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An empirical model to describe rapidity density and transverse momentum distributions

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Abstract. The distribution of rapidity density and transverse momentum is formulated empirically and analytically. It describes the data quite well over the wide energy range of $\sqrt{s} = 22.4 - 7000$ GeV.

1 Introduction

We formulate the transverse momentum and rapidity density distributions of produced particles in multiple particle production (MPP) empirically and analytically. Compared with the current models which use a theoretical idea and the simulation technique, present model has following advantages.
(1) It describes all existing data concerned reasonably.
(2) Consequently it is the most suitable one to be used in analyzing high energy cosmic-ray events.
(3) We can see the least necessary ingredients to describe the MPP.

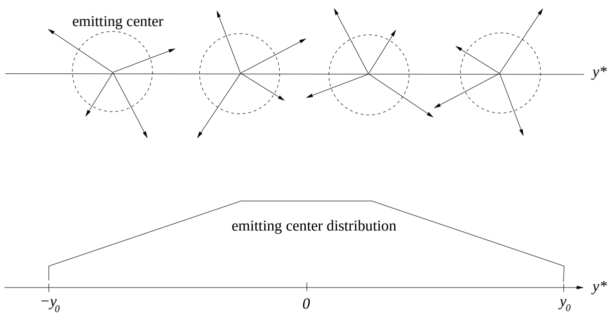

Figure 1. Distribution of the emitting centers on the rapidity axis in CMS. The parameter y_0 is defined as $y_0 = \ln(\sqrt{s}/M) - \ln a_2$ where M is nucleon mass and a_2 an adjustable parameter. (See 2.1.) The produced particles are emitted isotropically from several emitting centers. The energy distribution of produced particles is that of Tsallis type, eq.(2), on respective emitting centers.

2 Transverse momentum and rapidity density distributions

2.1 Assumptions

We enumerate the assumptions to formulate the rapidity density distribution, d^2N_{ch}/dy^*dp_T, of produced particles in this section, the details of which are found in Ref.[1].
(1) All produced particles are pions. (The assumption affects slightly the rapidity density distribution.)
(2) Produced particles are emitted isotropically from several centers.(Fig. 1) The emitting centers are distributed on the rapidity axis in the center of mass system (CMS) with the distribution $g(y')dy'$ of

$$g(y') = \begin{cases} c & (0 \le y' < by_0) \\ c - \dfrac{c(1-d)}{y_0(1-b)}(y' - by_0) & (by_0 \le y' < y_0) \\ 0 & (y' \ge y_0) \end{cases} \qquad (1)$$

where the values of the parameters $b = d = 0.25$ are determined by trial and error by fitting the (pseudo-)rapidity distributions to the data at various energies. The parameter $c = 2/y_0[2b + (1-b)(1-d)]$ is determined by the normalization of the distribution. The parameter y_0, defined as $y_0 = \ln(\sqrt{s}/M) - \ln a_2$ (M : nucleon mass), is determined through fitting the (pseudo-)rapidity density distributions to the data at various energies by adjusting the value of the parameter a_2.
(3) The energy distribution of produced particles $f(p)dp$ is that of Tsallis type on the respective emitting centers. That is,

$$f(p) = C\frac{p^2}{T_0^3}\left[1 + \frac{q-1}{T_0}p\right]^{-q/(q-1)} \qquad (q > 1) \qquad (2)$$

where the values of the parameters T_0 and q are determined so as to reproduce the transverse momentum (abbreviated as p_T hereafter) distributions at various energies.

Then the rapidity density distribution is

$$\frac{d^2N_{ch}}{dy^*dp_T} = a_1 y_0 \int_{-y_0}^{y_0} \frac{p_T E}{2p^2} f(p) g(y') dy' \qquad (3)$$

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where $E = \mu\cosh(y^* - y')$, $p = \sqrt{E^2 - m^2}$, $\mu = \sqrt{p_T^2 + m^2}$ (m : pion mass). (The quantity with an asterisk is the one in CMS.) The distribution can be converted to the pseudo-rapidity density distribution and x distribution ($x^* \equiv 2p^*_\parallel/\sqrt{s}$) easily.

