

Constraints on direct acceleration of UHECRs in astrophysical sources

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Abstract. A toy model of the population of numerous non-identical extragalactic sources of ultra-high-energy cosmic rays is presented. In the model, the acceleration of cosmic-ray particles is direct (not diffusive) and takes place in magnetospheres of supermassive black holes in galactic nuclei, the key parameter of acceleration being the black-hole mass. We use astrophysical data to describe the population of these cosmic-ray accelerators and confront the model with cosmic-ray and gamma-ray data.

1. INTRODUCTION

There are numerous hints which might help to constrain possible models of presently unknown sources of ultra-high-energy (UHE) cosmic rays (CRs). The observation [1–3] of the spectrum steepening consistent with the Greisen–Zatsepin–Kuzmin [4, 5] (GZK) cutoff, together with the global isotropy of the arrival directions and the fact that the UHE particles are not expected to be confined by the Milky-Way magnetic field suggest that the bulk of cosmic rays at energies $\gtrsim 10^{19}$ eV come from outside of the Galaxy. On the other hand, the absence of clustering at $E \gtrsim 5 \times 10^{19}$ eV [6] translates into the number density of sources $n \gtrsim 10^{-4} \text{ Mpc}^{-3}$, which means that sources of these extreme particles should not be exceptional.

On the other hand, the geometrical (Hillas) criterion [7] together with estimates of radiative energy losses of particles being accelerated (e.g. [8, 9]) leave just a few candidate classes of sources capable of acceleration of particles to UHE energies [10]. The conventional diffusive (e.g., relativistic or non-relativistic shock) acceleration may work only in ultrarelativistic jets, hot spots and lobes of exceptional active galaxies (powerful radio galaxies and blazars) which are not that abundant in the nearby Universe. For very special field configurations when synchrotron losses are suppressed and the curvature radiation dominates, possible acceleration sites include also immediate neighbourhood of supermassive black holes (SMBHs) in the galactic nuclei. We will concentrate on the latter option here.

A natural guess is that, contrary to what is often assumed in simulations (see however Ref. [11]), numerous UHECR sources are not identical – there should be less and more powerful accelerators where the maximal energies, injection spectra and fluxes of accelerated particles are different. While, for numerous sources, the assumption of equal fluxes is well justifiable (in the sense that only the mean flux of a large sample of sources is important and this mean flux does not vary significantly from one region in the Universe to another) and the injection spectrum is often fixed by the acceleration model, the maximal energies are expected to vary significantly, having a serious impact on the observed spectrum [12].

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In this work, we discuss a toy model of numerous and different sources of UHECRs and, within certain assumptions, confront it with the experimental data.

2. A TOY MODEL OF PARTICLE ACCELERATION IN THE BLACK-HOLE MAGNETOSPHERE

A toy model of particle acceleration in the black-hole magnetosphere was proposed by Neronov et al. [13, 14], where the reader may find its detailed description. The key idea is that, for a stationary rotating black hole without electric charge embedded into the external magnetic field, homogeneous at the horizon distance scale, there exists a region near the rotation axis, where particles moving along magnetic lines are accelerated by the electric field. The energy gain is determined by the solution to the field equations but is often irrelevant since particles cannot achieve this maximal energy because of inevitable energy losses associated with the accelerated motion of the particle. The particle energy is determined by the balance between the energy losses and the energy gain per unit time. It was shown in Ref. [14] that protons can be accelerated to the energies of about 10^{20} eV only if the magnetic field is almost aligned with the rotation axis. In this case, only the curvature radiation is relevant for an accelerated particle, and not the synchrotron one. We should also note here that we do not consider energy losses related to interactions of accelerated particles in the source. The maximal energy of an accelerated particle is

$$\begin{aligned}\mathcal{E}_{\text{curv}} &= \left(\frac{3}{2}\right)^{1/4} \frac{A}{Z^{1/4}} \frac{m}{e^{1/4}} E^{1/4} R^{1/2}, \\ \mathcal{E}_{\text{curv}} &\approx 1.23 \times 10^{22} \text{ eV} \frac{A}{Z^{1/4}} \left(\frac{B_0}{1 \text{ G}}\right)^{1/4} \left(\frac{R}{1 \text{ kpc}}\right)^{1/2} \kappa^{1/4},\end{aligned}\quad (1)$$

