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On the occurrence of Closed Timelike Curves and the observer's point of view

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Abstract. The existence of Closed Timelike Curves (CTCs) in a generic spacetime is often associated with a non-physical choice of coordinates and can be cured by limiting the admissibility of such coordinates. Lichnerowicz conditions, for instance, represent a criterion for admissibility. The result, however, is a very restrictive limitation which may imply "removal" of important regions (with respect to the peculiarity of phenomena which may happen there) of the spacetime manifold. We consider here the point of view of a family of observers (Fundamental Slicing Observers, FSO) having their world lines orthogonal to the surfaces of constant coordinate time. We say that the time coordinate has not a global character if the associated FSO change their causality condition in the domain of validity of the coordinates themselves. Furthermore, in those regions where FSO have no more timelike world lines, CTCs are present and one may think of special devices or investigation tools apt to operationally detect them. We will discuss in detail theoretical approaches involving (scalar) waves or photons.

1 Introduction

Consider a generic spacetime with metric g in a coordinate system $x^\alpha = (t, x^a)$.[1] Let the spacetime admit a non-null foliation, i.e. an integrable distribution of hypersurfaces which can be either timelike or spacelike, but not lightlike (this particular case can be discussed separately and is not relevant for the analysis presented below) and assume that such a foliation is parametrized by the coordinate time, i.e. the "slices" are the t =const. hypersurfaces. There exists then a congruence of curves which is orthogonal to the foliation itself and is geometrically characterized by the vanishing of the vorticity associated with the unit normal vector field.

More precisely, in the spacetime regions where the t =const. foliation is spacelike, the congruence of orthogonal curves is timelike and defines a family of test observers, which we term Fundamental

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[1] Notation and conventions adopted here are those of [1]. Metric signature is $-+++$, greek indices run from 0 to 3 whereas latin ones from 1 to 3.

Slicing Observers (FSO), as stated above. Differently, in those regions where the foliation is timelike, the congruence of orthogonal curves is spacelike and cannot be associated anymore with test observers. Limiting situations such that the foliation is *always* or *never* spacelike also exist (even for the same spacetime with the metric expressed in different coordinate systems).

However, as a typical feature, in a given coordinate domain, the t =const. foliation can change its causality property, (equivalently, the FSO with 4-velocity field orthogonal to the foliation, i.e., aligned with dt can have a horizon) in a "critical region." Here, any closed curve contained on a t =const. slice is a Closed Timelike Curve (CTC).

The existence of CTCs can be then equally studied from different points of view:

1. In terms of coordinates, in the sense that a non-physical choice of coordinates may allow for the existence of critical regions (including CTCs), whereas better choices may exclude their formation;

2. In terms of FSO, forming an irrotational congruence of timelike world lines but having a horizon.

The first approach seems to be much more studied and understood than the second one. The main result here is due to Lichnerowicz [2] and may be summarized by conditions on the choice of admissible coordinates as follows: *A coordinate system in order to be admissible should imply that the metric tensor $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$ are negative definite matrices (if the metric signature is the one we are using here, $-+++$), i.e., all their principal minors are negative*. Clearly, this condition on the rank 1 minor $g^{tt} < 0$ of the inverse metric implies that $dt \cdot dt < 0$, i.e., the normal to the foliation is *always* timelike.

The important point here is that Lichnerowicz conditions exclude CTCs, whereas the standard coordinate conditions do not in general exclude them. Much has been written about CTCs and time travel in general relativity—see, for example, [3–6], the recent review [7] and the references quoted therein. An interesting physical problem arises when an observer making measurements aims at locating CTCs in the spacetime region accessible to him. Which "scouting device" should be used? How can one face with the problem of the existence of CTCs from an operational point of view?

As usual, we may learn much about this topic from black hole or black hole-related stationary axisymmetric spacetimes.

2 Fundamental slicing observers

In a stationary axisymmetric spacetime (also orthogonally transitive and reflection-symmetric, with coordinate adapted to the temporal and rotational Killing vectors), with metric [8]

$$\begin{aligned} ds^2 &= g_{\alpha\beta}dx^\alpha dx^\beta \\ &= -N^2dt^2 + g_{ab}(dx^a + N^a dt)(dx^b + N^b dt) \\ &= g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{\phi\phi}d\phi^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 \,, \end{aligned} \tag{1}$$

where N is the lapse factor and N^a the shift vector defined by

$$N = \frac{1}{\sqrt{-g^{tt}}}\,, \qquad N_a = g_{ta}\,, \quad N^a = g^{ab}N_b\,, \tag{2}$$

and the metric components only depend on (r, θ). The shift vector is directed along ∂_ϕ, i.e., $N^a = N^\phi \delta^a_\phi$, and the FSO are also ZAMOs (Zero Angular Momentum Observers), with four-velocity given by

$$u^\flat_{(\mathrm{ZAMO})} = -N dt\,, \qquad u_{(\mathrm{ZAMO})} = \frac{1}{N}(\partial_t - N^\phi \partial_\phi)\,, \tag{3}$$

with the symbol $\flat$ denoting the fully covariant form of a tensor.