2.2 p_T distributions

The p_T distribution at the zenith angle $\theta^* = 90°$, expressed in terms of the invariant cross section, is

$$E\frac{d^3\sigma}{d^3p}\bigg|_{\theta^*=90°} = \frac{\sigma_{inel}}{2\pi p_T}\left(\frac{d^2N_{ch}}{dy^*dp_T}\right)_{y^*=0}$$
$$= \frac{\sigma_{inel}}{4\pi}a_1 y_0 \int_{-y_0}^{y_0} \frac{\mu\cosh(y')}{p^2} f(p)g(y')dy' \qquad (4)$$

where $p = \sqrt{[\mu\cosh(y')]^2 - m^2}$. The distribution includes three parameters T_0, q and a_1 since the integration is almost independent of the parameter a_2 at high energies, *i.e.* $\ln(\sqrt{s}/M) \gg 3$. We calculate the p_T distributions for several pairs of values of the parameters (T_0, q) to look for the best fitting one to the data. The value of the parameter a_1 can be obtained through the empirical relation of the quantity ρ_0, the pseudo-rapidity density at $\eta^* = 0$,

$$\rho_0 \equiv \int_0^\infty \left(\frac{d^2N_{ch}}{d\eta^* dp_T}\right)_{\eta^*=0} dp_T$$
$$= 2.716 - 0.307\ln(s) + 0.0267\ln^2(s) \qquad (5)$$

obtained by CMS Collaboration[2], since it includes the parameters T_0, q and a_1. The distributions describe the p_T distribution well over the whole region of the data at any incident energy. (See Fig. 2.)

The values of the parameters T_0 and q in the energy distribution are obtained by fitting the p_T distributions to the data at the energies of $\sqrt{s}$ = 63, 200, 500, 900, 1800, 2360, 7000 GeV. The empirical energy dependences of them are

$$q = 0.0295\log_{10}\left(\frac{\sqrt{s}}{1.0\ \mathrm{GeV}}\right) + 1.020,$$
$$T_0 = \begin{cases} 0.122 & (\sqrt{s} \le \epsilon_0) \\ 0.0163\log_{10}\left(\frac{\sqrt{s}}{1.0\ \mathrm{GeV}}\right) & (\sqrt{s} > \epsilon_0) \end{cases} \qquad (6)$$

with $\epsilon_0 = 3.82 \times 10^2$ GeV.

2.3 Pseudo-rapidity density distributions

The pseudo-rapidity distribution

$$\frac{d^2N_{ch}}{d\eta^* dp_T} = \frac{p_T(e^{\eta^*} + e^{-\eta^*})}{\sqrt{p_T^2(e^{\eta^*} + e^{-\eta^*})^2 + 4m^2}} \frac{d^2N_{ch}}{dy^* dp_T} \qquad (7)$$

includes four parameters of T_0, q, a_1 and a_2. The former two have been determined already and the latter two are determined by fitting the distributions to the data. The distributions describe the data well over the whole region of the data at any incident energy. (See Fig. 3.)