where B_0 is the external magnetic field, $R \sim R_S/\chi$ is the curvature radius of magnetic-field lines, R_S is the Schwarzschild radius, Ze is the particle charge, Am is the particle mass (A is the atomic number and m is the nucleon mass) and κ is a coefficient between the electric field and the external magnetic field. We assume that all of the particles start with equal initial conditions and so all of them are accelerated to the same energy $\mathcal{E}_{\text{curv}}$.

The original model of Ref. [14] treated the magnetic field B_0 as a free parameter. However, the field is constrained and, in particular, cannot be too high (see Ref. [10] for a detailed discussion). The maximal value of the magnetic field is determined [15, 16] by the so-called Eddington limit, $B_{\text{Ed}} = 10^4 \left(\frac{M}{10^9 M_\odot}\right)^{-1/2} \text{ G}$. For finding the spectrum of cosmic rays in the frameworks of this model, we need to know the actual maximal particle energy as a function of black-hole mass M , rather than its upper limit. In a general case, we can parametrize the external magnetic field as $B_0 = k B_{\text{Ed}} \left(\frac{M}{10^9 M_\odot}\right)^\alpha$, where α and k are some parameters.

Several realistic models predict this kind of dependence, e.g. the Shakura–Syunyaev model [17, 18] ($k \approx 0.31$, $\alpha = 0$) or the model of Ref. [19] ($k \approx 0.0093$, $\alpha \approx -0.31$). We have

$$\mathcal{E}_{\text{curv}} \approx 2.9 \times 10^{20} \text{ eV} \frac{A}{Z^{1/4}} \left(\frac{M}{10^9 M_\odot}\right)^{\frac{3}{8} + \frac{\alpha}{4}} \left(\frac{\chi}{1^\circ}\right)^{-1/2} (k\kappa)^{1/4}.\quad (2)$$

During acceleration, the particle emits curvature photons. In what follows, we will need to obtain an upper bound on this emission. The peak energy of the photons is determined by the particle energy $\mathcal{E}_{\text{curv}}$,

$$\mathcal{E}_\gamma = \frac{3}{2} \frac{\mathcal{E}_{\text{curv}}^3}{m^3 R} \sim 50 \text{ TeV} \frac{A^3}{Z^{3/4}} \left(\frac{M}{10^9 M_\odot}\right)^{\frac{1}{8} + \frac{3\alpha}{4}} \left(\frac{\chi}{1^\circ}\right)^{-1/2} (k\kappa)^{3/4}.$$

The ratio of luminosities in photons, L_γ , and in cosmic rays, L_{CR} , may be estimated by comparing the total available potential difference in the acceleration region along the rotation axis to its fraction, spent on the particle acceleration:

$$\eta = \frac{L_\gamma}{L_{\text{CR}}} = \frac{\mathcal{E}_{\text{max}}}{\mathcal{E}_{\text{curv}}} = 3.12 \left(\frac{M}{10^9 M_\odot} \right)^{\frac{1}{8} + \frac{3z}{4}} \left(\frac{\chi}{1^\circ} \right)^{1/2} \frac{Z^{5/4}}{A} \zeta \kappa^{-1/4} k^{3/4}, \quad (3)$$

where $(\zeta \kappa^{-1/4}) \sim (0.1 - 2)$ for realistic black-hole parameters. In numerical calculations presented below, we use $\kappa^{1/4} = 0.7$ and $\zeta = 0.25$.

To summarize, the model we use assumes a monochromatic spectrum of accelerated particles with $\mathcal{E} = \mathcal{E}_{\text{curv}}$, Eq. (1), in each particular source. The value of $\mathcal{E}_{\text{curv}}$ depends, within the assumed magnetic-field model, on the SMBH mass M only (in what follows, we do not consider acceleration of other particles than protons). The overall flux from the source remains a free parameter.