The ZAMOs have an "operational characterization" in terms of the Sagnac effect (see, e.g., [9] and references therein) which relies on the periodicity assumption for the coordinate ϕ when it is spacelike to make closed loops: they are the unique circularly orbiting observer family for which Sagnac effect vanishes. Thus the circular CTC regions of these spacetimes have the property that they *exclude* the ZAMOs observers who see these effect as trivial. Actually, in a region where the ϕ coordinate is no longer spacelike, the interpretation of angular momentum breaks down, as does the entire setting for considering the Sagnac effect; so it is not surprising that the ZAMOs must be excluded. Let us recall below the details of the Sagnac effect.

2.1 Sagnac effect

The Sagnac effect refers to the asymmetry in the arrival times of a pair of oppositely rotating null circular orbits, beginning at the same time, as seen by a given circularly rotating observer in a given stationary spacetime.

Let (ζ_-, ζ_+) be the ordered pair of oppositely-signed coordinate angular velocities of such a pair of null circular orbits (with $K_\pm$ denoting the corresponding null four-momentum), and let ζ be the angular velocity of a rotating observer with four-velocity U (therefore distinct from this pair). Then

$$K_\pm = \Gamma_\pm(\partial_t + \zeta_\pm \partial_\phi)\,, \qquad U = \Gamma(\partial_t + \zeta\partial_\phi)\,, \tag{4}$$

where $\Gamma_\pm$ are arbitrary normalization factors but Γ is chosen so that $U \cdot U = -1$. Note that the lightlike condition for $K_\pm$ implies

$$g_{tt} + 2\zeta_\pm g_{t\phi} + \zeta_\pm^2 g_{\phi\phi} = 0\,, \tag{5}$$

so that

$$\zeta_+ + \zeta_- = -2\frac{g_{t\phi}}{g_{\phi\phi}} = -2N^\phi = 2\zeta_{(\mathrm{ZAMO})}\,, \qquad \zeta_+\zeta_- = \frac{g_{tt}}{g_{\phi\phi}}\,, \tag{6}$$

since N^ϕ is the only nonvanishing component of the shift vector.

One easily finds that the difference of the coordinate arrival times after one complete revolution with respect to the observer U are

$$\begin{aligned} \Delta t &= t_+ - t_- = 2\pi\,[1/(\zeta_+ - \zeta) - 1/(\zeta - \zeta_-)] \\ &= -4\pi[\zeta - \zeta_{(\mathrm{ZAMO})}]/[(\zeta - \zeta_-)(\zeta - \zeta_+)]\,, \end{aligned} \tag{7}$$

where we have identified with

$$\zeta_{(\mathrm{ZAMO})} = (\zeta_- + \zeta_+)/2 \tag{8}$$

the ZAMO angular velocity, by using Eq. (6). This shows the physical meaning of ZAMOs and Exercise 33.3 of [1] illustrates how one can in principle locate them by posing a rigid circulating mirror around a black hole.

The Sagnac time difference, as usually referred to in the literature, corresponds to the difference in the arrival times as seen by the static observer, which has zero angular velocity $\zeta = 0$, namely

$$\Delta t_{(\mathrm{sagnac})} = \Delta t|_{\zeta=0} = 4\pi({\zeta_-}^{-1} + {\zeta_+}^{-1})/2\,. \tag{9}$$

As a consequence,

- if a given observer measures the difference in the arrival times of two oppositely circulating photons and moves himself circularly in order to make such a difference as vanishing (in a stationary axisymmetric spacetime), then the observer realizes the condition of having zero angular momentum, i.e., of being a ZAMO or a FSO.

- if the observer is unable to find zero delay in the arrival times of the two photons, even approaching the speed of light in his own motion, then ZAMOs do not exist and one is *inside* a critical region where CTCs are expected to exist.

To see an explicit example, let us consider Gödel metric [10–13] in cylindrical coordinates $x^\alpha = (t, r, \phi, z)$

$$ds^2 = -\left(dt - \frac{2}{\Omega}\sinh^2\beta d\phi\right)^2 + dr^2 + \frac{2}{\Omega^2}\sinh^2\beta(1-\sinh^2\beta)d\phi^2 + dz^2\ , \tag{10}$$

where $\beta = \sqrt{2}\Omega r/2$ and $\Omega > 0$ is the rotational parameter of the fluid source. The hypersurface where $g_{\phi\phi}$ vanishes is the cylinder surrounding the z-axis of radius

$$r_0 = \frac{\sqrt{2}}{\Omega}\ln(1+\sqrt{2}) = \frac{\sqrt{2}}{\Omega}\text{arcsinh}\,1, \tag{11}$$

and CTCs exist outside this radius where the ZAMOs are no longer defined (while the static observers with four-velocity ∂_t exist everywhere).

