distribution functions. We first note that the function $R$, which resums the large logarithms in $b$-space, is the same as the one obtained for $Q_T$ resummation [15] and it has been computed to next-to-leading logarithmic (NNLL) accuracy. Secondly, the presence of the cosine function has important phenomenological consequences. In fact, the $\phi^*$ distribution does not show the typical Sudakov behaviour as $Q_T$, but rather tends to a constant plateau as $\phi^* \rightarrow 0$. The expression above is matched to the next-to-leading order (NLO) calculation, to provide an accurate estimate of the distributions for a vast range of $\phi^*$.

3 Phenomenology with $\phi^*$ at the Tevatron and the LHC

The DØ collaboration compared their data to theoretical predictions obtained from the program Resos. Thanks to the smaller experimental uncertainty the collaboration was able to discriminate between two different NP models, showing in particular that the data at forward rapidities disfavoured small-$x$ broadening, which had not been previously possible due to errors on the $Q_T$ spectrum, even with Tevatron Run-II data [22]. An independent calculation for the $\phi^*$ distribution was performed in [25]. NNLL+NLO perturbative predictions were compared to DØ data with a good agreement in all rapidity regions, once the theoretical uncertainties were faithfully estimated. As an example, we report the comparison between data and theory in Fig. 1 in the case of electrons, for central (left) and forward (right) rapidities.

The prediction for the $\phi^*$ distribution in proton-proton collision at 7 TeV is shown in Fig. 2. This was first computed in [26], where measurements by the LHC collaborations were encouraged. The ATLAS collaboration has recently performed a measurement of the $\phi^*$ distribution [27]. The results were presented at this conference for the first time, together with a comparison to our calculation. The data are well described by the NNLL+NLO prediction, within the quoted theoretical uncertainty, with no need of NP effects. One of the striking features of the comparison is the substantial difference between the magnitude of the experimental error and the estimated theoretical uncertainty, obtained varying the perturbative scales of the calculation. Despite the QCD calculation being very precise (NNLL+NLO), the theoretical uncertainty is almost an order of magnitude bigger than the experimental error. Therefore, future work to push the accuracy of both resummation and fixed-order calculation one order higher is desirable.

A measurement of the $\phi^*$ distribution has been recently performed by the LHCb collaboration, in the case of electron final-states, while a CMS measurement is in progress. Moreover, the LHCb collaboration plans to measure the muon channel. The large rapidity coverage of the LHCb detector makes this measurement particularly interesting. Additional effects may arise, such as the role of small-$x$ effects, which are neglected by conventional $Q_T$ resummations.

Finally, resummed results for these variables have been implemented in a computer program which will soon be released for public use.

References

Figure 1. Comparison of the theoretical NNLL+NLO prediction [25] for the \( \phi^* \) distribution to the experimental data collected by the DØ collaboration [22], in different rapidity bins.

Figure 2. The theoretical NNLL+NLO prediction [26] for the \( \phi^* \) distribution at the LHC (\( \sqrt{s} = 7 \) TeV).