3. POPULATION OF THE SOURCES AND THE OBSERVED SPECTRUM

The properties of a single source are determined, within the magnetic-field model we choose to study, by the SMBH mass (the dependence from the spin is weak). To reconstruct the UHECR spectrum one has to consider the population of SMBHs distributed in mass and luminosity. For simplicity we will assume that the *mean* SMBH luminosity in cosmic rays is related to its mass,

$$L_{\text{CR}} \propto M^\beta, \quad (4)$$

where β is an additional model parameter. Note that not every black hole can work as a source, because the source should possess some special properties (for example, small inclination angle, the vacuum gap larger than R_S , absence of numerous charged particles in the vicinity of the black hole which might imply a thin or even absent accretion disc). The fraction of sources where the mechanism works is also encoded in the mean luminosity, Eq. (4). The observed spectrum can be obtained by convolving the SMBH mass function with the (monochromatic) single-source spectrum and Eq. (4) and taking into account the propagation effects.

For our calculation, we use one of the most recent published redshift-dependent mass functions [20]. Of two functions presented there, we choose to use the one based on the stellar mass functions because it has smaller statistical uncertainties. The systematic uncertainties of the mass function may be judged from Ref. [21] and are well within the overall precision of our toy model.

Before reaching the Earth, the accelerated protons may interact with the cosmic microwave background resulting in the GZK suppression [4, 5] and in the so-called ‘‘dip’’ feature in the spectrum [22, 23]. To account for the propagation effects, we use the numerical code developed in Refs. [24]. The code also traces secondary particles produced in the interactions. It makes use of the kinetic-equation approach and calculates the propagation of nucleons, stable leptons and photons using the standard dominant processes.

Fig. 1 presents the predicted cosmic-ray fluxes in the best-fit model for the Auger spectrum [25] for different dependencies of the magnetic field B_0 on SMBH mass mentioned in previous section. Thered curves correspond to B_0 given by the Eddington limit, the green one describes the Shakura–Syunyaev model [17, 18] and the blue one corresponds to the model of Ref. [19]. The overall flux normalization is a free parameter. Besides we tried two values of angle $\chi = 1^\circ$ and 5° and varied the luminosity dependence (4) parameter β in the range $-1 < \beta < 2$. The best-fit parameter values are indicated on the plot. One can see that the first two models produce satisfactory spectral fits above 10 EeV.

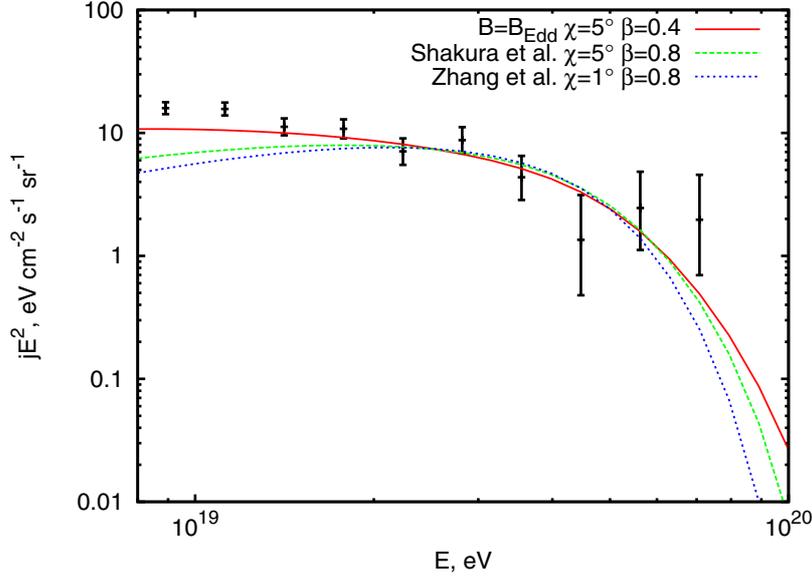


Figure 1. The cosmic-ray flux predicted by the model for three different assumptions about the SMBH magnetic field, see labels on the plot and explanations in the text, versus the Auger experimental data [25].