Reaching operationally (or effectively) the ZAMO condition for a given observer (who should also be aware of this) is, in principle, possible.

3 Causality and Wave Propagation

Consider the propagation of a (test) radiation field in a coordinate patch containing CTCs. These might leave an imprint on waves propagating in such a spacetime and a trace could then be observationally detectable. More specifically, let us consider the case of a massless field propagating in a region which contains a closed null geodesic. In the WKB approximation, we can imagine a wave packet moving essentially along the closed null geodesic.

Some unusual features for wave phenomena in gravitational fields that do not have a global time coordinate should be somehow expected. In fact, what happens is that a wave entering a region in which the time coordinate has changed its causality condition (i.e., dt from timelike becomes spacelike) would appear to get immediately reflected (see Ref. [14] for a detailed explanation).

An interesting case is again that of the Gödel universe, which is free of horizons and singularities: waves propagating in the Gödel universe exhibit such an imprint [15–18]. Gödel's metric in the original quasi-Cartesian coordinates $\{t, x, y, z\}$ is

$$ds^2 = -dt^2 - 2\sqrt{2}U dt dy + dx^2 - U^2 dy^2 + dz^2\ , \qquad U = e^{\sqrt{2}\Omega x}\ , \tag{12}$$

where ∂_t, ∂_y and ∂_z are three of the five Killing vectors of this spacetime. The determinant of the metric turns out to be $g = -U^2$ and the inverse metric is

$$(\partial_s)^2 = (\partial_t)^2 - 2\sqrt{2}U^{-1}(\partial_t)(\partial_y) + (\partial_x)^2 + U^{-2}(\partial_y)^2 + (\partial_z)^2\ . \tag{13}$$

Observers at rest with respect to the coordinates, i.e., with four-velocity $u = \partial_t$ follow geodesics of metric (12).

Propagation of test electromagnetic radiation in the Gödel universe has been investigated by many authors (see, e.g., [15, 19, 20]) and the results can be summarized saying that *waves cannot propagate parallel to the direction of rotation of this universe* (actually, waves can propagate only in one direction along the y axis). This feature is connected with the violation of causality. To see the imprint of acausality, it is necessary to compare these results with those on wave propagation in causal Gödel-type rotating universes that possess cosmic time. Indeed, there exist Gödel-type solutions of Einstein's equations that contain both causal and acausal universe models (see [21–24] and references therein). The metric of such Gödel-type solutions is of the form

$$ds^2 = -dt^2 - 2\eta R(t)\mathcal{U}dtdy + R^2(t)[dx^2 - (\eta^2 - 1)\mathcal{U}^2 dy^2 + dz^2]\,, \tag{14}$$

where $R(t)$ is the scale factor,

$$\mathcal{U} = e^{\lambda x}\,, \qquad \eta = \frac{2\Omega}{\lambda}\,. \tag{15}$$

Here $\lambda > 0$ is a constant parameter and $\Omega > 0$ is a vorticity parameter. The Gödel solution is recovered for $R = 1$, $\lambda = \sqrt{2}\Omega$ and $\eta = \sqrt{2}$. In general, Eq. (14) represents a spacetime of Petrov type D with three Killing vector fields given by $\partial_x - \lambda y \partial_y$, ∂_y and ∂_z, so that it is a spatially homogeneous universe of type III in the Bianchi classification. Let $U = \partial_t$ be the four-velocity vector of the family of static observers in the Gödel-type universe, so that this congruence has expansion and rotation, but no shear. In particular, the *rotation tensor* $\omega(U)_{\alpha\beta}$ can be expressed in terms of the exterior derivative of U as

$$dU^\flat = a(U) \wedge U + 2\omega(U)^\flat\,, \tag{16}$$

where $a(U) = \nabla_U U$ is the acceleration of this congruence given by $a(U)^\flat = -\eta \dot{R}\mathcal{U}dy$, with $\dot{R} = dR/dt$. It follows that the only nonzero components of $\omega(U)_{\alpha\beta}$ are given by $\omega(U)_{xy} = -\omega(U)_{yx} = -\Omega R\mathcal{U}$, so that the vorticity invariant of the congruence U results in

$$\omega(U) = \Big(\frac{1}{2}\omega(U)_{\alpha\beta}\omega(U)^{\alpha\beta}\Big)^{1/2} = \frac{\Omega}{R(t)}\,. \tag{17}$$

Moreover, the expansion scalar $\Theta(U) = \nabla_\alpha U^\alpha$ is given by $3\dot{R}/R$. Note that in the special case of stationary Gödel-type universes with $R = 1$, the observers U follow geodesics world lines and their vorticity invariant reduces to Ω. Various sources have been considered for universes of Gödel type [21–24] and the the simplest choice corresponds to two-fluid cosmological models [25].