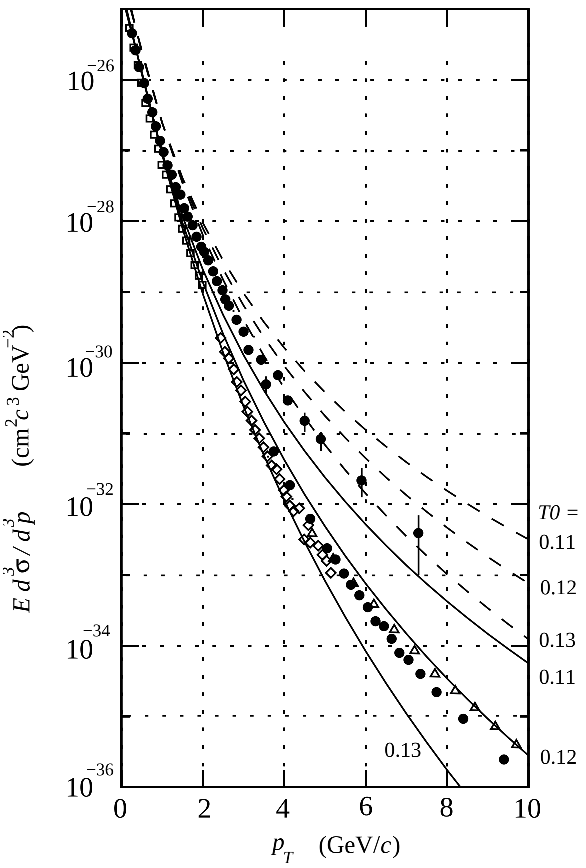

Figure 2. The p_T distributions at $\sqrt{s}$ = 63 (lower)[3] and 500 (upper) GeV[4]. Experimental data are for a half of the charged hadrons (or π^0's) and hence the curves are a half of eq.(4). The solid lines (at 63 GeV) and chain lines (at 500 GeV) are for the values of the parameters in the table below. The values of q are determined so as to reproduce the average p_T's at respective energies for the values of T_0.

T_0 (GeV/c)	q $\sqrt{s}$ = 63 GeV	$\sqrt{s}$ = 500 GeV
0.11	1.098	1.128
0.12	1.075	1.109
0.13	1.055	1.090

The best fitting curves are for the values (0.122, 1.071) and (0.125, 1.100) of the parameters (T_0, q) (T_0 in GeV/c) at $\sqrt{s}$ = 63 GeV and 500 GeV, respectively.

The values of the parameters a_1 and a_2 are obtained by fitting the pseudo-rapidity density distributions to the data at the energies of $\sqrt{s}$ = 22.4, 200, 540, 630, 900, 1800, 2360, 7000 GeV. The empirical energy dependences of them are

$$a_1 = 1.85\left(\frac{\sqrt{s}}{100\ \mathrm{GeV}}\right)^{0.225}, \quad a_2 = 0.940\left(\frac{\sqrt{s}}{10\ \mathrm{GeV}}\right)^{\alpha} \qquad (8)$$
$$(\alpha = 0.317 \text{ and } 0.178)$$

The two exponents of the parameter a_2 are for the maximum and minimum dependences since the values of a_2 are distributed widely due to the limited data in the forward region of the rapidity.

3 Discussions

Present model describes the rapidity density and p_T distributions quite well over the whole region of the data in the wide energy range of $\sqrt{s}$ = 22.4 – 7000 GeV. Consequently it reproduces the energy dependence of the average p_T, that of the charged multiplicity and that of the pseudo-rapidity density at $\eta^* = 0$. It describes the p_T averages at the rapidity $y^* = 5.01 - 6.59$ in the forward region, obtained by UA7 Collaboration[6] at $\sqrt{s}$ = 630 GeV, too.

The inelasticity, expected by the model, either increases or decreases with the energy from the assumed value of $K = 0.5$ at $\sqrt{s}$ = 10 GeV for the minimum and maximum dependences of the parameter a_2 (See 2.3.), *i.e.* $K = 0.95$ and 0.18 at $\sqrt{s} = 10^5$ GeV, respectively.

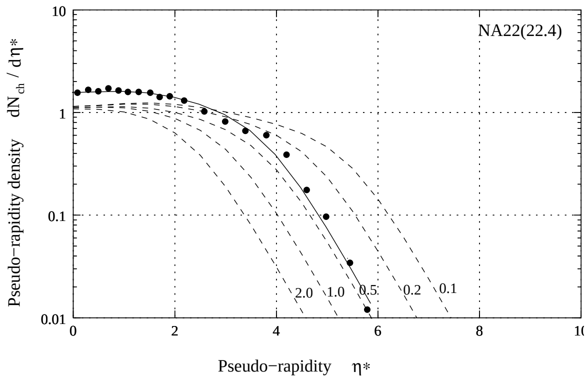

Figure 3. Pseudo-rapidity density distribution at $\sqrt{s}$ = 22.4 GeV. The chain lines are for the value of the parameters $a_1 = 1.0$ and $a_2 = 0.1, 0.2, 0.5, 1.0, 2.0$ (attached to the curves). The solid curve of $a_1 = 1.38$ and $a_2 = 0.5$ is the best fitting one to the data by EHS-NA22 Collaboration.[5]

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