4. CONSTRAINTS

In this section, we discuss additional consistency checks of the model. They include estimates of the accompanying gamma radiation which should not be in conflict with the measured diffuse gamma-ray background, estimates of the concentration of sources and of the luminosity of a single source. We will see that the model passes these tests. For order-of-magnitude estimates in this section, we assume $B_0 \sim B_{\text{Ed}}$.

Concentration of the sources. Let us check that the local concentration of sources of cosmic rays with energies $\mathcal{E} \gtrsim 6 \times 10^{19}$ eV is not in conflict with the lower limit [6] based on the statistics of clustering. To this end, we integrate the SMBH mass function over the range of masses corresponding to these energies. The dependence of the particle energy from the black-hole mass is given by Eq. (2). Black-hole masses corresponding to particle energies $\mathcal{E} \gtrsim 6 \times 10^{19}$ eV are $M \gtrsim 10^7 M_\odot$. Integrating the mass function in this range of masses, we obtain

$$n = \int_{M_{\min}}^{M_{\max}} \frac{dn}{d \log M} d \log M.$$

The integral is saturated at its lower limit that is M_{\max} can be taken arbitrary high to obtain the estimate $n \sim 10^{-3} \frac{1}{\text{Mpc}^3}$. As we see, the total concentration of sources is larger than the clustering lower bound of $10^{-4} \frac{1}{\text{Mpc}^3}$. As we have discussed above, only a fraction of the calculated concentration n corresponds to the true concentration of the sources. Our estimate tells us that this fraction should be not less than a few per cent which is reasonable.

Luminosity of a single source. A simple estimate of the luminosity of a single source may be obtained as follows. Take the observed flux of cosmic rays with energies $\mathcal{E} \gtrsim 6 \times 10^{19}$ eV from the recent data [2, 3], $F \sim 2.5 \times 10^5 \frac{\text{eV}}{\text{m}^2 \cdot \text{s}}$. This flux is produced by the sources situated in the GZK sphere, where the GZK horizon radius is $R_{\text{GZK}} \sim 130 \text{ Mpc}$ for energies $\mathcal{E} \gtrsim 6 \times 10^{19}$ eV, see e.g. [26]. We assume that all these sources have approximately equal cosmic-ray luminosities $L_0 [\frac{\text{eV}}{\text{s}}]$, independent from the source black-hole mass. The flux from every single source, situated at the distance d from us,

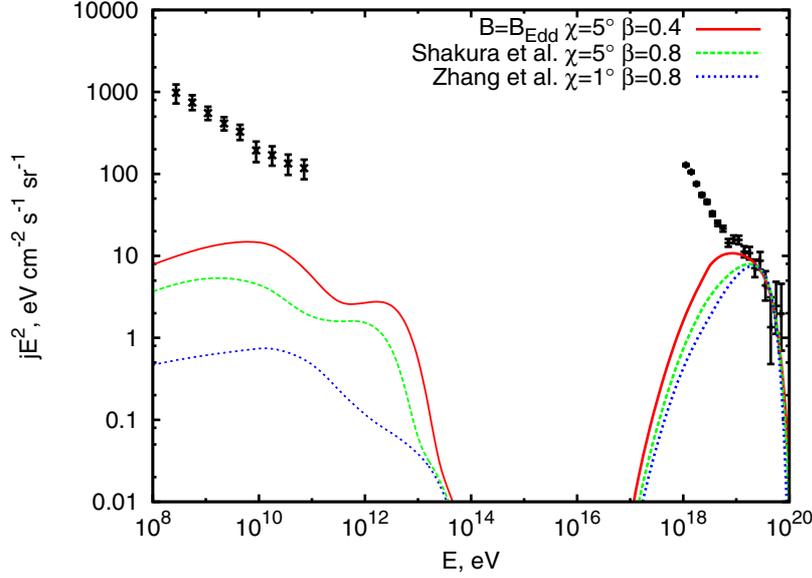


Figure 2. Gamma-ray fluxes predicted by the same models as shown in Fig. 1.