For metric (14), we find $g = -R^6\mathcal{U}^2$ and hence the inverse metric is given by

$$(\partial_s)^2 = (\eta^2 - 1)(\partial_t)^2 - 2\eta R^{-1}\mathcal{U}^{-1}(\partial_t)(\partial_y) + R^{-2}[(\partial_x)^2 + \mathcal{U}^{-2}(\partial_y)^2 + (\partial_z)^2]\,. \tag{18}$$

Note that $U = \partial_t$ is always timelike whereas the observers whose four-velocity is the unit vector orthogonal to the coordinate time t =const. hypersurfaces, i.e. the FSO with four-velocity $n = -1/\sqrt{1 - \eta^2}dt$ are timelike only for $\eta < 1$. Thus the solutions are *causal* for $\eta < 1$ and *acausal* for $\eta > 1$. (The special case $\eta = 1$ constitutes a limiting situation). A complete treatment of the propagation of electromagnetic waves in this Gödel-type universe can be found in Refs. [16, 17]. In the following section, we investigate the propagation of radiation parallel to the direction of rotation of a *stationary* Gödel-type universe with $R = 1$.

3.1 Waves in Stationary Gödel-Type Universes

Consider the propagation of scalar waves in metric (14) with $R = 1$, for simplicity. We seek a solution of the equation

$$\nabla_\alpha \partial^\alpha \Psi - \hat{m}_0^2 \Psi = 0 \tag{19}$$

in the form

$$\Psi = e^{i(-\omega t + k_2 y + k_3 z)}\psi(x)\,, \tag{20}$$

where $\hat{m}_0 = m_0/\hbar$, m_0 is the mass of the scalar particle and ω, k_2 and k_3 are constant propagation parameters. The function $\psi(x)$ thus satisfies the equation

$$\lambda^2\chi^2 \frac{d^2\psi}{d\chi^2} - [(\eta^2 - 1)\omega^2 + k_3^2 + \hat{m}_0^2]\psi = 0\,, \tag{21}$$

where we have introduced $\mathcal{U}^{-1} = \chi$ as a new variable and imposed the requirement that the wave travel in the z direction ($k_2 = 0$). For $x : -\infty \to \infty$, the solutions for ψ can be written as χ^σ with σ satisfying the following relation

$$\lambda^2\sigma(\sigma - 1) = (\eta^2 - 1)\omega^2 + k_3^2 + \hat{m}_0^2\,. \tag{22}$$

The only finite solution for ψ in the range $\chi : 0 \to \infty$ is a *constant*, provided

$$(\eta^2 - 1)\omega^2 + k_3^2 + \hat{m}_0^2 = 0\,. \tag{23}$$

Therefore, if $\eta < 1$, so that the stationary Gödel-type universe is *causal*, we have

$$\Psi = \psi_0 e^{i(-\omega t + k_3 z)}\,, \qquad \omega^2 = \frac{k_3^2 + \hat{m}_0^2}{1 - \eta^2}\,, \tag{24}$$

where ψ_0 is a constant; otherwise, for $\eta \geq 1$ wave propagation parallel to the rotation axis is impossible. That is, waves can only propagate parallel to the rotation axis of *causal* stationary Gödel-type universes.

What happens to a packet of scalar radiation produced in the neighborhood of a generic point $z = z_0$ on the rotation axis in the acausal ($\eta \geq 1$) Gödel-type universe? It follows from Eq. (23) that the radiation is then confined in space around z_0 (details in Ref. [14]).

4 Concluding remarks

A spacetime admitting Closed Timelike Curves (CTCs) necessarily admits a timelike foliation (at least in a bounded region), i.e., a timelike slicing. Associated with a foliation there is the family of observers (Fundamental Slicing Observers, FSO) whose world lines are orthogonal to the hypersurfaces of the foliation itself. However, when the foliation is (or becomes to be) timelike, these observers do not exist anymore. In a stationary axisymetric spacetime FSO are also ZAMOs, and ZAMOs have a physical characterization in terms of Sagnac effect: they are the only circularly rotating observer families who see zero Sagnac effect for a pair of oppositely rotating photons. If it not possible to find observers who see zero Sagnac effect, then timelike foliations of spacetime exist and also CTCs. From a completely different point of view, wave propagation is forbidden (or strongly limited) in a spacetime admitting CTCs. This property can also be used to locate CTCs: waves of any kind suffer for immediate damping as soon as they enter in a region containing CTCs.

Both these “scouting devices" may allow for the identification of CTCs, in principle at least. No doubt that they should receive special attention in future works.

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