is $F_0 = L_0/(4\pi d^2)$. Thus for the total flux we have

$$F = \int_0^{R_{\text{GZK}}} F_0 n dV = L_0 \cdot n \cdot R_{\text{GZK}}.$$

Using $n \sim 10^{-3} \frac{1}{\text{Mpc}^3}$, we obtain $L_0 \sim 10^{39} \frac{\text{erg}}{\text{s}}$. The corresponding luminosity in photons is $L_\gamma = \eta \cdot L_0$, where η is given by Eq. (3). One has $L_\gamma \sim 10^{39} \left(\frac{M}{10^9 M_\odot}\right)^{1/8} \frac{\text{erg}}{\text{s}}$. This value is much smaller than the typical bolometric luminosity of an AGN, $L_{\text{AGN}} \sim (10^{41} - 10^{43}) \frac{\text{erg}}{\text{s}}$. Taking the concentration of the sources of order of the lower limit, $n \sim 10^{-4} \frac{1}{\text{Mpc}^3}$, does not result in a conflict as well.

Diffuse gamma-ray background. While individual cosmic-ray sources have quite low gamma-ray luminosities, one may wonder about the total emission of all sources in the Universe (beyond the GZK sphere). The emitted curvature photons have energies of order a few TeV and interact with the infrared background radiation to produce electromagnetic cascades in which the energy of the leading gamma rays downgrade to the GeV band. Electrons in the cascade are deflected by cosmic magnetic fields so distant sources contribute to the diffuse gamma-ray background. We have performed a numerical simulation of the secondary gamma-ray flux. Taking into account the injection spectrum of the curvature photons which is similar to the synchrotron one [27] and the relation (3) between the gamma-ray and cosmic-ray fluxes from a single source, we used the same code to describe the propagation of the accompanying gamma rays and to calculate the observed gamma-ray flux. The result is presented in Fig. 2.

One may see that the diffuse gamma-ray upper limit is satisfied.

5. CONCLUSIONS AND DISCUSSION

We have constructed and studied a toy model of UHECR acceleration in the vicinity of numerous and various supermassive black holes in centers of galaxies. The model assumes that:

- cosmic-ray particles are accelerated by the regular electric field within a few R_S from the SMBH [14]; the field configuration is given by the solutions of Refs. [28, 29] and is fully

determined by the SMBH mass M , its angular momentum a and the magnetic-field normalization B_0 ;

- all cosmic-ray particles accelerated near a given SMBH have similar initial conditions and therefore all are accelerated up to one and the same energy limited by the curvature-radiation losses; this maximal energy is calculated in the model and depends on M only (provided B_0 is a given, model-dependent, function of M ; dependence from a smooths this monochromatic spectrum insignificantly; interaction losses are not considered);
- the *mean* flux of a source (which accounts for the fraction of the sources where this mechanism does work) depends from M in a power-like manner; the normalization and the exponent are two free parameters of the model (the best fit to the cosmic-ray spectrum indicates that this dependence is weak);
- the concentration of sources is determined by the redshift-dependent SMBH mass function taken from astrophysical literature.

Within these assumptions and given the $B_0(M)$ relation is fixed, the model has two free parameters which we find by fitting the cosmic-ray spectrum at the Earth to the experimental data. With parameters fixed in this way, we subject the model to several further tests which it passes successfully: (1) the concentration of sources is large enough to satisfy the constraints from absence of clustering in UHECR arrival directions; (2) the luminosity of a particular source, determined by the flux normalization and concentration, is not too high; (3) secondary gamma rays from distant sources do not overshadow the measured GeV diffuse gamma-ray background.

We note that in the low-energy part of the spectrum, the contribution of the mechanism we discuss is insufficient to explain the observed spectrum due to the depletion of the SMBH mass function at low masses. It is tempting to speculate that this depletion is compensated by a huge contribution of the SMBH in our own Galaxy which, indeed, has the appropriate mass.

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